Solution to Midterm of Phys. 108 (1998)

Physics 108 Midterm Examination (Spring, 1998)

1. **Air bubble in a glass plate:** (15 points)

   An air bubble is trapped inside a glass plate with an index of refraction of $n = 1.5$ and a thickness of $d = 1$ cm. The diameter of the bubble D is slightly smaller than 1 cm so that the thickness of the glass wall that separates the bubble and the air outside the glass plate can be neglected when compared to $d = 1$ cm. Such a glass plate can be considered as a pair of identical lenses. Assume that the index of refraction of the air both inside and outside of the glass plate is $n_0 = 1$. Find the sign and magnitude of the focal length of one of them.
   (Hint: you do not need to use matrices)

1. First lens:

$r_1 = \infty$, $r_2 = +0.5\,\text{cm}$

$n_1 = 1.0$

$n_2 = 1.5$

$$\frac{1}{f} = \frac{n_2 - n_1}{n_1}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) = \frac{1.5-1}{1}\left(0 - \frac{1}{0.5}\right) = -1\ (\text{cm}^{-1})$$

$\therefore f = -1\,\text{cm}$

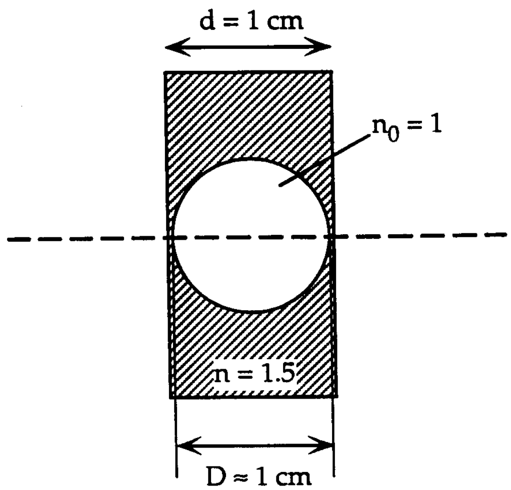
Second lens:

$r_1 = -0.5\,\text{cm}$, $r_2 = \infty$

$n_1 = 1.0$, $n_2 = 1.5$

$$\frac{1}{f} = \frac{n_2 - n_1}{n_1}\left(\frac{1}{-0.5} - 0\right) = -1\ (\text{cm}^{-1})$$

$\therefore f = -1\,\text{cm}$



2. **Combination of thin lenses:**

   As shown in the following figure, two positive thin lenses are separated by a distance of $d = 30$ cm. The first one has a focal length of $f_1 = +10$ cm. The second one has a focal length of $f_2 = +20$ cm. A small object with a height y_0 is placed on the left-side of the first lens at a distance of 20 cm.
   (1) (20 points) Use thin lens equation to determine the final image position of the object measured from the center of the second lens.
   (2) (10 points) Use ray diagrams to construct the image of the object after the second lens.
   (3) (10 points) Find the system ABCD matrix for this pair of lenses.
   (4) (10 points) Find the principal points F_1, F_2, H_1, and H_2 and mark in the figure.

2-(1). Using

$$\frac{1}{-x_0} + \frac{1}{x_i} = \frac{1}{f_1}$$

$x_0 = -20\,\text{cm}$, $f_1 = +10\,\text{cm}$

$\Rightarrow x_i = +20\,\text{cm}$ from the first lens (f_1) but $x_0' = -10\,\text{cm}$ from the second lens (f_2). From

$$-\frac{1}{x_0'} + \frac{1}{x_i'} = \frac{1}{f_2}$$

and $f_2 = +20\,\text{cm}$, we have

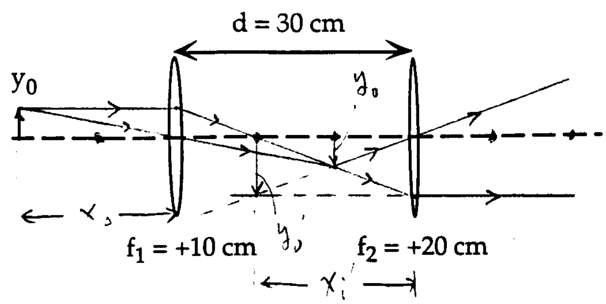
$x_i' = -20\,\text{cm}$ from the second lens, i.e., 20 cm on the left-side of the second lens.

2-(3) $M_{f_1} = \begin{pmatrix} 1 & 0 \\ -\frac{1}{10} & 1 \end{pmatrix}$

$T_d = \begin{pmatrix} 1 & 30 \\ 0 & 1 \end{pmatrix}$

$M_{f_2} = \begin{pmatrix} 1 & 0 \\ -\frac{1}{20} & 1 \end{pmatrix}$

2-(2)



$M = M_{f_2} T_d M_{f_1}$

$= \begin{pmatrix} 1 & 0 \\ -\frac{1}{20} & 1 \end{pmatrix}\begin{pmatrix} 1 & 30 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ -\frac{1}{10} & 1 \end{pmatrix}$

$= \begin{pmatrix} -2 & 30 \\ 0 & -\frac{1}{2} \end{pmatrix}$

$= \begin{pmatrix} A & B \\ C & D \end{pmatrix}$

$\therefore p = \infty$, $q = \infty$, $f_1 = \infty$, $f_2 = \infty$.

3. **Michelson interferometer**

A collimated beam is incident onto a Michelson interferometer as shown in the following figure. The beam has two frequency components: one is blue with a wavelength $\lambda_1 = 4500\text{Å}$ and the other one is green with a wavelength $\lambda_2 = 5000\text{Å}$. The interferometer is in the air so that the indices of refraction for both frequency components are equal to 1. Initially, the interferometer is adjusted such that the difference of the two optical paths is zero, $2(x_2 - x_1) = 0$. As the optical path difference $2\Delta x \equiv 2(x_2 - x_1)$ increases, the intensity at the detector corresponding to each one of the two frequency components varies periodically.

(1) (10 points) Write down the intensity at the detector for each of the two frequency components versus $\Delta x \equiv x_2 - x_1$.

(2) (10 points) Find the separation Δx between two successive minima in the intensity for both frequency components.

(3) (15 points) Since the separations between two successive minima for the two components are different, what is the smallest value of $\Delta x \equiv x_2 - x_1$ at which a minimum at $\lambda_2 = 5000\text{Å}$ meets with a minimum at $\lambda_1 = 4500\text{Å}$ again ?

3-(1):

$I(\lambda_1, \Delta x) = \frac{I(\lambda_1)}{2}\left(1 - \cos\frac{4\pi}{\lambda_1}\Delta x\right)$;

$I(\lambda_2, \Delta x) = \frac{I(\lambda_2)}{2}\left(1 - \cos\frac{4\pi}{\lambda_2}\Delta x\right)$ ※

3-(2):

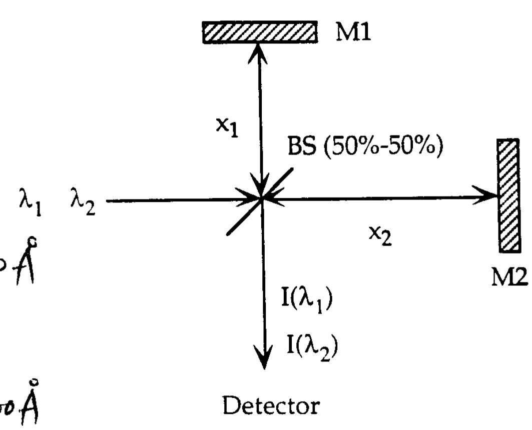
For λ_1, minima occur at

$\frac{4\pi}{\lambda_1}(x_2 - x_1) = 2\pi m$

$\Rightarrow \Delta x\big|_{\lambda_1} = \frac{\lambda_1}{2} = 2250\text{Å}$

By the same token,

$\Delta x\big|_{\lambda_2} = \frac{\lambda_2}{2} = 2500\text{Å}$



From

$l_f = A l_0 + B\alpha_0$

$= -2l_0 + 30\alpha_0$

$\alpha_f = C l_0 + D\alpha_0$

$= -\frac{1}{2}\alpha_0$

$\therefore x_i' = -\frac{l_f}{\alpha_f} = 2\left(30 + 2\left(-\frac{l_0}{\alpha_0}\right)\right)$

$= 2(30 - 40)$

$= -20$ cm ※

(3). $(m+1)\lambda_1 = m\lambda_2 \Rightarrow m = 9$, $\Delta x = \frac{m}{2}\lambda_2 = 22{,}500\text{Å}$ ※