Solutions to Physics 108 MT (2016)

1-(a) From the equation for the focal length of a convex glass lens in air

$$\frac{1}{f} = (n_g - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = (n_g - 1)\frac{2}{R}$$

$$= (1.5 - 1)\cdot 2/R = \frac{1}{R}$$

$$R = f = +5\,cm$$

1-(b) The angular magnification

$$M_A = 1 + \frac{d_0}{f} = 1 + \frac{25\,cm}{5\,cm} = 6$$

1-(c) The equation of a thin lens when $n_1 \neq n_2 \neq n_3$ is

$$\frac{n_1}{s} + \frac{n_3}{s'} = \frac{n_2 - n_1}{R_1} + \frac{n_3 - n_2}{R_2}$$

Since $R_1 = -R_2 = R = 5\,cm$, $n_1 = n_w = 4/3$, $n_3 = 1$, $n_2 = n_g = 3/2$, we have

$$\frac{n_w}{s} + \frac{1}{s'} = \frac{1}{R}\left(2n_g - (n_w + 1)\right)$$

$$2n_g - (n_w + 1) = 3 - \left(\tfrac{4}{3} + 1\right)$$

$$= 3 - \frac{7}{3} = \frac{2}{3}$$

$$\therefore \frac{(4/3)}{s} + \frac{1}{s'} = \frac{2}{15}$$

$$f_1 = s\Big|_{s' = +\infty} = +10 \text{ cm}$$

$$f_2 = s'\Big|_{s = +\infty} = +7.5 \text{ cm}$$

F(d) Let $s' = -d_0$, then $s = +100/13$ cm

$$M_A = \left(\frac{y'}{d_0}\right) \Big/ \frac{y_0}{d_0} = \frac{y'}{y_0} = -\frac{s'}{s} n_w$$

$$= +4\frac{1}{3}$$

2-(a) Since

$$f = \frac{D^2 - L^2}{4D} \le \frac{D^2}{4D} = \frac{D}{4}$$

where D is the separation between a real object and a real image, and L is the separation between two possible location of a converging lens with f, the focal length f needs to be smaller or equal to $D/4$ as $L^2 > 0$.

Now $D = 60\,cm$, we can only use one lens with $f = +10\,cm$.

From $L^2 = D^2 - 4Df = 60^2 - 4 \times 60 \times 10$

$$= 1200\,cm^2$$

$$\therefore L = \sqrt{1200}\,cm$$

Thus the object distance should be either

$$s = \frac{D-L}{2} = \frac{60\,cm - \sqrt{12} \cdot 10\,cm}{2} = 12.68\,cm$$

or

$$s' = \frac{D+L}{2} = \frac{60\,cm + \sqrt{12} \cdot 10\,cm}{2} = 47.32\,cm$$

2-(b) Now for any of the two lenses,

$$f > \frac{D'}{4} = \frac{30\,cm}{4} = 7.5\,cm.$$

We can use two 10-cm focal length lenses one right after another with no gap. Such a combination of two 10-cm lenses has an effective focal length,

$$f' = \frac{10\,cm}{2} = 5\,cm < \frac{D'}{4} = 7.5\,cm$$

Now from the part 2-(a)

$$s = \frac{1}{2}(D' - L') = \frac{1}{2}(30\,cm - \sqrt{3}\cdot 10\,cm) = 6.34\,cm$$

or

$$s = \frac{1}{2}(D' + L') = \frac{1}{2}(30\,cm + \sqrt{3}\cdot 10\,cm) = 23.66\,cm$$

3-(a) From

$$\frac{1}{s}+\frac{1}{s'}=-\frac{2}{R}$$

if $s \ll |R|$, then

$$s'=\frac{1}{-\frac{1}{s}-\frac{2}{R}} \simeq -s$$

regardless the value of R. And the linear magnification

$$m=\frac{y_i}{y_o}=-\frac{s'}{s}=+1$$

3-(b) Now $s=+8$ cm, $R=-20$ cm,

$$\frac{1}{s'}=-\frac{2}{R}-\frac{1}{s}=-\frac{1}{40}$$

$\therefore s'=-40$ cm (behind the mirror)

$$m=\frac{y_i}{y_o}=-\frac{s'}{s}=+5.$$

(Like the one in a better hotel bathroom)

4-(a) The total height change along the 2-cm width is

$$h = \alpha \cdot 2\text{cm}.$$

If we see 20 fringes, and the height change between neighboring fringes is δh

$$\delta(\phi_2 - \phi_1) = 2\pi = \frac{4\pi}{\lambda} \cdot \delta h \cdot n_{air} \cdot \cos\theta$$

At normal incidence, $\theta = 0$, $n_{air} = 1$

$$\therefore \delta h = \frac{\lambda_0}{2} = 0.25\,\mu\text{m} = 2.5 \times 10^{-5}\,\text{cm}$$

$$\therefore \alpha = \frac{h}{2\text{cm}} \simeq \frac{20 \cdot \delta h}{2\text{cm}} = 2.5 \times 10^{-4}\,\text{rad}$$

4-(b) The phase difference between S_1 and S_2 as a function of θ is

$$(\phi_2 - \phi_1)_\theta = \frac{2\pi}{\lambda} \cdot d \cdot \sin\theta$$

For $(\phi_2 - \phi_1)_\theta = 6\pi$

$$\theta = \sin^{-1}\left(\frac{3\lambda}{d}\right)$$

4-(c) Let δ be the extra phase of S_1 relative to S_2. Then

$$\left(\phi_2 - \phi_1\right)_\delta = \frac{2\pi d}{\lambda} \sin\theta - \delta$$

Let us consider the fringe consisting of all the points at which $(\phi_2 - \phi_1) = 0$. Then the location of these points (bright fringes) is given by

$$\sin\theta = \left(\frac{\lambda}{2\pi d}\right)\cdot\delta + \left(\phi_2 - \phi_1\right)\Big|_{\delta=0} \cdot \frac{\lambda}{2\pi d}$$

$$= \left(\frac{\lambda}{2\pi d}\right)\cdot\delta$$

As δ increases from zero to 3π, $\sin\theta$ increases from 0 to

$$\sin\theta = \left(\frac{\lambda}{2\pi d}\right)\cdot(3\pi) = \frac{3}{2}\left(\frac{\lambda}{d}\right)$$

The fringe moves upward by an angle

$$\theta = \sin^{-1}\left(\frac{3}{2}\frac{\lambda}{d}\right)$$

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