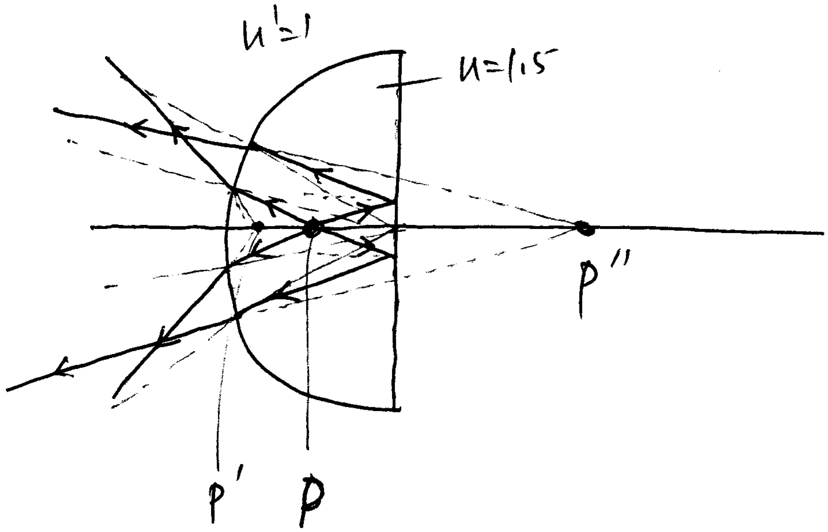
Solution to Physics 108 Midterm (1999)

1-(a) Two images of the air bubble (P' and P'')



1-(b). For the image P', we use

$$\frac{n}{s_o} + \frac{n'}{s_i} = \frac{n'-n}{r}$$

For the left-going ray as shown in the figure
$r = -2\text{ cm}$, $s_o = 1\text{ cm}$, $n' = 1$, $n = 1.5$,

$$\Rightarrow \quad s_i^{-1} = -\frac{n}{s_o} + \frac{n'-n}{r} = -1.5\text{ cm}^{-1} + \frac{-0.5}{-2}\text{ cm}^{-1}$$

$$= -1.25\text{ cm}^{-1}$$

$\Rightarrow \quad s_i = -0.8\text{ cm}$ (on the right side of the curved surface).

For image P'' after the first reflection, the virtual image is formed at

$$s_o = 3\,\text{cm}$$

on the right side of the curved surface. After refraction, the final image distance s_i is given by

$$s_i^{-1} = -\frac{n}{s_o} + \frac{n'-n}{r} = -\frac{1.5}{3}\,\text{cm}^{-1} + 0.25\,\text{cm}^{-1}$$

$$= -0.25\,\text{cm}^{-1}$$

$\Rightarrow$ $s_i = -4\,\text{cm}$ (again on the right side of the curved surface).

2-(a) the focal length of the first lens

$$\frac{1}{f_1} = \frac{n_1 - n_0}{n_0}\left(\frac{1}{r_1}\right) = (n_1 - 1)\frac{1}{r_1}$$

the focal length of the second lens

$$\frac{1}{f_2} = \frac{n_2 - n_0}{n_0}\left(-\frac{1}{r_3}\right) = (n_2 - 1)\frac{1}{r_3}$$

the total matrix for the lens combination is

$$M = M_{f_2}' \cdot M_{f_1} = \begin{pmatrix} 1 & 0 \\ -1/f_2 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ -\frac{1}{f_1} & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ -\left(\frac{1}{f_1} + \frac{1}{f_2}\right) & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{pmatrix}$$

$$\therefore \quad \frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} = (n_1 + n_2 - 2)\frac{1}{r_1}.$$

2-(a).

Let
$$u_1(\lambda_0+\delta\lambda) = u_1(\lambda_0) + \frac{du_1}{d\lambda_0}\delta\lambda$$
$$u_2(\lambda_0+\delta\lambda) = u_2(\lambda_0) + \frac{du_2}{d\lambda_0}\delta\lambda$$

then

$$\frac{1}{f(\lambda_0+\delta\lambda)} = \left(u_1(\lambda_0)+u_2(\lambda_0)-2\right)\frac{1}{r_1} + \left(\frac{du_1}{d\lambda_0}+\frac{du_2}{d\lambda_0}\right)\delta\lambda\cdot\frac{1}{r_1} + o(\delta\lambda^2)$$

If $\frac{du_1}{d\lambda_0} = -\frac{du_2}{d\lambda}$, then

$$\frac{1}{f(\lambda_0+\delta\lambda)} = \left(u_1(\lambda_0)+u_2(\lambda_0)-2\right)\frac{1}{r_1} + o(\delta\lambda^2)$$

almost independent of λ around λ_0.

3-(a). $$\phi_{20} - \phi_{10} = \frac{4\pi}{\lambda} n(\rho_0)(X_{20} - X_{10}) + \pi$$
$$= \frac{4\pi}{\lambda}(1.00029) \cdot (X_{20} - X_{10}) + \pi$$

3-(b) $$\phi_2 - \phi_1 = \frac{4\pi}{\lambda} n(\rho_0)(X_{20} - X_{10}) + \pi$$
$$+ \frac{4\pi}{\lambda} d\left(n(\rho) - n(\rho_0)\right)$$
$$= \phi_{20} - \phi_{10} + \frac{4\pi}{\lambda} d(0.00029) \frac{\rho - \rho_0}{\rho_0}$$

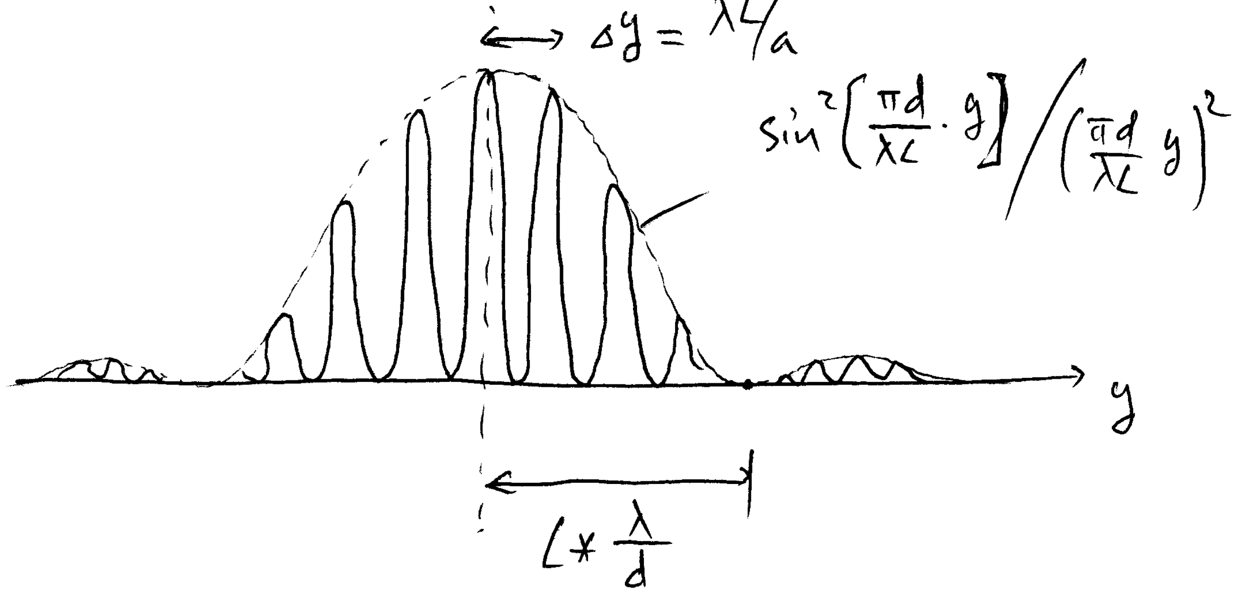
3-(c) Since
$$I_{det}(\rho) = \frac{I_{inc}}{2}\left(1 + \cos(\phi_2 - \phi_1)\right)$$
$$= \frac{I_{inc}}{2}\left[1 + \cos\left(\phi_{20} - \phi_{10} + \frac{4\pi}{\lambda} d(0.00029) \frac{\Delta\rho}{\rho_0}\right.\right.$$
$$= \frac{I_{inc}}{2}\left(1 + \cos(\phi_{20} - \phi_{10}) + \frac{4\pi}{\lambda} d(0.00029) \frac{\Delta\rho}{\rho_0} \cdot \left(-\sin(\phi_{20} - \phi_{10})\right)\right) + O\left(\frac{\Delta\rho}{\rho_0}\right)^2$$

We have

$$\boxed{\begin{aligned}\Delta I_{det}(\rho) &= I_{det}(\rho) - I_{det}(\rho_0)\\ &= -\frac{I_{inc}}{2}\sin(\phi_{20}-\phi_{10})\cdot\frac{4\pi}{\lambda}d\,(0.00029)\,\frac{\Delta\rho}{\rho_0}\end{aligned}}$$

4-(1) $\delta y = \dfrac{\lambda L}{a}$

4-(2) When the width of the slit d has to be considered we have



The envelope of $I(y)$ drops off as shown.