**23-2.** The critical angle is,

$$\sin\theta_c = \frac{n_1}{n_2} = \frac{1}{n} \Rightarrow n = \frac{1}{\sin\theta_c} = \frac{1}{\sin(33°33')} = 1.8094$$

**External Reflection:**

$$\tan\theta_p = \frac{n_2}{n_1} = n \Rightarrow \theta_p = \tan^{-1}(1.8094) = 61°4'$$

**Internal Reflection:**

$$\tan\theta_p' = \frac{n_1}{n_2} = \frac{1}{n} \Rightarrow \theta_p' = \tan^{-1}\left(\frac{1}{1.8094}\right) = 28°56'$$

**23-5.** In each of the 4-equations the desired form results from introducing $\theta_t$ through the relation $n = \sin\theta/\sin\theta_t$. For example:

$$r_{\mathrm{TE}} = \frac{\cos\theta - \sqrt{n^2 - \sin^2\theta}}{\cos\theta + \sqrt{n^2 - \sin^2\theta}} = \frac{\cos\theta - \sin\theta\sqrt{1/\sin^2\theta_t - 1}}{\cos\theta + \sin\theta\sqrt{1/\sin^2\theta_t - 1}} = \frac{\cos\theta - \sin\theta\sqrt{(\cos^2\theta_t)/\sin^2\theta_t}}{\cos\theta + \sin\theta\sqrt{(\cos^2\theta_t)/\sin^2\theta_t}}$$

$$r_{\mathrm{TE}} = \frac{\cos\theta\sin\theta_t - \sin\theta\cos\theta_t}{\cos\theta\sin\theta_t + \sin\theta\cos\theta_t} = -\frac{\sin(\theta - \theta_t)}{\sin(\theta + \theta_t)}$$

The other 4 relations are similarly shown to be true. Proceeding by first noting from above that,

$$\sqrt{n^2 - \sin^2\theta} = \sin\theta\cos\theta_t/\sin\theta_t$$

Then,

$$r_{\mathrm{TM}} = \frac{-n^2\cos\theta + \sqrt{n^2 - \sin^2\theta}}{n^2\cos\theta + \sqrt{n^2 - \sin^2\theta}} = \frac{-\frac{\sin^2\theta\cos\theta}{\sin^2\theta_t} + \frac{\sin\theta\cos\theta_t}{\sin\theta_t}}{\frac{\sin^2\theta\cos\theta}{\sin^2\theta_t} + \frac{\sin\theta\cos\theta_t}{\sin\theta_t}}$$

$$r_{\mathrm{TM}} = \frac{-\sin\theta\cos\theta + \cos\theta_t\sin\theta_t}{\sin\theta\cos\theta + \cos\theta_t\sin\theta_t} = \frac{-\sin\theta\cos\theta(\sin^2\theta_t \overset{1}{+} \cos^2\theta_t) + \cos\theta_t\sin\theta_t\left(\sin^2\theta \overset{1}{+} \cos^2\theta\right)}{\sin\theta\cos\theta(\sin^2\theta_t + \cos^2\theta_t) + \cos\theta_t\sin\theta_t(\sin^2\theta + \cos^2\theta)}$$

$$r_{\mathrm{TM}} = \frac{-\sin\theta\cos\theta\sin^2\theta_t - \sin\theta\cos\theta\cos^2\theta_t + \cos\theta_t\sin\theta_t\sin^2\theta + \cos\theta_t\sin\theta_t\cos^2\theta}{+\sin\theta\cos\theta\sin^2\theta_t + \sin\theta\cos\theta\cos^2\theta_t + \cos\theta_t\sin\theta_t\sin^2\theta + \cos\theta_t\sin\theta_t\cos^2\theta}$$

$$r_{\mathrm{TM}} = \frac{(\sin\theta\cos\theta_t - \cos\theta\sin\theta_t)(\sin\theta\sin\theta_t - \cos\theta\cos\theta_t)}{(\sin\theta\cos\theta_t + \cos\theta\sin\theta_t)(\sin\theta\sin\theta_t + \cos\theta\cos\theta_t)} = -\frac{\sin(\theta - \theta_t)\cos(\theta + \theta_t)}{\sin(\theta + \theta_t)\cos(\theta - \theta_t)}$$

$$r_{\mathrm{TM}} = -\frac{\tan(\theta - \theta_t)}{\tan(\theta + \theta_t)}$$

and,

$$t_{\mathrm{TE}} = \frac{2\cos\theta}{\cos\theta + \sqrt{n^2 - \sin^2\theta}} = \frac{2\cos\theta}{\cos\theta + \sin\theta\cos\theta_t/\sin\theta_t} = \frac{2\cos\theta\sin\theta_t}{\cos\theta\sin\theta_t + \sin\theta\cos\theta_t} = \frac{2\cos\theta\sin\theta_t}{\sin(\theta + \theta_t)}$$

Finally,

$$t_{\mathrm{TM}} = \frac{2n\cos\theta}{n^2\cos^2\theta + \sqrt{n^2 - \sin^2\theta}} = \frac{2\cos\theta\sin\theta/\sin\theta_t}{\frac{\sin^2\theta\cos\theta}{\sin^2\theta_t} + \frac{\sin\theta\cos\theta_t}{\sin\theta_t}} = \frac{2\cos\theta\sin\theta_t}{\cos\theta\sin\theta + \sin\theta_t\cos\theta_t}$$

The denominator in this expression is the same as the one that appeared in the work leading to the final form of $r_{\mathrm{TM}}$. Using the result derived there,

$$t_{\mathrm{TM}} = \frac{2\cos\theta\sin\theta_t}{\sin(\theta + \theta_t)\cos(\theta - \theta_t)}$$

**23-12.** For diamond $n = 2.42$.

(a) **External:**
TM: $\theta_p = \tan^{-1}(n) = \tan^{-1}(2.42) = 67.55°$, no critical angle.
TE: No Brewster angle, no critical angle.

(b) **Internal:**
TM: $\theta_c = \sin^{-1}(1/2.42) = 0.426 = 24.41°$, $\theta_p' = \tan^{-1}(1/2.42) = 0.392 = 22.46°$.
TE: $\theta_c = \sin^{-1}(1/2.42) = 0.426 = 24.41°$, no Brewster angle.

**23-13.** The reflectance and transmission are given by

$$R_{\mathrm{TE}} = \left( \frac{\cos\theta - \sqrt{n^2 - \sin^2\theta}}{\cos\theta - \sqrt{n^2 - \sin^2\theta}} \right)^2 = \left( \frac{\cos(50°) - \sqrt{1.6^2 - \sin^2(50°)}}{\cos(50°) + \sqrt{1.6^2 - \sin^2(50°)}} \right)^2 = 0.1385 = 13.85\%,$$

$$T_{\mathrm{TE}} = 1 - R_{\mathrm{TE}} = 86.15\%$$

$$R_{\mathrm{TM}} = \left( \frac{-n^2\cos\theta + \sqrt{n^2 - \sin^2\theta}}{n^2\cos\theta + \sqrt{n^2 - \sin^2\theta}} \right)^2 = \left( \frac{-1.6^2\cos(50°) + \sqrt{1.6^2 - \sin^2(50°)}}{1.6^2\cos(50°) + \sqrt{1.6^2 - \sin^2(50°)}} \right)^2 = 0.00623 = 0.623\%$$

$$T_{\mathrm{TM}} = 1 - R_{\mathrm{TM}} = 99.38\%$$

**23-15.** (a) The critical angle is $\theta_c = \sin^{-1}(n_2/n_1) = \sin^{-1}(1/1.458) = 43.3°$

The polarizing angle for external reflection is is $\theta_p = \tan^{-1}(n_2/n_1) = \tan^{-1}(1.458/1) = 55.6°$

The polarizing angle for internal reflection is $\theta_p' = \tan^{-1}(1/1.458) = 34.4°$

(b) At normal incidence: $R_{TE} = r_{TE}^2 = \left(\frac{1-n}{1+n}\right)^2 = \left(\frac{1-1.1458^2}{1+1.1458^2}\right)^2 = 0.0347, T_{TE} = 1 - R_{TE} = 0.9653$

At $\theta = 45°$: $R_{TE} = \left( \frac{\cos\theta - \sqrt{n^2 - \sin^2\theta}}{\cos\theta + \sqrt{n^2 - \sin^2\theta}} \right)^2 = \left( \frac{\cos 45° - \sqrt{(1.458)^2 - \sin^2 45°}}{\cos 45° + \sqrt{(1.458)^2 - \sin^2 45°}} \right)^2 = 0.0821$

$T_{TE} = 1 - R_{TE} = 0.9179.$

(c) At normal incidence: $R_{TM} = R_{TE} = 0.0347, T_{TE} = 0.9653$

At $\theta = 45°$: $R_{TM} = \left( \frac{-n^2\cos\theta + \sqrt{n^2 - \sin^2\theta}}{n^2\cos\theta + \sqrt{n^2 - \sin^2\theta}} \right)^2 = \left( \frac{-(1.458)^2\cos 45° + \sqrt{1.458^2 - \sin^2 45°}}{(1.458)^2\cos 45° + \sqrt{1.458^2 - \sin^2 45°}} \right)^2$

$R_{TM} = 0.0067, T_{TM} = 0.9933$

(d) Use Eqs. (23-28) and (23-29):

For incident angles less than $\theta_p'$, $\phi_{TM} = \phi_{TE} = 0$. So for $\theta = 0°$ and $20°$, $\phi_{TM} - \phi_{TE} = 0$.

For $\theta = 40°$ which lies between $\theta_p'$ and $\theta_c$, $\phi_{TM} - \phi_{TE} = \pi - 0 = \pi$.

For $\theta > \theta_c$: with $n = 1/1.458$,

$$\phi_{TM} - \phi_{TE} = -2\tan^{-1}\left( \frac{\sqrt{\sin^2\theta - n^2}}{n^2\cos\theta} \right) + \pi + 2\tan^{-1}\left( \frac{\sqrt{\sin^2\theta - n^2}}{\cos\theta} \right)$$

$\theta = 50°$: $\phi_{TM} - \phi_{TE} = 2.43\,\mathrm{rad} = 139°$

$\theta = 70°$ $\phi_{TM} - \phi_{TE} = 2.65\,\mathrm{rad} = 152°$

$\theta = 90°$ $\phi_{TM} - \phi_{TE} = \pi = 180°$

**23-16.** (a) By trial and error, $\theta = \theta_{\text{rhomb}} = 59.857°$, so that with $n = 1/1.65$

$$2\,(\phi_{TE} - \phi_{TM}) = -\,2\tan^{-1}\!\left(\frac{\sqrt{\sin^2\theta - n^2}}{\cos\theta}\right) + 2\tan^{-1}\!\left(\frac{\sqrt{\sin^2\theta - n^2}}{n^2\cos\theta}\right) - \pi = -\,1.5\,\pi \Rightarrow \pi/2$$

(b) If $\theta = (1 + 0.05)\,59.857° = 62.850°$ :

$$2\,(\phi_{TE} - \phi_{TM}) = -\,2\tan^{-1}\!\left(\frac{\sqrt{\sin^2\theta - n^2}}{\cos\theta}\right) + 2\tan^{-1}\!\left(\frac{\sqrt{\sin^2\theta - n^2}}{n^2\cos\theta}\right) - \pi = -\,1.543\,\pi \Rightarrow 0.457\,\pi$$

If $\theta = (1 - 0.05)\,59.857° = 56.864°$ :

$$2\,(\phi_{TE} - \phi_{TM}) = -\,2\tan^{-1}\!\left(\frac{\sqrt{\sin^2\theta - n^2}}{\cos\theta}\right) + 2\tan^{-1}\!\left(\frac{\sqrt{\sin^2\theta - n^2}}{n^2\cos\theta}\right) - \pi = -\,1.461\,\pi \Rightarrow 0.539\,\pi$$

**23-20.** Consider Eq. (23-47)

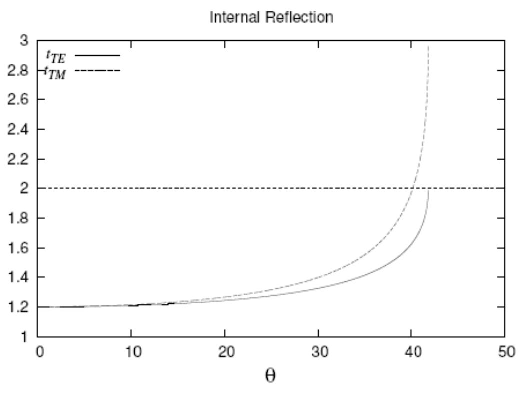
$$1 = r^2 + n\left(\frac{\cos\theta_t}{\cos\theta}\right)t^2 \quad n = n_2/n_1$$

(a) **External Reflection:** $n > 1$; $\theta_t < \theta_i$; $\cos\theta_t > \cos\theta$. Thus, $1 - r^2 = n\left(\frac{\cos\theta_t}{\cos\theta}\right)t^2 > t^2 \Rightarrow t^2 < 1 - r^2$
Since $r^2 < 1$, $t^2 < 1$.
**Internal Reflection:** $n < 1$; $\theta_t > \theta_i$; $\cos\theta_t < \cos\theta$. Thus, $1 - r^2 = n\left(\frac{\cos\theta_t}{\cos\theta}\right)t^2 < t^2 \Rightarrow t^2 > 1 - r^2$
No upper limit is imposed.

(c) For the angle of incidence equal to the critical angle, $\sin\theta = \sin\theta_c = n$. Then,

$$t_{\text{TE}} = \frac{2\cos\theta}{\cos\theta + \sqrt{n^2 - \sin^2\theta}} = \frac{2\cos\theta}{\cos\theta} = 2$$
$$t_{\text{TM}} = \frac{2\,n\cos\theta}{n^2\cos\theta + \sqrt{n^2 - \sin^2\theta}} = \frac{2}{n}$$

(c)




**23-19.** Given $n_I = 5.3$ at $\lambda = 589.3\,\text{nm}$. (a) $\alpha = 4\,\pi\,n_I/\lambda = 4\,\pi\,(5.3)/(589.3\,\text{nm}) = 0.113\,\text{nm}^{-1}$

(b) $I = I_0\,e^{-\alpha s}$. For $I = 0.01\,I_0$, $e^{-\alpha s} = 0.01 \Rightarrow s = (-1/\alpha)\ln(0.01) = 40.75\,\text{nm} = 0.069\,\lambda$

**23-21.** (a) The penetration depth is

$$|z|_{1/e} = \frac{1}{\alpha} = \frac{\lambda}{2\pi}\frac{1}{\sqrt{\sin^2\theta/n^2 - 1}} = \frac{0.546\ \mu\text{m}}{2\pi}\frac{1}{\sqrt{\sin^2(45°)/(1/1.6)^2 - 1}} = 0.164\ \mu\text{m}$$

(b) Since irradiance is proportional to the square of the field amplitude and with $\alpha = \frac{1}{|z|_{1/e}} = 6.089\ \mu\text{m}^{-1}$,

$$\frac{I}{I_0} = e^{-2\alpha|z|} = e^{-2(6.089\ \mu\text{m}^{-1})(1\mu\text{m})} = 5.1 \times 10^{-6}$$

**25-1.** (a) Consider,

$$K \equiv n^2 = (n_R + i\, n_I)^2 = n_R^2 - n_I^2 + 2\, i\, n_I n_R = K_R + i\, K_I$$
$$K_R = n_R^2 - n_I^2,\ K_I = 2\, n_I n_R$$

Solving these two relations for $n_R$ and $n_I$ proceeds as,

$$K_I = 2\, n_I n_R = 2\, n_I \sqrt{K_R^2 + n_I^2}$$
$$K_I^2 = 4\, n_I^2 (K_R^2 + n_I^2)$$
$$4\, n_I^4 + 4K_R^2 n_I^2 - K_I^2 = 0$$
$$n_I^2 = \frac{-4\, K_R \pm \sqrt{16\, K_R^2 + 16\, K_I^2}}{8} = \frac{-K_R \pm \sqrt{K_R^2 + K_I^2}}{2}$$

To make $n_I^2 > 0$, choose the $+$ sign. Thus,

$$n_I = \left[\frac{-K_R + \sqrt{K_R^2 + K_I^2}}{2}\right]^{1/2}$$

$$n_R^2 = K_R + n_I^2 = \frac{2\, K_R}{2} + \frac{-K_R + \sqrt{K_R^2 + K_I^2}}{2} = \frac{K_R + \sqrt{K_R^2 + K_I^2}}{2}$$

$$n_R = \left[\frac{K_R + \sqrt{K_R^2 + K_I^2}}{2}\right]^{1/2}$$

(b) If $K_I \approx K_R$,

$$n_R = \left[\frac{K_I + \sqrt{2\, K_I^2}}{2}\right]^{1/2} = \sqrt{K_I}\left(\frac{1 + \sqrt{2}}{2}\right)^{1/2} = 1.099\, \sqrt{K_I}$$

$$n_I = \left[\frac{-K_I + \sqrt{2\, K_I^2}}{2}\right]^{1/2} = \sqrt{K_I}\left(\frac{-1 + \sqrt{2}}{2}\right)^{1/2} = 0.455\, \sqrt{K_I}$$