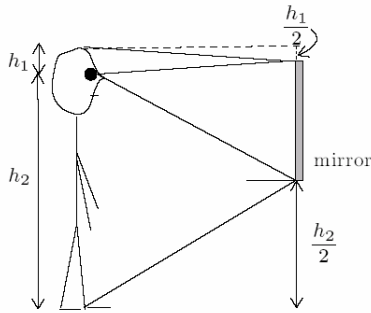**2-4.** See the figure below. Let the height of the person be $h = h_1 + h_2$.
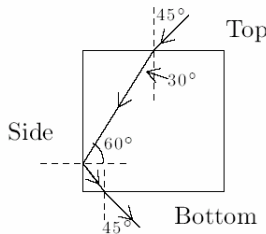

The person must be able to see the top of his head and the bottom of his feet. From the figure it is evident that the mirror height is:

$$h_{\mathrm{mirror}} = h - h_1/2 - h_2/2 = h/2$$

The mirror must be half the height of the person. So for a person of height six ft person, the mirror must be 3 ft high.

2-5. Refer to the figure.

At Top: $(1)\sin 45 = \sqrt{2}\sin\theta' \Rightarrow \theta' = 30$
At Side: $\sqrt{2}\sin 60° = (1)\sin\theta'$, $\sin\theta' = \sqrt{1.5} > 1$
Thus total internal reflection occurs.
At Bottom: reverse of Top: $\theta' = 45°$

2-6. The microscope first focuses on the scratch using direct rays. Then it focuses on the image I_2 formed in a two-step process: (1) reflection from the bottom to produce an intermediate image I_1 and (2) refraction through the top surface to produce an image I_2. Thus, I_1 is at $2t$ from top surface, and I_2 is at the apparent depth for I_1, serving as the object: $s' = \dfrac{2t}{n}$ or $n = \dfrac{2t}{s'} = \dfrac{3}{1.87} = 1.60$
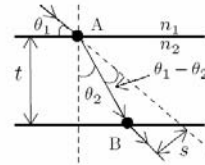
2-8. Referring to the figure one can see that,

$s = AB\sin(\theta_1 - \theta_2)$ and $AB = \dfrac{t}{\cos\theta_2}$. Therefore,

$s = \dfrac{t\sin(\theta_1 - \theta_2)}{\cos\theta_2}$. For $t = 3\,\mathrm{cm}$, $n_2 = 1.50$, $\theta_1 = 50°$,

Snell's law gives, $\sin\theta_2 = \dfrac{n_1}{n_2}\sin\theta_1 = \dfrac{1}{1.5}\sin 50°$.

Then, $\theta_2 = 30.71°$ and $s = \dfrac{3\sin(50° - 30.71°)}{\cos 30.71°} = 1.153\,\mathrm{cm}$.

2-9. Image of near end: $s = 60\,\mathrm{cm}$, $\dfrac{1}{60} = \dfrac{1}{s'} = \dfrac{1}{-40}$, $s' - 24\,\mathrm{cm}$

Image of far end: $s = 60 + 100\,\mathrm{cm}$, $\dfrac{1}{160} + \dfrac{1}{s'} = \dfrac{1}{-40}$, $s' = -32\,\mathrm{cm}$. So, $L' = \Delta s' = -24 - (-32) = 8\,\mathrm{cm}$

2-10. (a) See Figure 2-34 in the text. Image due to rays directly from bubble through plane interface:
$\dfrac{n_1}{s} + \dfrac{n_2}{s'} = 0$ or $\dfrac{1.5}{s} + \dfrac{1}{s'} = 0 \Rightarrow s' = -3.33\,\mathrm{cm}$.

(b) Image due to rays first reflected in spherical mirror, then refracted through plane interface:
reflection: $\dfrac{1}{2} + \dfrac{1}{s_1'} = -\dfrac{2}{R}$ and $\dfrac{1}{2.5} + \dfrac{1}{s_1'} = -\dfrac{2}{-7.5}$ $s_1' = -7.5\,\mathrm{cm}$

refraction: $\dfrac{n_1}{s} + \dfrac{n_2}{s_2'} = 0$ or $\dfrac{1.5}{15} + \dfrac{1}{s_2'} = 0$ $s_2' = -10\,\mathrm{cm}$
Thus the images are at 3.33 cm and 10 cm behind the interface.

2-32. Refer to Figure 2-39 in the text.

(a) Let the angle with the normal to the interface in each region of index of refraction n_i be θ_i. Then applying Snell's law sequentially at each interface leads to,

$$n_0 \sin\theta_0 = n_1 \sin\theta_1 = n_2 \sin\theta_2 \ldots = n_i \sin\theta_i \ldots = n_f \sin\theta_f$$

That is,

$$n_0 \sin\theta_0 = n_f \sin\theta_f$$

(b) In each medium the lateral displacement is $t_i \tan\theta_i$. The total lateral displacement y due to N media can be written as,

$$y = \sum_{i=1}^{n} t_i \tan\theta_i$$

where $\sin\theta_i = (n_0/n_i) \sin\theta_0$.

2-34. The focal length is the image position for incident parallel light rays (object at ∞). In all cases the following relation is to be used

$$\frac{n_1}{s} + \frac{n_2}{s'} = \frac{n_2 - n_1}{R} \Rightarrow \frac{n_1}{\infty} + \frac{n_2}{f} = \frac{n_2 - n_1}{R} \Rightarrow \frac{n_2}{f} = \frac{n_2 - n_1}{R}$$

For the situation in which the center of curvature in medium with $n = 4/3$:

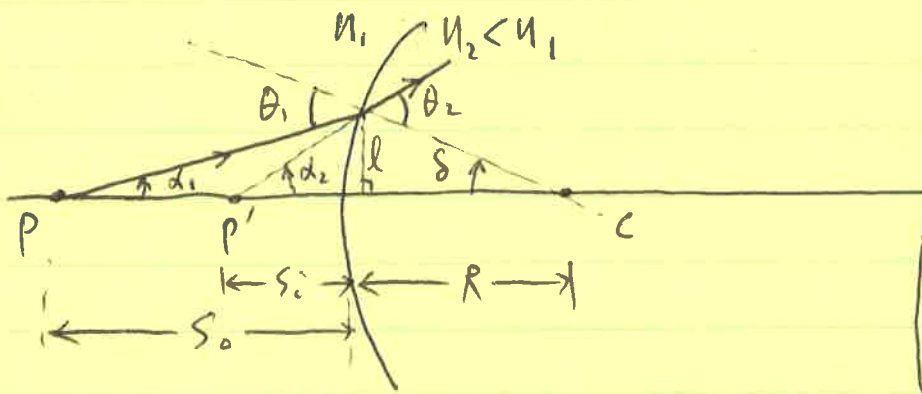For light incident from the medium of index 1: $\dfrac{4/3}{f} = \dfrac{4/3 - 1}{10}$ or $f = +40\,\text{cm}$

For light incident from the medium of index 4/3: $\dfrac{1}{f} = \dfrac{1 - 4/3}{-10}$ or $f = +30\,\text{cm}$

For the situation in which the center of curvature is in the medium with $n = 1$,

For light incident from the medium of index 4/3: $\dfrac{4/3}{\infty} + \dfrac{1}{f} = \dfrac{1 - 4/3}{10}$ or $f = -30\,\text{cm}$

For light incident from the medium of index 1: $\dfrac{1}{\infty} + \dfrac{4/3}{f} = \dfrac{4/3 - 1}{-10}$ or $f = -40\,\text{cm}$

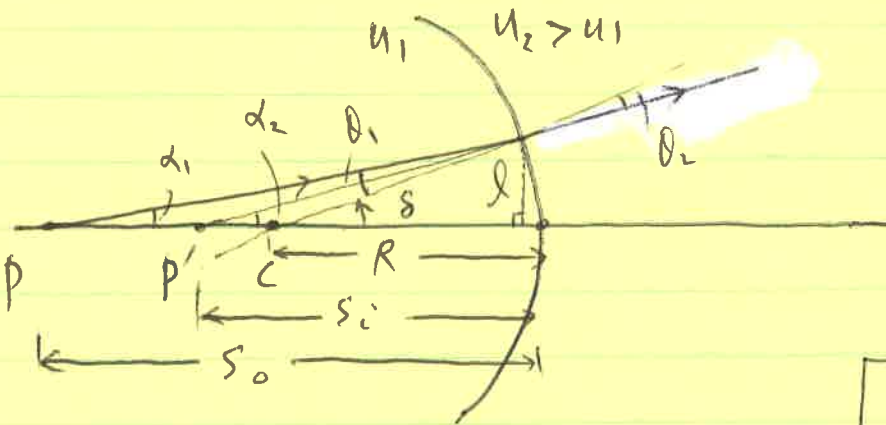# Refraction of a spherical surface under other circumstances

$$n_1 \theta_1 \simeq n_2 \theta_2$$

$$\theta_2 = |\alpha_2| + |\delta| = \frac{\ell}{s_i} + \frac{\ell}{R}$$

$$\boxed{\frac{n_1}{s_0} + \frac{n_2}{-s_i} = \frac{n_2 - n_1}{R}}$$
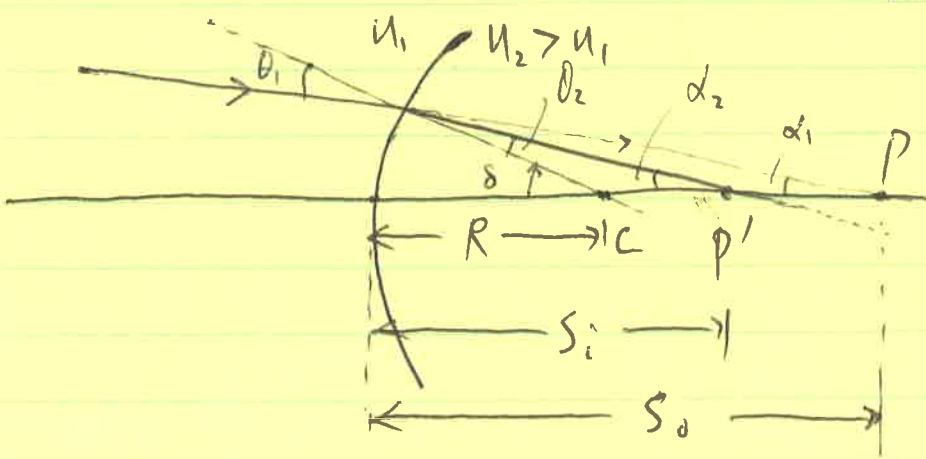
(virtual image)

$$n_1 \theta_1 \simeq n_2 \theta_2$$

$$\theta_1 = |\delta| - |\alpha_1| = \frac{\ell}{R} - \frac{\ell}{s_0}$$

$$\theta_2 = |\delta| - |\alpha_2| = \frac{\ell}{R} - \frac{\ell}{s_i}$$

$$\boxed{\frac{n_1}{s_0} + \frac{n_2}{-s_i} = \frac{n_2 - n_1}{-R}}$$

(virtual image)

$$n_1 \theta_1 \cong n_2 \theta_2$$

$$\theta_1 = |\delta| - |\alpha_1| = \frac{\ell}{R} - \frac{\ell}{s_0}$$

$$\theta_2 = |\delta| - |\alpha_2| = \frac{\ell}{R} - \frac{\ell}{s_i}$$

$$\boxed{\frac{n_1}{-s_0} + \frac{n_2}{s_i} = \frac{n_2 - n_1}{R}}$$