Solution to Physics 108 Midterm

1-(1) For refraction at the first surface with $r = \infty$, $s^{(1)} = 35$ cm, $n_1 = n_{air} = 1$, $n_2 = n_g = 2$, we have

$$\frac{n_1}{s^{(1)}} + \frac{n_2}{s'^{(1)}} = \frac{n_2 - n_1}{r} = 0$$

Thus

$$s'^{(1)} = -\left(\frac{n_2}{n_1}\right) s^{(1)} = -2 \times 35 \text{ cm} = -70 \text{ cm}$$ #

It is a virtual image at 70 cm to the left of the ~~surface~~ first surface. As a result, with respect to the second refraction surface, it is a real object with $s^{(2)} = |s'^{(1)}| + d = 80$ cm. With $r = 40$ cm, $n_1 = 2$, $n_2 = 1$

$$\frac{n_1}{s^{(2)}} + \frac{n_2}{s'^{(2)}} = \frac{n_2 - n_1}{r}$$

$$\Rightarrow \frac{2}{80} + \frac{1}{s'^{(2)}} = \frac{(-1)}{40}$$

$$\therefore s'^{(2)} = -20 \text{ cm}.$$

It is a virtual image at 20 cm to the left of the second refracting surface.

1-(2)

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = R_{21} T_1 R_{10}$$

$$= \begin{pmatrix} 1 & 0 \\ \frac{n_1 - n_2}{n_2 R_2} & \frac{n_1}{n_2} \end{pmatrix} \begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \frac{n_0 - n_1}{n_1 R_1} & \frac{n_0}{n_1} \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ \frac{2-1}{40} & 2 \end{pmatrix} \begin{pmatrix} 1 & 10 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1/2 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ \frac{1}{40} & 2 \end{pmatrix} \begin{pmatrix} 1 & 5 \\ 0 & 1/2 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 5 \\ \frac{1}{40} & \frac{9}{8} \end{pmatrix}$$

1-(3)

$$S_i = -\frac{AS_0 + B}{CS_0 + D} = -\frac{35+5}{\frac{35}{40} + \frac{9}{8}} = -20\text{ cm}$$

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2-(1) With $s_o = +10\,m$, $R = +20\,m$,

$$\frac{1}{s_o} + \frac{1}{s'} = -\frac{2}{R}$$

$$\Rightarrow \frac{1}{10} + \frac{1}{s'} = -\frac{2}{20}$$

$$\therefore s' = -5\,m$$

It is a virtual image at 5m to the right of the reflecting surface.

From $m = \frac{y_i}{y_o} = -\cancel{}\frac{s'}{s_o} = -\frac{(-5m)}{10m} = \frac{1}{2}$

$$y_i = m\,y_o = (1/2)\cdot y_o = +0.25\,m$$

It has the same orientation as the object.

2-(2) At a distance of 2m away, the observer now is at 2m + 5m = 7m from the image, with a linear size of $y_i = 0.25\,m$. Thus the angular size of y_i to the observer is

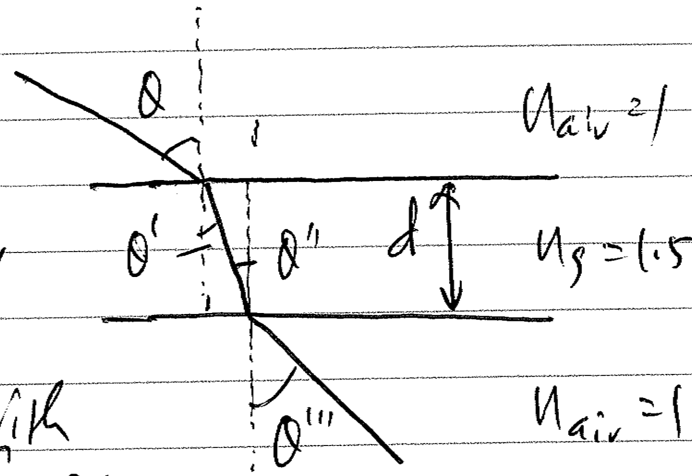
$$\alpha = \frac{y_i}{7m} = \frac{0.25\,m}{7\,m} = \frac{1}{28} \text{ radian}$$

3 - (1) From Snell's law,

$n_{air} \sin\theta = n_g \cdot \sin\theta'$

and

$n_g \sin\theta'' = n_{air} \sin\theta'''$



For a glass plate with the two side surfaces parallel to each other, we have $\theta' = \theta''$.
As a result,

$$n_{air} \sin\theta = n_g \sin\theta' = n_g \sin\theta'' = n_{air} \sin\theta'''$$

$$\therefore \theta''' = \theta = 60^\circ$$

3 - (2) The reflected light intensity as a function of wavelength λ is given by

$$I_{refl.}(\lambda, d) = I_{inc}(\lambda) \frac{1}{2} \left(1 - \cos \frac{4\pi n_{air}}{\lambda} \cos\theta''' d\right)$$

The dark lines are those wavelengths λ with

$$\frac{4\pi n_{air} d}{\lambda} \cos\theta''' = 2\pi \cdot m$$

or $\lambda = \frac{2d}{m} \cos\theta''' \cdot n_{air} = \frac{d}{m}$

In the visible range from 400 nm to 700 nm, we have the dark lines ($I(\lambda, d) \simeq 0$) at

$$\lambda = 625.00 \text{ nm} \quad (m=8)$$
$$= 555.56 \text{ nm} \quad (m=9)$$
$$= 500.00 \text{ nm} \quad (m=10)$$
$$= 454.55 \text{ nm} \quad (m=11)$$
$$= 416.67 \text{ nm} \quad (m=12)$$

3-(3) When the gap is increased to $d' = 5005$ nm, the dark lines appear at

$$\lambda' = \frac{d'}{m}$$

and in the visible range,

$$\lambda' = 625.62 \text{ nm} \quad (m=8)$$
$$556.11 \text{ nm} \quad (m=9)$$
$$500.50 \text{ nm} \quad (m=10)$$
$$455.00 \text{ nm} \quad (m=11)$$
$$417.08 \text{ nm} \quad (m=12)$$

The shift in the dark lines is on average

$$\Delta\lambda \equiv \lambda' - \lambda \simeq \frac{\Delta d}{10} \simeq 0.5 \text{ nm}$$