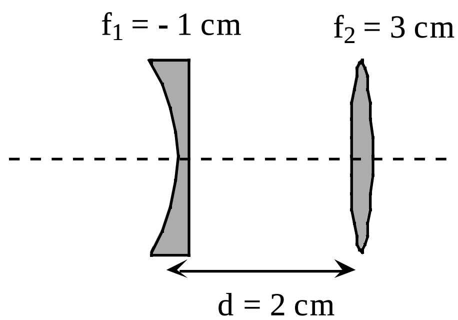
Physics 108 Final Exam (Sample#2)

1. **Geometric optics: (35 points)**

A beam exapnder is made of a diverging lens with $f_1 = -1\ \text{cm}$ and a converging lens with $f_2 = 3\ \text{cm}$, with the two lenses separated by $d = 2\ \text{cm}$.



(1) (15 points) Find the ABCD-matrix for such a lens system.
(2) (10 points) Show that the focal points for such a lens system are both at infinity.
(3) (10 points) A ray parallel to the optical axis,

$$\begin{pmatrix} \ell_0 \\ \alpha_0 \end{pmatrix} = \begin{pmatrix} \ell_0 \\ 0 \end{pmatrix}$$

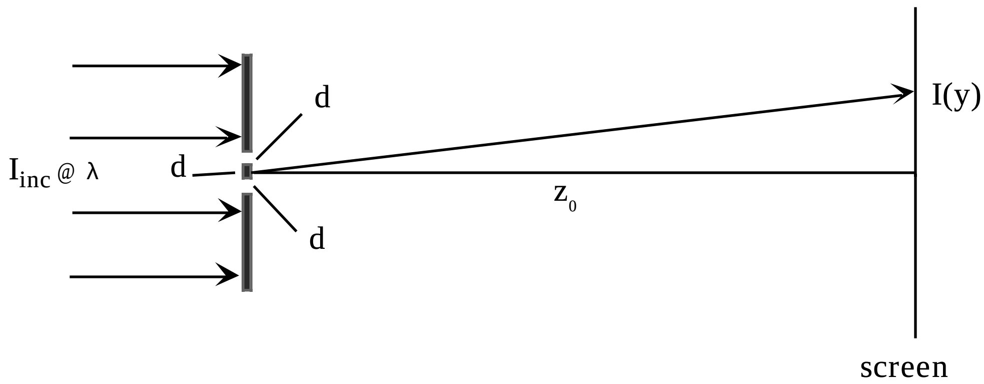
is incident from the left onto the expander. Show that the outgoing ray is also a parallel ray but with the height increased by a factor of 3, namely,

$$\begin{pmatrix} \ell_f \\ \alpha_f \end{pmatrix} = \begin{pmatrix} 3\ell_0 \\ 0 \end{pmatrix}$$

2. **Diffraction: (30 points)**

Two identical, long slits of width d are parallel to each other and along x-axis (into the paper). The separation between the lower edge of one slit and the upper edge of the other is also d. A collimated, monochromatic light of wavelength λ uniformly illuminates both slits at normal incidence as shown above. A screen is placed at a distance z_0 such that $d^2/\lambda z_0 << 1$. Using the coordinate system as shown in the figure,
(1) (15 points) find the intensity $I(y)$ on the screen if the upper silt is blocked;

(2) (15 points) find the intensity $I(y)$ on the screen if both slits are unblocked.

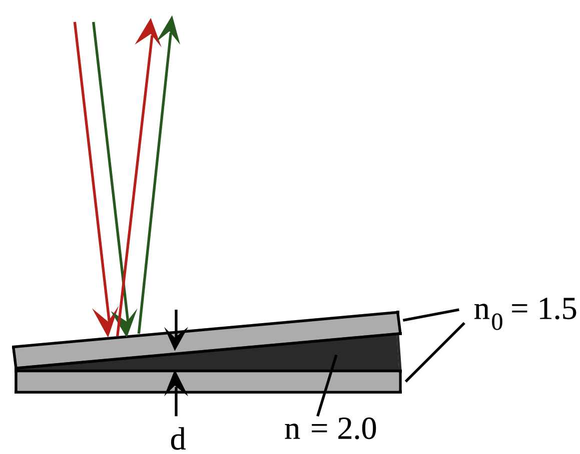


3. **Interference: (30 points)**

A wedge-shaped dielectric film with a refractive index of n = 2 is sandwiched between two identical rectangular glass slides with refractive indices of $n_0 = 1.5$. From the left to right, the thickness of the film increases linearly from zero. You use such an arrangement to observe the interference fringes. If the illuminating light source has two colors: one with a wavelength $\lambda_1 = 5000$ Å (green), and the other with a wavelength $\lambda_2 = 6000$ Å (red), when viewing the film near the normal incidence,

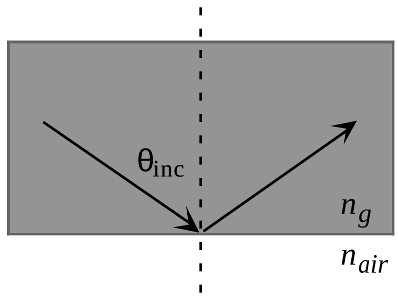
(1) (15 points) the fringes of which color have a smaller fringe-to-fringe spacing ?

(2) (15 points) at what thickness when a bright intensity fringe of one color overlaps with a dark intensity fringe of the other color for the first time?



4. **Fresnel equations: (45 points)**

A beam of light inside a glass with a refractive index $n_g = 1.4864$ is incident on a flat glass-air interface. The index of refraction of the air is taken to be $n_{air} = 1$.



(1) (10 points) Find the Brewster angle θ_B from the glass side.
(2) (10 points) Find the critical angle θ_C for the total internal reflection.
(3) (10 points) Explain why θ_C is always larger than θ_B.
(4) (15 points) Show that at θ_B, the electric field of p-polarization on the air side is larger than the incident electric field, i.e., $t_p(\theta_{inc} = \theta_B) > 1$.

5. **Polarization: (30 points)**

(1) (10 points) Find the Jones vector and indicate whether it is linearly polarized, or circularly polarized, or elliptically polarized for polarized light is given by

$$\begin{pmatrix} 1+i \\ 1-i \end{pmatrix}$$

(2) (10 points) The Jones matrix for a linear polarizer with its transmission axis (TA) making an angle θ with respect to x-axis is given by

$$M_\theta = \begin{pmatrix} \cos^2\theta & \sin\theta\cos\theta \\ \sin\theta\cos\theta & \sin^2\theta \end{pmatrix}$$

Write down the Jones matrix $M_{\theta+\pi/2}$ for a linear polarizer with its TA set at an angle of $\theta + \pi/2$ with respect to x-axis.

(3) (10 points) Show that

$$M_{\theta+\pi/2} M_\theta = M_\theta M_{\theta+\pi/2} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

6. **Thin film optics: (20 points)**

A dielectric film with a uniform thickness d = 100 Å and refractive index n = 2 is freely suspended in the air (n_{air} = 1). A collimated light beam with wavelength $\lambda = 5000\text{Å}$ is normally incident on the film.

(1) (10 points) Find the reflectance $R = |r|^2$ off the film.

(2) (10 points) Find the transmittance $T = |t|^2$ through the film.