

4-11. Generally, $\mathbf{k} \cdot \mathbf{r} = k_x x + k_y y + k_z z$.

(a) For propagation along the z -axis, $k_x = k_y = 0$. So $\mathbf{k} \cdot \mathbf{r} = k_z z$, with $k_z = 2\pi/\lambda$. The waveform can then be written as,

$$\psi = A \sin(\mathbf{k} \cdot \mathbf{r} - \omega t) = A \sin(k_z z - \omega t) = A \sin\left[\frac{2\pi}{\lambda}(z - vt)\right] = A \sin 2\pi(z/\lambda - vt)$$

(b) In this case, $k_z = 0$ and $k_x = k_y = |k|/\sqrt{2} = \frac{1}{\sqrt{2}} \frac{2\pi}{\lambda}$. The general form of the wave is then,

$$\psi = A \sin(\mathbf{k} \cdot \mathbf{r} \pm \omega t) = A \sin(k_x x + k_y y \pm \omega t) = A \sin\left[\frac{2\pi}{\sqrt{2}\lambda}(x + y \pm vt)\right] = A \sin 2\pi\left(\frac{x}{\sqrt{2}\lambda} + \frac{y}{\sqrt{2}\lambda} \pm vt\right).$$

If one is interested in the wave displacement only on the line $x = y$,

$$\psi = A \sin 2\pi\left(\frac{2x}{\sqrt{2}\lambda} \pm vt\right).$$

(c) In this case $\mathbf{k} = \frac{k}{\sqrt{3}}(\hat{x} + \hat{y} + \hat{z})$ and $\mathbf{k} \cdot \mathbf{r} = \frac{k}{\sqrt{3}}(x + y + z)$, with $k = 2\pi/\lambda$. The waveform is then,

$$\psi = A \sin(\mathbf{k} \cdot \mathbf{r} \pm \omega t) = A \sin\left[\frac{k}{\sqrt{3}}(x + y + z) \pm \omega t\right] = A \sin\left[\frac{2\pi}{\sqrt{3}\lambda}(x + y + z \pm vt)\right]$$

4-12. Let $\tilde{z} = a + ib$ where a and b are real.

(a) $(\tilde{z} + \tilde{z}^*)/2 = (a + ib + a - ib)/2 = a = \text{Re}(\tilde{z})$

(b) $(\tilde{z} - \tilde{z}^*)/2i = (a + ib - a + ib)/2i = b = \text{Im}(\tilde{z})$

(c) Let $\tilde{z} = e^{i\theta} = \cos\theta + i\sin\theta$ and apply the result from (a): $\cos\theta = (e^{i\theta} + e^{-i\theta})/2$

(d) Let $\tilde{z} = e^{i\theta} = \cos\theta + i\sin\theta$ and apply the result from (b): $\sin\theta = (e^{i\theta} - e^{-i\theta})/2i$

4-13. (a) Note that $e^{i\pi/2} = \cos(\pi/2) + i\sin(\pi/2) = i$ so that

$$i A e^{i(\mathbf{k} \cdot \mathbf{r} - \omega t + \varphi_0)} = e^{i\pi/2} A e^{i(\mathbf{k} \cdot \mathbf{r} - \omega t + \varphi_0)} = A e^{i(\mathbf{k} \cdot \mathbf{r} - \omega t + \varphi_0 + \pi/2)}$$

(b) Similarly, $e^{i\pi} = \cos(\pi) + i\sin(\pi) = -1$ so that

$$-A e^{i(\mathbf{k} \cdot \mathbf{r} - \omega t + \varphi_0)} = e^{i\pi} A e^{i(\mathbf{k} \cdot \mathbf{r} - \omega t + \varphi_0)} = A e^{i(\mathbf{k} \cdot \mathbf{r} - \omega t + \varphi_0 + \pi)}$$

5-4. One could proceed directly by mimicking the development for the cosine waves leading to Eqs. (5-9) and (5-10). I choose to first convert the given fields to the cosine form and then using those equations. That is,

$$y_1 = 5 \sin(\omega t + \pi/2) = 5 \cos(\omega t) = 5 \cos(0 - \omega t)$$

$$y_2 = 7 \sin(\omega t + \pi/3) = 7 \cos(\omega t + \pi/3 - \pi/2) = 7 \cos(\omega t - \pi/6) = 7 \cos(\pi/6 - \omega t)$$

Then using Eqs. (5-9) and (5-10):

$$y_0 = \sqrt{5^2 + 7^2 + 2 \cdot 5 \cdot 7 \cos(\pi/6)} = 11.6$$

$$\tan \alpha = \frac{0 + 7 \sin(\pi/6)}{5 + 7 \cos(\pi/6)} = \alpha = 0.098 \pi$$

$$y = 11.6 \cos(0.098 \pi - \omega t) = 11.6 \cos(\omega t - 0.098 \pi) = 11.6 \sin(\omega t - 0.098 \pi + \pi/2)$$

$$y = 11.6 \sin(\omega t + 0.402 \pi)$$

$$7-4. \text{ (a) visibility} = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}} = \frac{I_1 + I_2 + 2\sqrt{I_1 I_2} - (I_1 + I_2 - 2\sqrt{I_1 I_2})}{I_1 + I_2 + 2\sqrt{I_1 I_2} + I_1 + I_2 - 2\sqrt{I_1 I_2}} = \frac{4\sqrt{I_1 I_2}}{2(I_1 + I_2)} = \frac{2\sqrt{I_1 I_2}}{I_1 + I_2}$$

So if $I_1 = N I_2$,

$$\text{visibility} \equiv \mathcal{V} = \frac{2\sqrt{N} I_2}{(N+1) I_2} = \frac{2\sqrt{N}}{N+1}$$

(b) Solving the above relation for N ,

$$N = \left[\frac{\sqrt{1 - \mathcal{V}^2} + 1}{\mathcal{V}} \right]$$

For $\mathcal{V} = 0.96$, $N = 1.78$. For $\mathcal{V} = 0.90$, $N = 2.55$. For $\mathcal{V} = 0.8$, $N = 4$. For $\mathcal{V} = 0.5$, $N = 13.9$

7-11. See Figure 7-26 in the text. Constructive interference occurs at screen locations,

$$y = \frac{m \lambda (d + L)}{a} = \frac{m \lambda (d + 2d)}{a} = \frac{m \lambda (3d)}{2d \alpha (n - 1)}$$

$$\alpha = \frac{3}{2} \frac{\lambda}{n - 1} \frac{\Delta m}{\Delta \lambda} = \frac{3}{2} \frac{589.3 \times 10^{-9}}{1.5 - 1} \frac{1}{3 \times 10^{-4}} = 0.005893 \text{ rad} = 0.3376^\circ = 20.3'$$

7-14. The condition for a minimum in the reflecting light is

$$\Delta_p + \Delta_r = 2 n t = (m + 1/2) \lambda$$

For the two wavelengths then

$$2 n t = (m_1 + 1/2) \lambda_1 = (m_2 + 1/2) \lambda_2$$

$$\frac{m_1 + 1/2}{m_2 + 1/2} = \frac{\lambda_2}{\lambda_1} = \frac{675}{525} = 1.2875$$

By trial and error, this relation is satisfied with $m_1 = 4$ and $m_2 = 3$. Then,

$$t = \frac{(m_1 + 1/2) \lambda_1}{2 n} = \frac{(4.5) (525 \text{ nm})}{2 (1.30)} = 908.65 \text{ nm}$$

7-19. See Figure 7-28 that accompanies this problem in the text. The dark lines are wavelengths for which destructive interference occurs on reflection. These satisfy,

$$m \lambda = 2 n t \cos \theta_t$$

Here the film is the air layer. The angle in the air film is the same as the incident angle of 45° since the angle that the ray emerges from the top glass slide into the air film is the same as the angle at which the ray entered the glass slide from the top ambient air. Then,

$$\lambda_m = \frac{2 n t \cos (45^\circ)}{m} = \frac{2 \cdot 1 \cdot (10^4 \text{ nm}) 0.707}{m} = \frac{14,142 \text{ nm}}{m}$$

There are 15 orders in the visible with m ranging from 21 to 35. The dark lines occur at: $\lambda_{21} = 673.4 \text{ nm}$, $\lambda_{22} = 642.8 \text{ nm}$..., $\lambda_{35} = 404.1 \text{ nm}$.

7-20. Refer to Figure 7-15b in the body of the text. Constructive interference will occur for

$$(m + 1/2) \lambda = 2 t$$

Here, t is the thickness of the air wedge at a given horizontal position. The 40th bright fringe corresponds to $m = 39$ since the first bright fringe occurs for $m = 0$. Then, for the 40th fringe,

$$39.5 \lambda = 2 t \Rightarrow t = \frac{(39.5)(589 \times 10^{-7})}{2} \text{ cm} = 1.16 \times 10^{-3} \text{ cm}$$

7-23. Refer to Figure 7-17 in the text. Using Eq. (7-38),

$$2t_m + \Delta_r = 2t_m + \lambda/2 = m\lambda \Rightarrow t_m = (m - 1/2)\lambda/2$$

The 10th bright fringe occurs for $m = 10$, so that

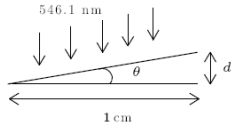
$$t_{10} = 9.5\lambda/2 = (9.5/2)(546.1 \times 10^{-6} \text{ mm}) = 2.59 \times 10^{-3} \text{ mm}$$

Using Eq. (7-39), the radius of curvature R of the lens surface can be found:

$$R = \frac{r_{10}^2 + t_{10}^2}{2t_{10}} = \frac{(7.89/2)^2 + (2.59 \times 10^{-3})^2}{2(2.59 \times 10^{-3})} = 3000 \text{ mm} = 3 \text{ m}$$

8-1. $\lambda = (2\Delta d/\Delta m) = (2 \cdot 0.014 \text{ cm})/523 = 4.6 \times 10^{-5} \text{ cm} = 436 \text{ nm}$

8-2. Straight fringes are due to a wedge between one mirror and the image of the other ($M2$ and $M1'$ in Figure 8-1). Interference then occurs as from reflection by an air wedge.



There are 12 fringes/cm so there are 11 fringe spaces/cm.

$$m\lambda = 2d \Rightarrow d = m\lambda/2 = (11/2)\lambda = 5.5\lambda.$$

$$\theta = t/(1 \text{ cm}) = 5.5(5.461 \times 10^{-6})/1 = 3.00 \times 10^{-4} \text{ cm} = 0.0172^\circ = 1'2''$$

8-3. The optical path difference due to the insertion of the thin sheet of width t and index n is

$$\Delta = m\lambda = 2(nt - t) = 2t(n - 1)$$

$$t = \frac{m\lambda}{2(n - 1)} = \frac{35(589 \times 10^{-9} \text{ m})}{2(1.434 - 1)} = 23.75 \times 10^{-6} \text{ m} = 23.75 \mu\text{m}$$