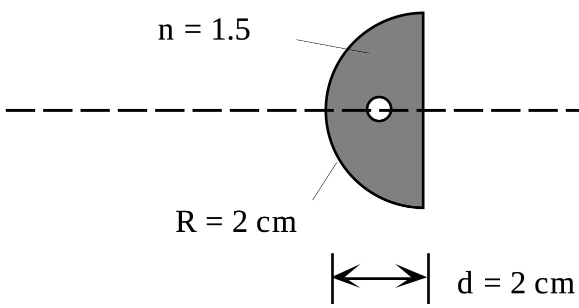
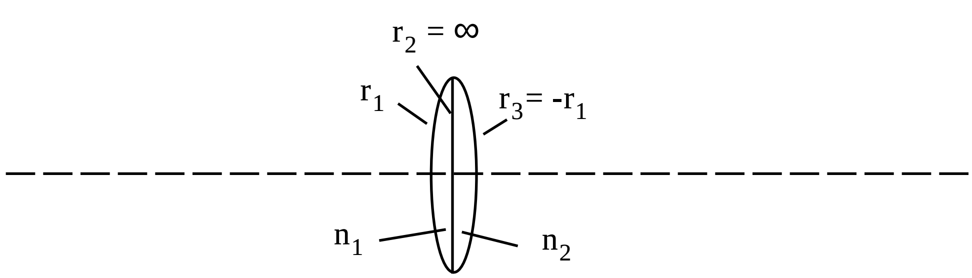
Physics 108 Midterm Examination (Spring, 1999)

1. **Image formation:**
   A semi-spherical plano-convex lens with an index of refraction $n = 1.5$ has the dimensions as shown below. The flat surface of the lens is coated so that it acts as a flat mirror. We can neglect the reflection at the curved surface of the lens. A small air bubble of radius $r = 0.1$ cm is trapped in the middle of the lens.



   (a) (10 points) How many air bubbles does one see from the left side?
   (b) (20 points) Find the image positions of the bubble viewed from the left side of the lens in the paraxial approximation.

2. **Combination of lenses:**
   Two plano-convex *thin* lenses with same physical dimensions but different indices of refraction are attached together as shown in the following figure. This combination of two lenses is placed in the air.



   (a) (15 points) Derive the focal length of this lens combination as a function of r_1, n_1, and n_2 using the matrix method and thin-lens approximation.
   (b) (10 points) Show that if $dn_1/d\lambda = -dn_2/d\lambda$ about a wavelength λ_0, the focal length of this lens combination is approximately independent of the wavelength around λ_0.
   (Hint: expand the indices of refraction about λ_0)

3. **Michelson interferometer:**

A Michelson interferometer is placed in the air with a constant density ρ_0. A gas cell of a length d = 1 cm is inserted in the second arm of the interferometer. The index of refraction of the gas in the cell varies with its density ρ as

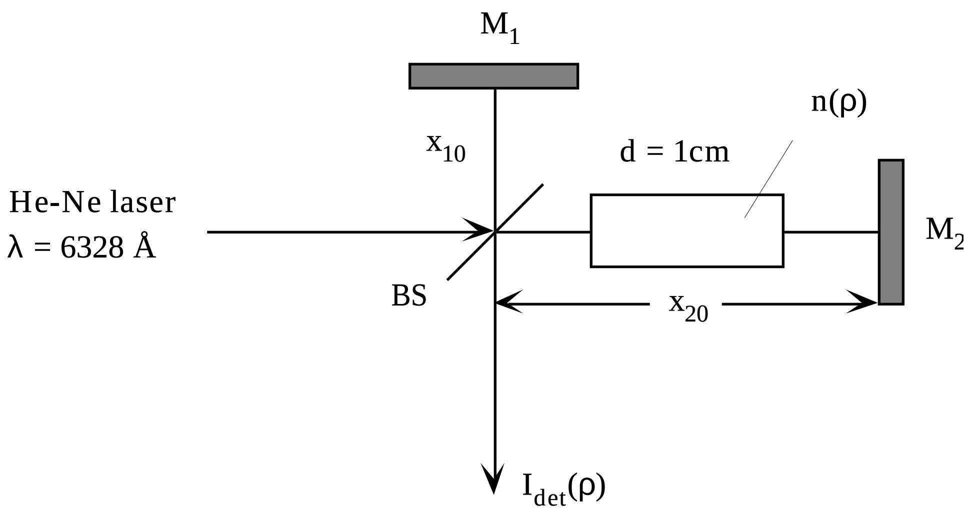
$$n(\rho) = 1 + 0.00029\left(\frac{\rho}{\rho_0}\right) = 1.00029 + 0.00029\left(\frac{\rho-\rho_0}{\rho_0}\right)$$

As ρ changes, the light intensity at the detector $I_{det}(\rho)$ changes.

(a) (10 points) With the gas cell removed, find the phase difference $\Phi_{20} - \Phi_{10}$ between the two arms of the interferometer in terms of x_{10}, x_{20}, and λ.

(b) (10 points) With the gas cell inserted, find the phase difference $\Phi_2 - \Phi_1$ between the two arms in terms of x_{10}, x_{20}, d, $(\rho-\rho_0)/\rho_0$ and λ (you may ignore the effect of the cell wall).

(c) (15 points) Treating $(\rho-\rho_0)/\rho_0$ as a small number, find the intensity variation $\Delta I_{det}(\rho) \equiv I_{det}(\rho) - I_{det}(\rho_0)$ at the detector in terms of $(\rho-\rho_0)/\rho_0$, x_{10}, x_{20}, d, λ, and the incident intensity I_{inc}.



4. **Fraunhofer diffraction and Young's interference fringes:**

A light beam at wavelength λ is normally incident onto two idential *thin* slits that are separated by a distance *a*. At a long distance *L* away from the slits a screen is used to observe Young's interference fringes.

(1) (5 points) What is the spacing between two neighboring fringes ?

(2) (5 points) What happens to the intensity of these fringes if the width of the slits *d* is taken into consideration ?