14-2. In general $\tilde{\boldsymbol{E}} = \left[E_{0x}e^{i\varphi_x}\hat{x}\hat{\boldsymbol{x}} + E_{0y}\,e^{i\varphi_y}\,\hat{\boldsymbol{y}}\right]e^{i(kz-\omega t)} = \begin{bmatrix} E_{0x}e^{i\varphi_x} \\ E_{0y}\,e^{i\varphi_y} \end{bmatrix} e^{i(kz-\omega t)} = \tilde{\boldsymbol{E}}_0$

(a) $\tilde{\boldsymbol{E}} = [E_0\hat{\boldsymbol{x}} - E_0\hat{\boldsymbol{y}}]e^{i(kz-\omega t)} \Rightarrow \tilde{\boldsymbol{E}}_0 = E_0\begin{bmatrix} 1 \\ -1 \end{bmatrix} \Rightarrow \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -1 \end{bmatrix}$. Linearly polarized at $-45°$.

(b) $\tilde{\boldsymbol{E}} = [E_0\hat{\boldsymbol{x}} + E_0\hat{\boldsymbol{y}}]e^{i(kz-\omega t)} \Rightarrow \tilde{\boldsymbol{E}}_0 = E_0\begin{bmatrix} 1 \\ 1 \end{bmatrix} \Rightarrow \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$. Linearly polarized at $45°$.

(c) $\tilde{\boldsymbol{E}} = \left[E_0\hat{\boldsymbol{x}} + E_0 e^{-i\pi/4}\hat{\boldsymbol{y}}\right]e^{i(kz-\omega t)} \Rightarrow \tilde{\boldsymbol{E}}_0 = E_0\begin{bmatrix} 1 \\ e^{-i\pi/4} \end{bmatrix} \Rightarrow \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ \frac{1}{\sqrt{2}}(1-i) \end{bmatrix}$. Then,

$$\tan 2\alpha = \frac{2E_{0x}E_{0y}\cos\varepsilon}{E_{0x}^2 - E_{0y}^2} \to \infty \Rightarrow 2\,\alpha = 90°,\, \alpha = 45°$$

Right elliptically polarized at $45°$.

(d) $\tilde{\boldsymbol{E}} = \left[E_0\hat{\boldsymbol{x}} + E_0 e^{i\pi/2}\hat{\boldsymbol{y}}\right]e^{i(kz-\omega t)} \Rightarrow \tilde{\boldsymbol{E}}_0 = E_0\begin{bmatrix} 1 \\ e^{i\pi/2} \end{bmatrix} \Rightarrow \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ i \end{bmatrix}$. Left-circularly polarized.

14-3. In general $\tilde{\boldsymbol{E}} = \left[E_{0x}e^{i\varphi_x}\hat{\boldsymbol{x}} + E_{0y}\,e^{i\varphi_y}\,\hat{y}\hat{\boldsymbol{y}}\right]e^{i(kz-\omega t)} = \begin{bmatrix} E_{0x}e^{i\varphi_x} \\ E_{0y}\,e^{i\varphi_y} \end{bmatrix} e^{i(kz-\omega t)} = \tilde{\boldsymbol{E}}_0 e^{i(kz-\omega t)}$

(a) $\tilde{\boldsymbol{E}} = (2\,E_0\hat{\boldsymbol{x}} + 0\hat{\boldsymbol{y}})e^{i(kz-\omega t)} \Rightarrow \tilde{\boldsymbol{E}}_0 = 2\,E_0\begin{bmatrix} 1 \\ 0 \end{bmatrix}$. Linearly polarized along the x-direction. Velocity is in the $+z$-direction. The amplitude is $A = 2\,E_0\sqrt{1^2+0^2} = 2\,E_0$.

(b) $\tilde{\boldsymbol{E}} = (3E_0\hat{\boldsymbol{x}} + 4E_0\hat{\boldsymbol{y}})e^{i(kz-\omega t)} \Rightarrow \tilde{\boldsymbol{E}}_0 = E_0\begin{bmatrix} 3 \\ 4 \end{bmatrix}$. The polarization direction makes the angle α with the x-axis where,

$$\alpha = \tan^{-1}(4/3) = 53°$$

The wave is traveling in the $+z$-direction with amplitude $A = \sqrt{3^2+4^2}\,E_0 = 5\,E_0$.

(c) $\tilde{\boldsymbol{E}} = 5\,E_0(\hat{\boldsymbol{x}} - i\hat{\boldsymbol{y}})e^{i(kz+\omega t)} \Rightarrow \tilde{\boldsymbol{E}}_0 = 5E_0\begin{bmatrix} 1 \\ -i \end{bmatrix}$. The propagation is in the $+z$-direction. The wave is right-circularly polarized with amplitude. The electric field vector traces out a circle of radius $5\,E_0$.

14-4. (a) $\tilde{\boldsymbol{E}}_1 = E_{01}(\hat{\boldsymbol{x}} - \hat{\boldsymbol{y}})e^{i(kz-\omega t)} \Rightarrow \tilde{\boldsymbol{E}}_{01} = 2\,E_{01}\begin{bmatrix} 1 \\ -1 \end{bmatrix}$. This is linearly polarized along $-45°$

$\tilde{\boldsymbol{E}}_2 = E_{02}\left(\sqrt{3}\hat{\boldsymbol{x}} + \hat{\boldsymbol{y}}\right)e^{i(kz-\omega t)} \Rightarrow \tilde{\boldsymbol{E}}_{02} = E_{02}\begin{bmatrix} \sqrt{3} \\ 1 \end{bmatrix}$. This is linearly polarized along $\alpha = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = 30°$

The angle between the two is $75°$.

(b) $\tilde{\boldsymbol{E}}_{01}\cdot\tilde{\boldsymbol{E}}_{02} = E_{01}\,E_{02}(\sqrt{3}-1) = (\sqrt{2}\,E_{01})\,(\sqrt{3+1^2}\,E_{02})\cos(\theta_{12}) \Rightarrow \cos\theta_{12} = \frac{\sqrt{3}-1}{2\sqrt{2}} \Rightarrow \theta_{12} = 75°$

14-13. See Figure 14-13 that accompanies the statement of this problem in the text. Using the Jones formalism,

$$\underbrace{\begin{bmatrix} 1 & 0 \\ 0 & -i \end{bmatrix}}_{\substack{\text{QWP} \\ \text{SA, hor}}}\underbrace{\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}}_{\substack{\text{LP} \\ \text{at } 45°}}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -i & -i \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ -i \end{bmatrix} \quad \text{Right circular polarization}$$

14-14. Using the Jones formalism,

$$\underbrace{\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}}_{\substack{\text{LP} \\ \text{TA vert}}}\underbrace{\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}}_{\text{HWP}}\underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}}_{\substack{\text{LP} \\ \text{TA hor}}}\underbrace{\begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}}_{\substack{\text{QWP} \\ \text{FA hor}}}\underbrace{\begin{bmatrix} 1 \\ 1 \end{bmatrix}}_{\substack{\text{LP} \\ \text{at } 45°}} = \begin{bmatrix} 0 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \text{No light}$$

14-17. Consider the action of the matrix on a general Jones vector,

$$\begin{bmatrix} 1 & i \\ -i & 1 \end{bmatrix}\begin{bmatrix} A \\ B+iC \end{bmatrix} = \begin{bmatrix} A+iB-C \\ -iA+B+iC \end{bmatrix} = (A-C+iB)\begin{bmatrix} 1 \\ -i \end{bmatrix}: \text{ Right circular polarization}$$

For a left-circular polarizer try,

$$\begin{bmatrix} 1 & -i \\ i & 1 \end{bmatrix}\begin{bmatrix} A \\ B+iC \end{bmatrix} = \begin{bmatrix} A-iB+C \\ iA+B+iC \end{bmatrix} = (A+C-iB)\begin{bmatrix} 1 \\ +i \end{bmatrix}: \text{ Left circular polarization}$$

14-18. Note that,

$$\underbrace{\begin{bmatrix} 1 \\ \pm i \end{bmatrix}}_{\text{Circular}} + \underbrace{\begin{bmatrix} \cos\alpha \\ \sin\alpha \end{bmatrix}}_{\text{Linear}} = \begin{bmatrix} \cos\alpha+1 \\ \sin\alpha \pm i \end{bmatrix} = \underbrace{\begin{bmatrix} A \\ B \pm iC \end{bmatrix}}_{\text{Elliptical}}$$

6-(a) $\begin{pmatrix} -i \\ +i \end{pmatrix} = (-i)\begin{pmatrix} 1 \\ -1 \end{pmatrix}$, linearly polarized with $\alpha = -45°$.

6-(b) $\begin{pmatrix} -1+i \\ 1+i \end{pmatrix} = (-1+i)\begin{pmatrix} 1 \\ \frac{1+i}{-1+i} \end{pmatrix} = (-1+i)\begin{pmatrix} 1 \\ -i \end{pmatrix}$,

Right-circularly polarized light.

6-(c) $\begin{pmatrix} -1-i \\ 1+i \end{pmatrix} = (-1-i)\begin{pmatrix} 1 \\ -1 \end{pmatrix}$, linearly polarized with $\alpha = -45°$.

6-(d) $\begin{pmatrix} i \\ 4 \end{pmatrix} = i\begin{pmatrix} 1 \\ -4i \end{pmatrix}$, elliptically polarized

(1) Simply a linear polarizer with TA $\parallel$ y

$$M_{Linear}(TA \parallel \hat{y}) = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

or

$$M_{Linear}(\theta = 90^\circ) = \begin{pmatrix} \cos^2\theta & \cos\theta \sin\theta \\ \cos\theta \sin\theta & \sin^2\theta \end{pmatrix}$$

Then with $\tilde{E}_{inc} = \begin{pmatrix} \alpha \\ \beta e^{i\Delta\phi_0} \end{pmatrix}$,

$$\tilde{E}_{out} = M_{Linear}(TA \parallel \hat{y}) \tilde{E}_{inc}$$

$$= \beta e^{i\Delta\phi_0} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

The total energy is reduced from

$$(\alpha^2 + \beta^2) = 1$$

to $\beta^2 \leq \alpha^2 + \beta^2 = 1$ $\divideontimes$

(2) Starting with a linear polarizer with TA at 45° from x-axis,

$$M_{linear}(\theta=45°) = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

then the output from the linear polarizer

$$\tilde{E}_{out}^{(1)} = M_{lin}(\theta=45°)\,\tilde{E}_{in}$$

$$= \frac{1}{\sqrt{2}}\left(\alpha+\beta e^{i\Delta\phi_0}\right)\begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix}$$

Now for a left-circular polarized light, we need to add $+\pi/2$ phase to the y-component. We use a $\lambda/4$-plate so oriented that

$$M_{\lambda/4} = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/2} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$$

$$\tilde{E}_{out}^{(2)} = M_{\lambda/4}\,\tilde{E}_{out}^{(1)} = \frac{1}{\sqrt{2}}\left(\alpha+\beta e^{i\Delta\phi_0}\right)\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ i \end{pmatrix}$$

$$\left|\frac{1}{\sqrt{2}}\left(\alpha+\beta e^{i\Delta\phi_0}\right)\right|^2 \leq 1$$

$\cancel{\times}$

If you start from the answer of Part (1)

$$\tilde{E}^{(1)}_{out} = \beta e^{i\Delta\phi_0} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

If we have a $\lambda/4$ plate set at $\theta = 45°$, then

$$M_{\lambda/4}(\theta = +45°) = \frac{1}{2} \begin{pmatrix} 1+i & 1-i \\ 1-i & 1+i \end{pmatrix}$$

then

$$\tilde{E}^{(2)}_{out} = \beta e^{i\Delta\phi_0} \begin{pmatrix} 1-i \\ 1+i \end{pmatrix} \cdot \frac{1}{2}$$

$$= \beta e^{i\Delta\phi_0} e^{-i\pi/4} \begin{pmatrix} 1 \\ i \end{pmatrix} \frac{1}{\sqrt{2}}$$

is a left-circularly polarized

If we start from $\begin{pmatrix} \alpha \\ \beta e^{i\Delta\phi_0} \end{pmatrix}$ and $M_{LL} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

then $\tilde{E}^{(1)}_{out} = \alpha \begin{pmatrix} 1 \\ 0 \end{pmatrix}$. We use $\lambda/4$ at $-45°$:

$$M_{\lambda/4}(-45°) = \frac{1}{2} \begin{pmatrix} 1+i & i-1 \\ i-1 & 1+i \end{pmatrix} \Rightarrow \tilde{E}^{(2)}_{out} \sim \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \end{pmatrix}$$

(3) First use a wave-plate with arbitrarily adjustable phase

$$M_{wp}(\Delta) = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\Delta} \end{pmatrix}$$

and choose $\Delta = -\Delta\phi_0$, then

$$\tilde{E}_{out}^{(1)} = M_{wp}(\Delta = -\Delta\phi_0)\,\tilde{E}_{inc}$$

$$= \begin{pmatrix} \alpha \\ \beta \end{pmatrix}, \quad \tan\theta = \beta/\alpha$$

Then use a half-wave plate to rotate the linear polarized $\tilde{E}_{out}^{(1)}$ into

$$\tilde{E}_{out}^{(2)} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad \left(M_{\lambda/2} = \begin{pmatrix} -\sin\theta & \cos\theta \\ \cos\theta & +\sin\theta \end{pmatrix} \right)$$

Followed by a $\lambda/4$-plate at $+45^\circ$

$$M_{\lambda/4}(+45^\circ) = \begin{pmatrix} 1+i & 1-i \\ 1-i & 1+i \end{pmatrix} \frac{1}{2}$$

$$\Rightarrow \tilde{E}_{out}^{(3)} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \end{pmatrix}, \text{ no loss of power.}$$

15-2. The polarizing angle is given by the relation, $\tan\theta_p = \dfrac{n_2}{n_1}$. So for $n_{\text{air}} = 1$ and $n_{\text{diam}} = 2.42$

$$\text{Internal reflection:}\ \ \theta_p = \tan^{-1}\left(\frac{n_{\text{air}}}{n_{\text{diam}}}\right) = \tan^{-1}\left(\frac{1}{2.42}\right) = 22.5^\circ$$
$$\text{External reflection:}\ \ \theta_p = \tan^{-1}\left(\frac{n_{\text{diam}}}{n_{\text{air}}}\right) = \tan^{-1}\left(\frac{2.42}{1}\right) = 67.5^\circ$$

15-4. $\dfrac{\lambda}{2} = t\,(\Delta n)$ or $t = \dfrac{\lambda}{2\,\Delta n} = \dfrac{632.8\times 1 -^{-7}\ \text{cm}}{2\,(1.599 - 1.594)} = 0.063\text{mm}$

15-8. The angular offset between successive polarizers is $\theta = 90^\circ/N$. Applying Malus' law N times in succession,

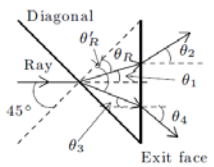
$$I_T = I_0\left(\cos^2\theta\right)^N = I_0\left[\cos\left(90^\circ/N\right)\right]^{2N} = 0.9\,I_0$$
$$\left[\cos\left(90^\circ/N\right)\right]^{2N} = 0.9$$

A numerical solution indicates that N is between 23 and 24. For $N = 24$, $I_T = 0.9022\,I_0$.

15-9. Using, $\lambda/4 = (\Delta n)\ t$,

$$t = \frac{\lambda}{4\,\Delta n} = \frac{589.3\times 10^{-6}\,\text{mm}}{4\,(1.5534 - 1.5443)} = 0.0162\text{mm}$$

15-10. See Figure 15-24 that accompanies the statement of this problem in the text. Also refer to the figure below for the labeling of the various angles:



At the diagonal interface:

$$E_p\,\text{component from } n_\parallel \text{ to } n_\perp\text{: } 1.4864\sin 45 = 1.6584\sin\theta_R\ \text{ or } \theta_R = 39.329^\circ$$
$$E_s\,\text{component from } n_\perp \text{ to } n_\parallel\text{: } 1.6584\sin 45 = 1.4864\sin\theta_R'\ \text{ or } \theta_R' = 52.086^\circ$$

On exit:

$$\text{Upper ray: } \theta_1 = 45 - \theta_R = 5.671^\circ;\ 1.6584\sin 5.671^\circ = (1)\sin\theta_2 \text{ or } \theta_2 = 9.432^\circ$$
$$\text{Lower ray: } \theta_3 = \theta_R' - 45 = 7.086^\circ;\ 1.4864\sin 7.086^\circ = (1)\sin\theta_4 \text{ or } \theta_4 = 10.566^\circ$$
$$\text{Deviation: } \theta_2 + \theta_4 = 9.432^\circ + 10.566^\circ = 19.997^\circ \approx 20^\circ$$

15-12. See Figure 15-25 that accompanies the statement of the problem in the text.
(a) The incident angle is the polarizing angle,

$$\tan\theta_p = \frac{n_2}{n_1} = \frac{1.33}{1} \Rightarrow \theta_p = 53.12^\circ$$

(b) The angle θ_R the refracted ray makes with the normal to the air/water interface is

$$\theta_R = \sin^{-1}\left(\frac{\sin\theta_P}{1.333}\right) = 36.877^\circ$$

The polarizing angle for the water/glass interface is, $\theta_P' = \tan^{-1}\left(\dfrac{1.50}{1.333}\right) = 48.37^\circ$

If the glass surface was parallel to the water surface the angle of incidence on the glass would be $\theta_R = 36.877^\circ$. However, for complete polarization off the glass, θ_P' must be 48.37°. Thus the glass must be tilted by $48.37^\circ - 36.88^\circ = 11.5^\circ$ relative to the water surface.