Physics 108 Final Exam (Spring Quarter, 2015)

1. **Lenses**

In a summer research project, you are asked to image an object with a linear magnification of m = - 2 on a screen that is 60 cm away from the object.

1) (**15 points**) If you use one thin lens to achieve this goal, what should the focal length of the lens be?
2) (**10 points**) If you use two identical thin lenses that are placed one right after the other (no separation between the two lenses) to reduce spherical aberration or cost as they are available on eBay, what should the focal length of the lenses be now?

2. **Diffraction:**

You have a perfect eye-sight such that your ability to distinguish objects afar is only limited by diffraction. Assume that your pupil is open to d = 5 mm in diameter to allow the light in, the focal length of your eye (treated as a thin lens) is f = 15 mm, and the refractive index inside your eye is $n_w = 4/3$. Use $\lambda_0 = 500$ nm for the visible light.

1) (**15 points**) Without the aid of a binocular, what is the minimum separation of two bright point-like features on an airplane that you can barely distinguish when the plane is L = 1000 m above where you stand?
2) (**5 points**) If you use a binocular with angular magnification of 50, what is the minimum separation of two bright features on the plane that you can resolve now?

3. **Interference**:

Two flat glass plates are set parallel to each other and separated by an air gap of unknown value *d*. When illuminated with an extended, monochromatic light λ = 600 nm from above, you can see concentric rings starting from normal incidence due to the interference effect. Assume that the center of these rings is completely dark.

1) (**10 points**) How does the light intensity change as a function of the viewing angle θ away from the normal incidence, λ, and *d*, in general?
2) (**10 points**) If the first dark ring from the center appears at a viewing angle of θ = 5° (1° = 0.0175 rad), what is the value of *d*?

4. **Optical constants:**

For transparent materials such as glass and water, optical dielectric constants and corresponding refractive indices are approximated by

$$\varepsilon(\omega) = n^2(\omega) = 1 + \frac{\omega_p^2}{\omega_0^2 - \omega^2},$$

where ω_0 is the angular frequency of the natural oscillation of bound electrons about positively charged ions, and it is in the ultraviolet range. ω_p is the plasmon frequency, also in the violet range. ω is the angular frequency of the light in the visible range. It is known that ω_0 for water is larger than ω_0 for glass and this is why $n_{water} < n_{glass}$ in general.

1) (**5 points**) When a beam of white light is incident from the glass side onto a glass-air interface, which color is totally reflected first and which color is totally reflected last? (Explain your answer instead of simply stating it.)
2) (**10 points**) When a beam of white light in the air is incident angle $\theta_1 > 0$ on a surface of water or glass, is the angular spread of the colors in the water larger or smaller than the spread in a glass? (again, explain your answer)

5. **Reflection and Transmission Coefficients: - Fresnel Equations**

The reflection of an unpolarized light from shiny surfaces of a concrete floor or smooth piece of metal or a table top is partly polarized and can be used to determine the transmission axis of a linear polarizer and to make anti-glare glasses using liner polarizers. The reason is in the answer to the following question.

1) (**15 points**) For transparent materials (also true for absorbing material), show that the reflectance for s-polarized component $R_s = |r_s|^2$ is always larger than the reflectance for p-polarized component $R_p = |r_p|^2$ for $\theta_1 \leq \theta_{1B}$. (Hint: use the fact that $n_1 < n_2$ and expressions for reflection coefficients.)
2) (**extra 10 points**) Show that $R_s = |r_s|^2 \geq R_p = |r_p|^2$ is also true for $\theta_1 > \theta_{1B}$.

6. **Total internal reflection and polarizing angle**

1) (**5 points**) For $n_1 > n_2$, show that the Brewster angle θ_{1B} is always smaller than the critical angle θ_{1C} for total internal reflection.
2) (**10 points**) Calcite is a uni-axial crystal such that when the electric field is linearly polarized along the optic axis (OA) of calcite, it experiences a refractive index of $n_e = 1.486$; and when the linearly polarized electric field is perpendicular to the OA of calcite, the refractive index is $n_o = 1.658$. Find critical angles for such two linearly polarized components of a light beam at a calcite-air interface. (Why bother? *By setting the incidence angle in between these two critical angles, one can produce linearly polarized beams out of an arbitrarily polarized beam of light.*)

7. **Jones Vectors**

(**20 points**) Specify the state of polarization for the following *un*-normalized Jones vectors by first making them normalized Jones vectors,

(a) $\begin{bmatrix} 1 \\ \sqrt{3} \end{bmatrix}$; (b) $\begin{bmatrix} -i \\ -1 \end{bmatrix}$; (c) $\begin{bmatrix} 1+i \\ -1+i \end{bmatrix}$; (d) $\begin{bmatrix} i \\ 1+i \end{bmatrix}$.

8. **Jones Matrices**:

A beam of light with unknown state of polarization serves as the start to produce light beams of well-defined polarization with the aid of one or a combination of polarizing devices at your disposal (linear polarizer, quarter-wave-plates, half-wave-plates, wave-plates with any other phase shifts than 90° or 180°, rotator, etc)

$$\tilde{E}_{inc} = \begin{bmatrix} \alpha \\ \beta e^{i\Delta\Phi_0} \end{bmatrix}$$

1) (**10 points**) What device or combination of devices will you use to produce a linearly polarized light along y-axis if losing some of the incident beam energy is permitted? (Show the Jones matrix or matrices, how you set it or them up, and how the final Jones vector is what you are looking for.)
2) (**10 points**) What device or combination of devices will you use to produce a left-circularly polarized light if losing some of the incident energy is permitted? (Show the Jones matrix or matrices, how you set it or them up, and how the final Jones vector is what you are looking for.)
3) (**extra 10 points**) What device or combination of devices will you use to produce a linearly polarized light along y-axis without losing any of the incident beam energy (assuming that there is no reflection loss)?