Solution to Physics 108 Final - 2001

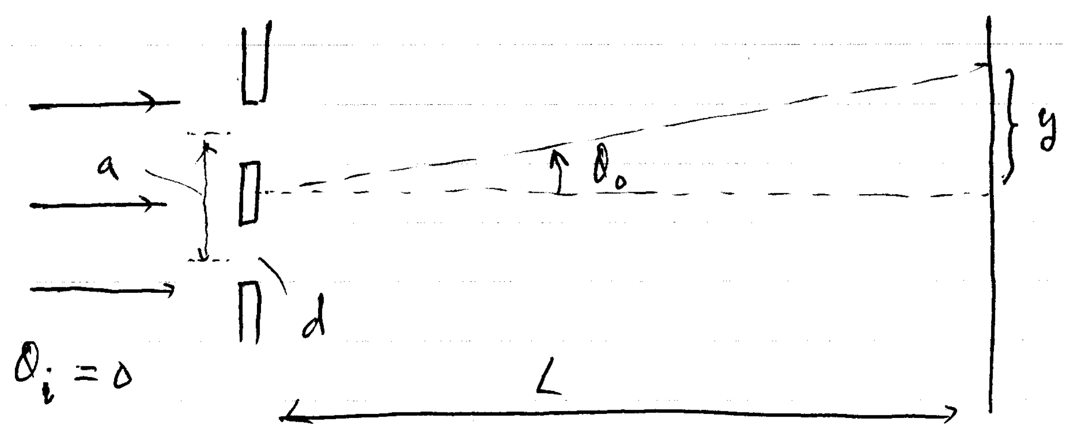
1.-(1)

$$I\left(\theta_0 = \frac{y}{L}\right) = I_{inc}\left(\frac{d^2}{\lambda L}\right) \frac{\sin^2\left(\frac{\pi d y}{\lambda L}\right)}{\left(\frac{\pi d y}{\lambda L}\right)^2} \cdot \frac{\sin^2\left(\frac{2\pi a y}{\lambda L}\right)}{\sin^2\left(\frac{\pi a y}{\lambda L}\right)}$$

1-(2) When $a = d$,

$$I\left(\theta_0 = \frac{y}{L}\right) = I_{inc}\left(\frac{d^2}{\lambda L}\right) \frac{\sin^2\left(\frac{2\pi d y}{\lambda L}\right)}{\left(\frac{\pi d y}{\lambda L}\right)^2}$$

$$= I_{inc} \cdot \frac{(2d)^2}{\lambda L} \cdot \frac{\sin^2\left(\frac{\pi 2 d y}{\lambda L}\right)}{\left(\frac{\pi 2 d y}{\lambda L}\right)^2}$$



2 - (1) From $n_{air} \sin\theta_b = n \sin\theta_g = n \cos\theta_b$

$\Rightarrow \tan\theta_b = n$, $\theta_b = \tan^{-1} n = 57.2°$

2 - (2) Since the two surfaces of the glass slide are parallel, the refraction angle θ_g as the beam enters the slide is the same as the incidence angle as the beam is incident on the second surface. As a result, from the Snell's law, the ~~refr~~ refraction angle as the beam exits the second surface is again θ_b.

The reflection coefficient for the p-polarized component off the second surface is

$$"r_{21}" = \frac{n \cos\theta_b - n_{air} \cos\theta_g}{n \cos\theta_b + n_{air} \cos\theta_g} = 0$$

as

$$n_{air} \cos\theta_g = n \cos\theta_b$$

or $\tan\theta_b = n$

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2-(3): At the Brewster angle θ_b, the p-polarized component goes through the slide without any reflection loss, while the s-pol. component experiences reflection loss twice. The reflectance off each of the two surfaces is

$$|r_s|^2 = \left(\frac{n_{air}\cos\theta_b - n\cos\theta_g}{n_{air}\cos\theta_b + n\cos\theta_g}\right)^2$$

$$= \left(\frac{\cos\theta_b - n\sin\theta_g}{\cos\theta_b + n\sin\theta_g}\right)^2$$

$$= \left(\frac{n^2-1}{n^2+1}\right)^2 = \left(\frac{1.4025}{3.4025}\right)^2$$

$$= 0.17$$

Total reflection loss from each slide is $2|r_s|^2$. After N such a slide, the intensity in s-polarized component is

$$I_s(N) = I_{s0}\left(1 - |r_s|^2\right)^{2N} \cong I_{s0}\, e^{-2N|r_s|^2}$$

while $I_p(N) = I_{p0} = I_{s0}$

Let

$$\frac{I_p(N)}{I_s(N)} \cong e^{2N|V_s|^2} = 10^5$$

then

$$N = \frac{\ln(10^5)}{2|V_s|^2} \cong 34.$$

3-(1): For $n_g = 1.4864$, from Snell's law,

$$n_g \sin\theta_i = n_{air} \sin\theta_t$$

$$\theta_t = \sin^{-1}(n_g \sin\theta_i) = \sin^{-1}(1.4864 * \sin 40^\circ)$$

$$= 72.8^\circ$$

$$R = |r_s|^2 = \left(\frac{n_g \cos 40^\circ - n_{air} \cos\theta_t}{n_g \cos 40^\circ + n_{air} \cos\theta_t} \right)^2$$

$$= \left(\frac{1.1386 - 0.295}{1.1386 + 0.295} \right)^2 = 0.35$$

For $n_g = 1.6584$, $n_g \sin\theta_i = 1.6584 * \sin 40^\circ > 1$ meaning that $\theta_i = 40^\circ$ exceeds the critical angle θ_c. The light is totally reflected:

$$R = |r_s|^2 = 1$$

3-(2) For the transmitted electric field,

$$\left|E_t^{(s)}(z)\right| = \left|E_t^{(s)}(0)\right| e^{-\frac{2\pi}{\lambda}\sqrt{n_g^2 \sin^2\theta_i - 1}\cdot z} \quad \begin{array}{c} 8.4\times10^{-11} \\ \shortparallel \end{array}$$

with $z = 10\lambda$, $\left|E_t^{(s)}(z)\right| / \left|E_t^{(s)}(0)\right| = e^{-20\pi\cdot\sqrt{n_g^2 \sin^2\theta_i - 1}}$

4. From $n_f = n_0 = n_{air} = 1$ and

$$t = \frac{2n_0}{M_{11}n_0 - M_{21} - M_{12}n_0 n_f + n_f M_{22}},$$

$$M = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} = \begin{pmatrix} \cos\phi & i\sin\phi/n_g \\ i n_g \sin\phi & \cos\phi \end{pmatrix}$$

$$\phi = \frac{2\pi n_g d}{\lambda},$$

we have

$$t = \frac{2}{\cos\phi - i n_g \sin\phi - i\sin\phi/n_g + \cos\phi}$$

$$T = |t|^2 = \frac{1}{\cos^2\phi + \sin^2\phi \cdot \frac{(n_g + 1/n_g)^2}{4}}$$

$$= \frac{1}{1 + \sin^2\phi \cdot \frac{(n_g^2 - 1)^2}{4n_g^2}} = \frac{1}{1 + \rho^2 \sin^2\phi}$$

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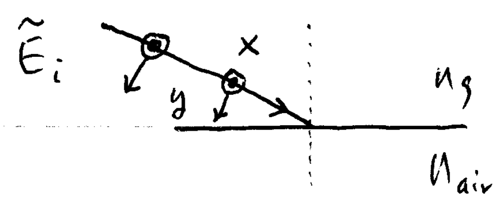


5-(1)

$$\tilde{E}_i = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \text{ in the first x-y frame}$$

5-(2) From $n_g \sin 54.6° = n_{air} \sin \tilde{\theta} = \sin \tilde{\theta}$,

$$\Rightarrow \cos \tilde{\theta} = \sqrt{1 - n_g^2 \sin^2 54.6°} = i \sqrt{n_g^2 \sin^2 54.6° - 1}$$

$$r_s = \frac{n_g \cos 54.6° - i\sqrt{n_g^2 \sin^2 54.6° - 1}}{n_g \cos 54.6° + i\sqrt{n_g^2 \sin^2 54.6° - 1}} = e^{i\phi_s}$$

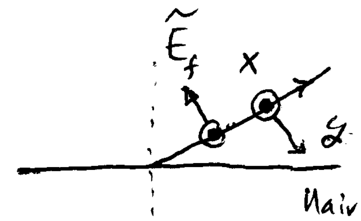
with $\phi_s = -78.7° = -1.374$ rad.

$$r_p = \frac{i n_g \sqrt{n_g^2 \sin^2 54.6° - 1} - \cos 54.6°}{i n_g \sqrt{n_g^2 \sin^2 54.6° - 1} + \cos 54.6°} = e^{i\phi_p}$$

with $\phi_p = 56.23° = 0.98$ rad

5-(3)

$$\tilde{E}_f = \frac{1}{\sqrt{2}} \begin{pmatrix} e^{i\phi_s} \\ -e^{i\phi_p} \end{pmatrix} \text{ in the new x-y frame}$$



5-(4)

$$M = \begin{pmatrix} 1 & 0 \\ 0 & e^{i(\phi_p - \phi_s) + i\pi} \end{pmatrix}$$

$$\cong \begin{pmatrix} 1 & 0 \\ 0 & e^{i\,1.75\pi} \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & e^{-i\pi/4} \end{pmatrix}^{*}$$

* With two such reflections,

$$MM = \begin{pmatrix} 1 & 0 \\ 0 & e^{-i\pi/2} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -i \end{pmatrix}$$

6-(1):

$$M = M_{f_4} M_{f_3} T_d M_{f_2} M_{f_1}$$

$$= \begin{pmatrix} 1 & 0 \\ -\frac{1}{f_4} & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -\frac{1}{f_3} & 1 \end{pmatrix} \begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -\frac{1}{f_2} & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -\frac{1}{f_1} & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ -\frac{f_3+f_4}{f_3 \cdot f_4} & 1 \end{pmatrix} \begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -\frac{f_1+f_2}{f_1 \cdot f_2} & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ -\frac{1}{5} & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \frac{1}{2} & 1 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{3}{2} & 1 \\ \frac{1}{5} & \frac{4}{5} \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

6-(2). From

$$\begin{pmatrix} l_f \\ \alpha_f \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} l_o \\ \alpha_o \end{pmatrix}; \qquad s_o = \frac{l_o}{\alpha_o} = 10$$

we have

$$l_f = A l_o + B \alpha_o \ ; \quad \alpha_f = C l_o + D \alpha_o.$$

From the fourth lens, the image position is given by

$$s_i = -\frac{l_f}{\alpha_f} = -\frac{A l_o + B\alpha_o}{C l_o + B \alpha_o}$$

$$= -\frac{A(l_o/\alpha_o) + B}{C(l_o/\alpha_o) + D}$$

$$= -\frac{(3/2)\,10 + 1}{(1/5)\,10 + 4/5}$$

$$= -\frac{16}{2\frac{4}{5}} = -5.7 \text{ cm}.$$

The image is a virtual one, located at 5.7 cm behind f_4 lens #