

23-2. The critical angle is,

$$\sin \theta_c = \frac{n_1}{n_2} = \frac{1}{n} \Rightarrow n = \frac{1}{\sin \theta_c} = \frac{1}{\sin(33^\circ 33')} = 1.8094$$

External Reflection:

$$\tan \theta_p = \frac{n_2}{n_1} = n \Rightarrow \theta_p = \tan^{-1}(1.8094) = 61^\circ 4'$$

Internal Reflection:

$$\tan \theta'_p = \frac{n_1}{n_2} = \frac{1}{n} \Rightarrow \theta'_p = \tan^{-1}\left(\frac{1}{1.8094}\right) = 28^\circ 56'$$

23-5. In each of the 4-equations the desired form results from introducing θ_t through the relation $n = \sin \theta / \sin \theta_t$. For example:

$$r_{\text{TE}} = \frac{\cos \theta - \sqrt{n^2 - \sin^2 \theta}}{\cos \theta + \sqrt{n^2 - \sin^2 \theta}} = \frac{\cos \theta - \sin \theta \sqrt{1/\sin^2 \theta_t - 1}}{\cos \theta + \sin \theta \sqrt{1/\sin^2 \theta_t - 1}} = \frac{\cos \theta - \sin \theta \sqrt{(\cos^2 \theta_t)/\sin^2 \theta_t}}{\cos \theta + \sin \theta \sqrt{(\cos^2 \theta_t)/\sin^2 \theta_t}}$$

$$r_{\text{TE}} = \frac{\cos \theta \sin \theta_t - \sin \theta \cos \theta_t}{\cos \theta \sin \theta_t + \sin \theta \cos \theta_t} = -\frac{\sin(\theta - \theta_t)}{\sin(\theta + \theta_t)}$$

The other 4 relations are similarly shown to be true. Proceeding by first noting from above that,

$$\sqrt{n^2 - \sin^2 \theta} = \sin \theta \cos \theta_t / \sin \theta_t$$

Then,

$$r_{\text{TM}} = \frac{-n^2 \cos \theta + \sqrt{n^2 - \sin^2 \theta}}{n^2 \cos \theta + \sqrt{n^2 - \sin^2 \theta}} = \frac{-\frac{\sin^2 \theta \cos \theta}{\sin^2 \theta_t} + \frac{\sin \theta \cos \theta_t}{\sin \theta_t}}{\frac{\sin^2 \theta \cos \theta}{\sin^2 \theta_t} + \frac{\sin \theta \cos \theta_t}{\sin \theta_t}}$$

$$r_{\text{TM}} = \frac{-\sin \theta \cos \theta + \cos \theta_t \sin \theta_t}{\sin \theta \cos \theta + \cos \theta_t \sin \theta_t} = \frac{-\sin \theta \cos \theta (\sin^2 \theta_t + \cos^2 \theta_t) + \cos \theta_t \sin \theta_t (\sin^2 \theta + \cos^2 \theta)}{\sin \theta \cos \theta (\sin^2 \theta_t + \cos^2 \theta_t) + \cos \theta_t \sin \theta_t (\sin^2 \theta + \cos^2 \theta)}$$

$$r_{\text{TM}} = \frac{-\sin \theta \cos \theta \sin^2 \theta_t - \sin \theta \cos \theta \cos^2 \theta_t + \cos \theta_t \sin \theta_t \sin^2 \theta + \cos \theta_t \sin \theta_t \cos^2 \theta}{\sin \theta \cos \theta \sin^2 \theta_t + \sin \theta \cos \theta \cos^2 \theta_t + \cos \theta_t \sin \theta_t \sin^2 \theta + \cos \theta_t \sin \theta_t \cos^2 \theta}$$

$$r_{\text{TM}} = \frac{(\sin \theta \cos \theta_t - \cos \theta \sin \theta_t)(\sin \theta \sin \theta_t - \cos \theta \cos \theta_t)}{(\sin \theta \cos \theta_t + \cos \theta \sin \theta_t)(\sin \theta \sin \theta_t + \cos \theta \cos \theta_t)} = -\frac{\sin(\theta - \theta_t)\cos(\theta + \theta_t)}{\sin(\theta + \theta_t)\cos(\theta - \theta_t)}$$

$$r_{\text{TM}} = -\frac{\tan(\theta - \theta_t)}{\tan(\theta + \theta_t)}$$

and,

$$t_{\text{TE}} = \frac{2 \cos \theta}{\cos \theta + \sqrt{n^2 - \sin^2 \theta}} = \frac{2 \cos \theta}{\cos \theta + \sin \theta \cos \theta_t / \sin \theta_t} = \frac{2 \cos \theta \sin \theta_t}{\cos \theta \sin \theta_t + \sin \theta \cos \theta_t} = \frac{2 \cos \theta \sin \theta_t}{\sin(\theta + \theta_t)}$$

Finally,

$$t_{\text{TM}} = \frac{2 n \cos \theta}{n^2 \cos^2 \theta + \sqrt{n^2 - \sin^2 \theta}} = \frac{2 \cos \theta \sin \theta / \sin \theta_t}{\frac{\sin^2 \theta \cos \theta}{\sin^2 \theta_t} + \frac{\sin \theta \cos \theta_t}{\sin \theta_t}} = \frac{2 \cos \theta \sin \theta_t}{\cos \theta \sin \theta_t + \sin \theta_t \cos \theta_t}$$

The denominator in this expression is the same as the one that appeared in the work leading to the final form of r_{TM} . Using the result derived there,

$$t_{\text{TM}} = \frac{2 \cos \theta \sin \theta_t}{\sin(\theta + \theta_t)\cos(\theta - \theta_t)}$$

23-12. For diamond $n = 2.42$.

(a) **External:**

TM: $\theta_p = \tan^{-1}(n) = \tan^{-1}(2.42) = 67.55^\circ$, no critical angle.

TE: No Brewster angle, no critical angle.

(b) **Internal:**

TM: $\theta_c = \sin^{-1}(1/2.42) = 0.426 = 24.41^\circ$, $\theta'_p = \tan^{-1}(1/2.42) = 0.392 = 22.46^\circ$.

TE: $\theta_c = \sin^{-1}(1/2.42) = 0.426 = 24.41^\circ$, no Brewster angle.

23-13. The reflectance and transmission are given by

$$R_{TE} = \left(\frac{\cos \theta - \sqrt{n^2 - \sin^2 \theta}}{\cos \theta + \sqrt{n^2 - \sin^2 \theta}} \right)^2 = \left(\frac{\cos(50^\circ) - \sqrt{1.6^2 - \sin^2(50^\circ)}}{\cos(50^\circ) + \sqrt{1.6^2 - \sin^2(50^\circ)}} \right)^2 = 0.1385 = 13.85\%,$$

$$T_{TE} = 1 - R_{TE} = 86.15\%$$

$$R_{TM} = \left(\frac{-n^2 \cos \theta + \sqrt{n^2 - \sin^2 \theta}}{n^2 \cos \theta + \sqrt{n^2 - \sin^2 \theta}} \right)^2 = \left(\frac{-1.6^2 \cos(50^\circ) + \sqrt{1.6^2 - \sin^2(50^\circ)}}{1.6^2 \cos(50^\circ) + \sqrt{1.6^2 - \sin^2(50^\circ)}} \right)^2 = 0.00623 = 0.623\%$$

$$T_{TM} = 1 - R_{TM} = 99.38\%$$

23-15. (a) The critical angle is $\theta_c = \sin^{-1}(n_2/n_1) = \sin^{-1}(1/1.458) = 43.3^\circ$

The polarizing angle for external reflection is $\theta_p = \tan^{-1}(n_2/n_1) = \tan^{-1}(1.458/1) = 55.6^\circ$

The polarizing angle for internal reflection is $\theta'_p = \tan^{-1}(1/1.458) = 34.4^\circ$

(b) At normal incidence: $R_{TE} = r_{TE}^2 = \left(\frac{1-n}{1+n} \right)^2 = \left(\frac{1-1.1458^2}{1+1.1458^2} \right)^2 = 0.0347, T_{TE} = 1 - R_{TE} = 0.9653$

At $\theta = 45^\circ$: $R_{TE} = \left(\frac{\cos \theta - \sqrt{n^2 - \sin^2 \theta}}{\cos \theta + \sqrt{n^2 - \sin^2 \theta}} \right)^2 = \left(\frac{\cos 45^\circ - \sqrt{(1.458)^2 - \sin^2 45^\circ}}{\cos 45^\circ + \sqrt{(1.458)^2 - \sin^2 45^\circ}} \right)^2 = 0.0821$

$T_{TE} = 1 - R_{TE} = 0.9179.$

(c) At normal incidence: $R_{TM} = R_{TE} = 0.0347, T_{TE} = 0.9653$

At $\theta = 45^\circ$: $R_{TM} = \left(\frac{-n^2 \cos \theta + \sqrt{n^2 - \sin^2 \theta}}{n^2 \cos \theta + \sqrt{n^2 - \sin^2 \theta}} \right)^2 = \left(\frac{-(1.458)^2 \cos 45^\circ + \sqrt{1.458^2 - \sin^2 45^\circ}}{(1.458)^2 \cos 45^\circ + \sqrt{1.458^2 - \sin^2 45^\circ}} \right)^2$

$R_{TM} = 0.0067, T_{TM} = 0.9933$

(d) Use Eqs. (23-28) and (23-29):

For incident angles less than θ'_p , $\phi_{TM} = \phi_{TE} = 0$. So for $\theta = 0^\circ$ and 20° , $\phi_{TM} - \phi_{TE} = 0$.

For $\theta = 40^\circ$ which lies between θ'_p and θ_c , $\phi_{TM} - \phi_{TE} = \pi - 0 = \pi$.

For $\theta > \theta_c$: with $n = 1/1.458$,

$$\phi_{TM} - \phi_{TE} = -2 \tan^{-1} \left(\frac{\sqrt{\sin^2 \theta - n^2}}{n^2 \cos \theta} \right) + \pi + 2 \tan^{-1} \left(\frac{\sqrt{\sin^2 \theta - n^2}}{\cos \theta} \right)$$

$$\theta = 50^\circ: \quad \phi_{TM} - \phi_{TE} = 2.43 \text{ rad} = 139^\circ$$

$$\theta = 70^\circ: \quad \phi_{TM} - \phi_{TE} = 2.65 \text{ rad} = 152^\circ$$

$$\theta = 90^\circ: \quad \phi_{TM} - \phi_{TE} = \pi = 180^\circ$$

23-16. (a) By trial and error, $\theta = \theta_{\text{rhomb}} = 59.857^\circ$, so that with $n = 1/1.65$

$$2(\phi_{TE} - \phi_{TM}) = -2 \tan^{-1} \left(\frac{\sqrt{\sin^2 \theta - n^2}}{\cos \theta} \right) + 2 \tan^{-1} \left(\frac{\sqrt{\sin^2 \theta - n^2}}{n^2 \cos \theta} \right) - \pi = -1.5 \pi \Rightarrow \pi/2$$

(b) If $\theta = (1 + 0.05) 59.857^\circ = 62.850^\circ$:

$$2(\phi_{TE} - \phi_{TM}) = -2 \tan^{-1} \left(\frac{\sqrt{\sin^2 \theta - n^2}}{\cos \theta} \right) + 2 \tan^{-1} \left(\frac{\sqrt{\sin^2 \theta - n^2}}{n^2 \cos \theta} \right) - \pi = -1.543 \pi \Rightarrow 0.457 \pi$$

If $\theta = (1 - 0.05) 59.857^\circ = 56.864^\circ$:

$$2(\phi_{TE} - \phi_{TM}) = -2 \tan^{-1} \left(\frac{\sqrt{\sin^2 \theta - n^2}}{\cos \theta} \right) + 2 \tan^{-1} \left(\frac{\sqrt{\sin^2 \theta - n^2}}{n^2 \cos \theta} \right) - \pi = -1.461 \pi \Rightarrow 0.539 \pi$$

23-20. Consider Eq. (23-47)

$$1 = r^2 + n \left(\frac{\cos \theta_t}{\cos \theta} \right) t^2 \quad n = n_2/n_1$$

(a) **External Reflection:** $n > 1$; $\theta_t < \theta_i$; $\cos \theta_t > \cos \theta$. Thus, $1 - r^2 = n \left(\frac{\cos \theta_t}{\cos \theta} \right) t^2 > t^2 \Rightarrow t^2 < 1 - r^2$
 Since $r^2 < 1$, $t^2 < 1$.

Internal Reflection: $n < 1$; $\theta_t > \theta_i$; $\cos \theta_t < \cos \theta$. Thus, $1 - r^2 = n \left(\frac{\cos \theta_t}{\cos \theta} \right) t^2 < t^2 \Rightarrow t^2 > 1 - r^2$
 No upper limit is imposed.

(c) For the angle of incidence equal to the critical angle, $\sin \theta = \sin \theta_c = n$. Then,

$$t_{TE} = \frac{2 \cos \theta}{\cos \theta + \sqrt{n^2 - \sin^2 \theta}} = \frac{2 \cos \theta}{\cos \theta} = 2$$

$$t_{TM} = \frac{2 n \cos \theta}{n^2 \cos \theta + \sqrt{n^2 - \sin^2 \theta}} = \frac{2}{n}$$

(c)



