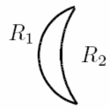
2-16. The plane side of the lens has $R_1 = \infty$. The radius of curvature R_2 of the convex side is then found from the lensmaker's equation:

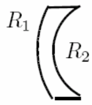
$$\frac{1}{f}=\frac{n_2-n_1}{n_1}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)\Rightarrow\frac{1}{25}=\frac{1.52-1}{1}\left(\frac{1}{\infty}-\frac{1}{R_2}\right)\Rightarrow R_2=-13\,\text{cm}$$

2-17. In general the lensmaker's equation gives, $\dfrac{1}{f}=\dfrac{n_2-n_1}{n_1}\left(\dfrac{1}{R_1}-\dfrac{1}{R_2}\right)$

For the positive meniscus lens shown to the right, $R_1=5\,\text{cm}$ and $R_2=10\,\text{cm}$.
Then, $\dfrac{1}{f}=\dfrac{1.50-1}{1}\left(\dfrac{1}{5}-\dfrac{1}{10}\right)\Rightarrow f=+20\,\text{cm}$



For the negative meniscus lens shown to the right, $R_1=10\,\text{cm}$ and $R_2=5\,\text{cm}$.
For this case, $\dfrac{1}{f}=\dfrac{1.50-1}{1}\left(\dfrac{1}{10}-\dfrac{1}{5}\right)\Rightarrow f=-20\,\text{cm}$



2-20. See Figure 2-36 in the text. Consider the three media as a sequence of three thin lenses. Each has a focal length given by the lensmaker's equation, and the equivalent focal length is given Eq. (2-33) as,

$\dfrac{1}{f_{\text{eq}}}=\dfrac{1}{f_1}+\dfrac{1}{f_2}+\dfrac{1}{f_3}$. Then, $\dfrac{1}{f_1}=(1.5-1)\left(\dfrac{1}{\infty}-\dfrac{1}{-15}\right)\Rightarrow f_1=30\,\text{cm}$,

$\dfrac{1}{f_2}=(1.65-1)\left(\dfrac{1}{-15}-\dfrac{1}{15}\right)$ or $f_2=-\dfrac{150}{13}\,\text{cm}$, and $\dfrac{1}{f_3}=\text{same as for } f_1: f_3=30\,\text{cm}$. Then,

$\dfrac{1}{f_{\text{eq}}}=\dfrac{1}{30}+\dfrac{-13}{150}+\dfrac{1}{30}$ and so $f_{\text{eq}}=-50\,\text{cm}$.

2-22. Refer to Figure 2-37 in the text.

(b) Lens heading towards mirror: $\dfrac{1}{3f/2}+\dfrac{1}{s'}=\dfrac{1}{-f}$ or $s'=-3f/5$. $m_1=-\dfrac{s'}{s}=-\dfrac{-3f/5}{3f/2}=2/5$

Mirror:

$s=3f+3f/5=18f/5\Rightarrow\dfrac{5}{18f}+\dfrac{1}{s'}=\dfrac{1}{f}\Rightarrow s'=18f/13,\ m_2=-\dfrac{s'}{s}=-(18f/13)/(18f/5)=-5/13$

Lens after reflection:

$s=3f-18f/13=21f/13\Rightarrow\dfrac{13}{21f}+\dfrac{1}{s'}=\dfrac{1}{-f}$ or $s'=21f/34,\ m_3=-s'/s=-(21f/34)/(21f/13)=\dfrac{13}{34}$

$m_T=\left(\dfrac{2}{5}\right)\left(-\dfrac{5}{13}\right)\left(\dfrac{13}{34}\right)=-17$. The image is inverted, $(21/34)\,f$ behind (right of) lens, inverted, and 1/17 original size.

2-27. See Figure 2-38 in the text. The applicable relations are:

Lens equations: $\dfrac{1}{s_1}+\dfrac{1}{s_1'}=\dfrac{1}{f}$ and $\dfrac{1}{s_2}+\dfrac{1}{s_2'}=\dfrac{1}{f}$,

Geometrical: $L=s_1+s_1'=s_2+s_2'$, $D=s_2-s_1=s_1'-s_2'$

Thus,

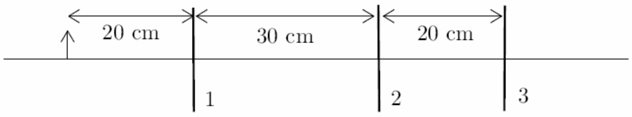
$$f=\frac{s_1s_1'}{s_1+s_1'}=\frac{s_1s_1'}{L}=\frac{s_2s_2'}{s_2+s_2'}=\frac{s_2s_2'}{L}\tag{1}$$

Because the lens equation can be satisfied the second time by simply interchanging object and image distances,

$$s_2=s_1' \text{ and } s_2'=s_1\tag{2}$$

Adding and subtracting the equations $L=s_2+s_2$ and $D=-s_1+s_2$, we get,
$L-D=2s$ and $L+D=2s_2$. Their product is by Eq. (1), $L^2-D^2=4s_1s_2$, or by Eq. (2), $L^2-D^2=4fL$.
Thus, $f=\dfrac{L^2-D^2}{4L}$.

2-23. The arrangement of the object and lenses is shown below.



(a) $f_1 = +10\,\text{cm},\ f_2 = +15\,\text{cm},\ f_3 = +20\text{cm}$

1st lens: $\frac{1}{20} + \frac{1}{s'} = \frac{1}{10}$ $\quad s' = 20 \quad m_1 = -20/20 = -1$

2nd lens: $\frac{1}{10} + \frac{1}{s'} = \frac{1}{15}$ $\quad s' = -30 \quad m_2 = -(-30)/10 = +3$

3rd lens: $\frac{1}{50} + \frac{1}{s'} = \frac{1}{20}$ $\quad s' = 100/3 \quad m_3 = -100/3\,(50) = -2/3$

$m_T = m_1 m_2 m_3 = +2$

(b) $f_1 = +10\,\text{cm},\ f_2 = -15\,\text{cm},\ f_3 = +20\text{cm}$

1st lens: $\frac{1}{20} + \frac{1}{s'} = \frac{1}{10}$ $\quad s' = 20 \quad m_1 = -20/20 = -1$

2nd lens: $\frac{1}{10} + \frac{1}{s'} = \frac{1}{-15}$ $\quad s' = -6 \quad m_2 = -(-6)/10 = +0.6$

3rd lens: $\frac{1}{26} + \frac{1}{s'} = \frac{1}{20}$ $\quad s' = 520/6 \quad m_3 = -520/(6 \times 26) = -\frac{10}{3}$

$m_T = m_1 m_2 m_3 = +2$

(c) $f_1 = -10\,\text{cm},\ f_2 = +15\,\text{cm},\ f_3 = -20\text{cm}$

1st lens: $\frac{1}{20} + \frac{1}{s'} = \frac{1}{-10}$ $\quad s' = 20/3 \quad m_1 = -(-20)/3\,(20) = \frac{1}{3}$

2nd lens: $\frac{3}{110} + \frac{1}{s'} = \frac{1}{15}$ $\quad s' = 330/13 \quad m_2 = -\frac{(330)\,(3)}{(13)\,(110)} = -\frac{9}{13}$

3rd lens: $\frac{-13}{70} + \frac{1}{s'} = \frac{1}{-20}$ $\quad s' = 140/19 \quad m_3 = -\frac{(140)\,(13)}{(19)\,(-70)} = \frac{26}{19}$

$m_T = m_1 m_2 m_3 = -6/19$

2-31. The distance between the object and the image is $D = s + s' = s + \frac{f\,s}{s - f}$. This is minimized when,

$\frac{dD}{ds} = 1 + \frac{(s-f)\,f - f\,s}{(s-f)^2} = 0 \Rightarrow s\,(s - 2\,f) = 0 \Rightarrow s = \not{0},\ 2\,f$. The minimum distance D occurs when $s = 2f$ and has the value $D = 2f + \frac{f\,(2f)}{2f - f} = 4f$. That is, in this configuration $s = s' = 2f$.

3-1. The *entrance pupil* is the aperture stop so no elements precede the aperture stop. The *exit pupil* is the image of the aperture stop formed by the lens. The position and size of the the exit pupil are found from the thin lens equation with the object being the aperture stop.

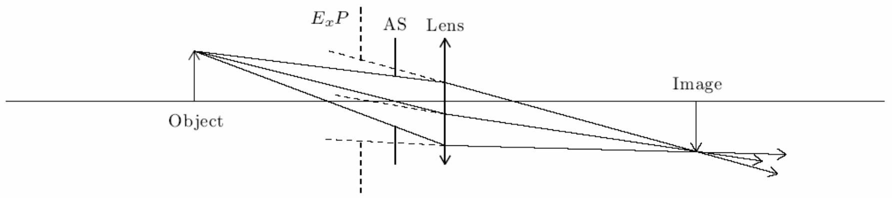
Exit Pupil and Size:

$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}\Rightarrow\frac{1}{2\,\mathrm{cm}}+\frac{1}{s'}=\frac{1}{5\,\mathrm{cm}}\Rightarrow s'=-3.33\,\mathrm{cm}$$
$$m=-\frac{s'}{s}=-\frac{3.33}{2}=\frac{5}{3}\Rightarrow \mathrm{size}=\frac{5}{3}\times 2\,\mathrm{cm}=3.33\mathrm{cm}$$

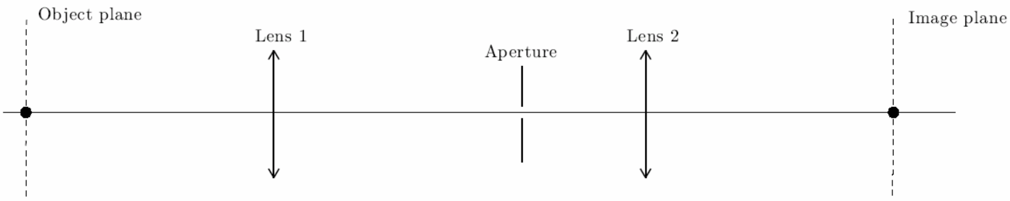
Image Position and Size:

$$\frac{1}{10\,\mathrm{cm}}+\frac{1}{s'}=\frac{1}{5\,\mathrm{cm}}\Rightarrow s'=10\mathrm{cm}$$
$$m=-s'/s=-10/10=-1\Rightarrow \mathrm{size}=1\times 2\,\mathrm{cm}=2\,\mathrm{cm},\mathrm{inverted}$$

The system is drawn to scale below.



3-4. (a) The vertical and horizontal scales in the figure differ by a factor of 4.



(b) **Image Plane:**

$$s_1'=\frac{f_1\,s_1}{s_1-f_1}=\frac{(40/3)\,(40)}{40-40/3}=20\,\mathrm{cm}\ (\text{right of Lens 1})\quad s_2'=\frac{f_2\,s_2}{s_2-f_2}=\frac{(20/3)\,(10)}{10-20/3}=20\,\mathrm{cm}\ (\text{right of Lens 2})$$

(c) **Candidates for AS:** Lens1, Aperture, or Lens 2.

Lens subtends at an angle of $\theta_{L1}=\frac{2}{40}=0.05\,\mathrm{rad}$.

Aperture image in Lens 1: $s'=\frac{20\,(40/3)}{20-40/3}=40\,\mathrm{cm}$, or at object plane, then $\theta_{\mathrm{Aperture}}=0°$

Lens 2 image through Lens 1: $s'=\frac{30\,(40/3)}{30-40/3}=24\,\mathrm{cm}$ left of Lens 1 or 16 cm right of object.

$$m=-s'/s=-24/30=-0.8\ \text{so size}=0.8\times 2=1.6\mathrm{cm},\ \ \theta_{L2}=\frac{1.6}{16}=0.1>0.05$$

Thus Lens 1 behaves as the AS. It is also the E_nP, being the first in line.

(d) **Exit Pupil E_xP:** Image of AS (i.e., Lens 1) in Lens 2:

$$s'=\frac{s\,f2}{s-f2}=\frac{30\,(20/3)}{30-20/3}=8.57\,\mathrm{cm},\text{right of Lens 2},\ \ m=-\frac{s'}{s}=-\frac{8.57}{30}=0.2857=2/7\ \text{so}\ D_{Exp}=\frac{2}{7}\times 2=\frac{4}{7}\,\mathrm{cm}$$

(e) **Field Stop:** Either Aperture or Lens 2, whichever subtends the smaller angle at center of E_nP=Lens 1:

$$\theta_{\mathrm{Aperture}}=\frac{0.5}{20}=0.025;\theta_{\mathrm{Lens\ 2}}=\frac{2}{30}=0.067$$

Thus the Aperture Stop behaves as the field stop (FS). *Continued on next page*

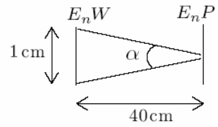
3-4. (e) *continued*.

Exit Window: Image of FS in following elements – lens Lens 2.

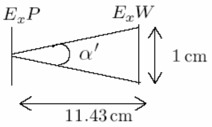
$$s' = \frac{(10)\,(20/3)}{10 - 20/3} = 20\,\text{cm, right of Lens 2, in image plane,}$$

$$m = -s'/s = 20/10 = -2 \text{ so } D_{E_x\omega} = 2 \times 0.5 = 1.0\,\text{cm}$$

(f) **Angular field of view:** Drawn not to scale,



$$\alpha = \frac{1}{40} = 0.0125\,\text{rad}$$
$$\alpha = 1.43°$$



$$\alpha' = \frac{1}{10 - 8.43} = 0.0875\,\text{rad}$$
$$\alpha' = 5.01°$$

3-16. Using Eq. (3-35),

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{L}{f_1 f_2} = \frac{1}{20} + \frac{1}{-8} - \frac{15}{(20)\,(-8)} \Rightarrow f = 53.3\,\text{cm}$$

If the film plane is in the image position for an object at infinity, its position can be found as follows. The image formed by the first lens of the object at infinity falls at the focal point of the first lens which is 5 cm past the second lens. This intermediate image serves as the object for the second lens. That is $s_2 = -5\,\text{cm}$. The final image position relative to the second lens can be found using the thin lens formula,

$$\frac{1}{s_2} + \frac{1}{s_2'} = \frac{1}{f_2} \Rightarrow \frac{1}{-5} + \frac{1}{s_2} = \frac{1}{-8} \Rightarrow s_2' = 13.33\,\text{cm}$$

That is, the film should be placed 13.33 cm past the negative lens of the combination. The image size h' of a distant object subtending an angle of 2° at the camera is found using the effective focal length of the lens.

$$\tan 2° = \frac{|h'|}{f} = \frac{h'}{53.33\,\text{cm}} \Rightarrow 1.86\,\text{cm}$$

Alternately one may find the image height as,

$$h' = m_1 m_2 h = -\frac{s_1'}{s_1}\left(-\frac{s_2'}{s_2}\right) = \frac{20\,\text{cm}}{s_1}\frac{13.33}{-5}h = (53.33\,\text{cm})\frac{h}{s_1} = (53.33\,\text{cm})\tan 2° = 1.86\,\text{cm}$$

3-22. The focal lengths of the two lenses are, $f_1 = f_2 = 3\,\text{cm}$ and they are separated by $L = 2.8\,\text{cm}$.

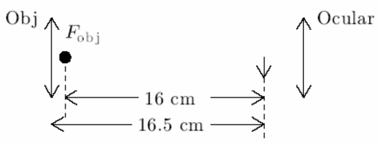
(a) The equivalent focal length is,

$$\frac{1}{f_{\text{eq}}} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{L}{f_1 f_2} = \frac{1}{3} + \frac{1}{3} - \frac{2.8}{9} \qquad f_{\text{eq}} = 2.8125\,\text{cm}$$

(b) The magnification is,

$$M = \frac{25}{f_{\text{eq}}} + 1 = \frac{25}{2.8125} + 1 = 9.9 \approx 10\times$$

3-23. (a) The system is sketched below.



$$M = m_{\text{obj}} \times M_{\text{oc}} = \frac{s'}{s} \times 10 \text{ where } s' = f + 16 = 16.5\,\text{cm}$$

$$s = \frac{s' f}{s' - f} = \frac{(16.5)\,(0.5)}{16.5 - 0.5} = \frac{8.25}{16}$$

$$|m| = \frac{s'}{s} = \frac{(16.5)\,(16)}{8.25} = 32,\; M = (32)\,(16) = 320\times$$

(b) $s = \dfrac{8.25}{16} = 0.516\,\text{cm}$

3-26. $\dfrac{1}{30} + \dfrac{1}{s'} = \dfrac{1}{0.2} \Rightarrow s' = 0.2013\,\text{m},\; m = \dfrac{s'}{s} = \dfrac{0.2013}{30} = 0.006711,\; h' = m\,h \Rightarrow h = \dfrac{h'}{m} = \dfrac{0.001}{0.006711} = 0.149\,\text{m} = 14.9\,\text{cm}$

3-31. See Figure 3-39 in the text. In the normal position, $s_{\text{oc}} = f_{\text{oc}} = 5\,\text{cm}$. But when $s'' = 25\,\text{cm}$,

$$s_{\text{oc}} = \frac{s''\,f_{\text{oc}}}{s'' - f_{\text{oc}}} = \frac{25 \cdot 5}{25 - 5}\,\text{cm} = 6.25\,\text{cm}$$

Thus, the ocular must be moved further from the objective by an amount $6.25\,\text{cm} - 5\,\text{cm} = 1.25\,\text{cm}$ in order to produce a real image on the screen 25 cm away.

3-32. Using Eq. (3-35) and the given information,

$$\frac{1}{f_{\text{eq}}} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{L}{f_1\,f_2} = \frac{1}{2\,\text{cm}} + \frac{1}{2\,\text{cm}} - \frac{2}{4\,\text{cm}} \Rightarrow f_{\text{eq}} = 2\,\text{cm}$$

(a) $M_\infty = \frac{25}{f_{\text{eq}}} = \frac{25}{2} = 12.5\,\times$ (b) $M_{\text{tel}} = \frac{f_{\text{obj}}}{f_{\text{oc}}} = \frac{30}{2} = 15\,\times$

(c) The exit pupil is the image of the AS or objective lens, formed by the ocular. Since the ocular consists of 2 lenses, one should argue as follows: To produce parallel rays *leaving* the ocular (image at ∞), the field lens must be at the focal point of the objective, 30 cm from the objective. Then the image there is 2 cm from the eye lens ($1/2 + 1/s' = 1/2$, $s' \to \infty$). Given this separation, the image of the objective in the *field* lens is

$$s_1' = \frac{30(2)}{30 - 2}\,\text{cm} = \frac{15}{7}\,\text{cm}$$

that is 1/7 cm *beyond* the eye lens. The image formed by the eye lens is then at,

$$s_2' = \frac{(-1/7)\,(2)}{-1/7 - 2}\,\text{cm} = 0.133\,\text{cm}$$

Thus the exit pupil falls 0.133 cm from the eye lens.
Using $M = D_{\text{obj}}/D_{E_xP}$ we also have $D_{E_xP} = \frac{4.5}{15}\,\text{cm} = 3\text{mm}$.

(d) $\theta = D_{\text{FL}}/L = 2/30 = (1/15)\,\text{rad} = 3.8°$.