Solutions to Physics 108 MT (2021)

1. For the first thin lens with $f^{(1)}$, given an object distance $S_o^{(1)}$, the image distance $S_i^{(1)}$ is given by

$$\frac{1}{S_o^{(1)}} + \frac{1}{S_i^{(1)}} = \frac{1}{f^{(1)}} \quad \cdots (1)$$

For the second lens with $f^{(2)}$, the image after the first lens serves as the object. Since the two lenses are close to each other, the object distance for the second lens is

$$S_o^{(2)} = -S_i^{(1)}.$$

Thus the image distance after the second lens is given by

$$\frac{1}{S_o^{(2)}} + \frac{1}{S_i^{(2)}} = \frac{1}{f^{(2)}} \quad \cdots (2)$$

(1) + (2):

$$\frac{1}{S_o^{(1)}} + \frac{1}{S_i^{(2)}} = \frac{1}{f^{(1)}} + \frac{1}{f^{(2)}}$$

Since $S_o^{(1)}$ is the object distance for the lens pair, $S_i^{(2)}$ is the image distance for the pair, i.e., $S_o \equiv S_o^{(1)}$, $S_i \equiv S_i^{(2)}$

$$\frac{1}{S_o} + \frac{1}{S_i} = \frac{1}{f^{(1)}} + \frac{1}{f^{(2)}}$$

$$\therefore \frac{1}{f} \equiv \frac{1}{f^{(1)}} + \frac{1}{f^{(2)}}$$

✱

The lens pair acts like one thin lens:

$$\frac{1}{S_o} + \frac{1}{S_i} = \frac{1}{f}$$

$$\frac{1}{f} = \frac{1}{f^{(1)}} + \frac{1}{f^{(2)}}$$

✱

2-(a) For the front surface, $S_o^{(1)} = +3\,\text{cm}$, $R_1 = +3\,\text{cm}$, $n_1 = 1$, $n_2 = n_g = 1.5$. From

$$\frac{n_1}{S_o^{(1)}} + \frac{n_2}{S_i^{(1)}} = \frac{n_2 - n_1}{R_1}$$

$$\Rightarrow \frac{1}{3\,\text{cm}} + \frac{1.5}{S_i^{(1)}} = \frac{1.5 - 1}{3\,\text{cm}}$$

$$\Rightarrow S_i^{(1)} = -9\,\text{cm}$$

2-(b) For the rear surface, $S_o^{(2)} = 6\,\text{cm} - (-9\,\text{cm}) = 15\,\text{cm}$. $n_2 = n_g = 1.5$; $n_3 = 1.0$, $R_2 = -3\,\text{cm}$ From

$$\frac{n_2}{S_o^{(2)}} + \frac{n_3}{S_i^{(2)}} = \frac{n_3 - n_2}{R_2}$$

$$\Rightarrow \frac{1.5}{15\,\text{cm}} + \frac{1}{S_i^{(2)}} = \frac{1 - (1.5)}{-3\,\text{cm}}$$

$$\Rightarrow S_i^{(2)} = +15\,\text{cm}.$$

2-(c) From the magnifications for the two refracting surfaces,

$$m_1 = -\frac{n_1 \cdot s_i^{(1)}}{n_2 \cdot s_o^{(1)}} \;;\quad m_2 = -\frac{n_2 \cdot s_i^{(2)}}{n_3 \cdot s_o^{(2)}}$$

$$n_3 = n_1 = 1$$

$$m = \frac{y_i}{y_o} = m_1 \cdot m_2 = \frac{s_i^{(1)}}{s_o^{(1)}} \cdot \frac{s_i^{(2)}}{s_o^{(2)}}$$

$$= \frac{-9\,cm}{+3\,cm} \cdot \frac{+15\,cm}{+15\,cm} = -3$$

$$\therefore\; y_i = -3\,y_o = -3\,mm.$$

The image is 3mm in height. It is inverted.

3-(a) It is the angular size when it is placed at $d_o = 25\,cm$ in front of the eye.
Thus

$$\alpha_o = \frac{y_o}{d_o} = \frac{1\,mm}{25\,cm} = 4 \times 10^{-3}\ rad.$$

3-(b) With a concave mirror, $r = -4\,cm$. Since $s_o = 1.9\,cm$, from

$$\frac{1}{s_o} + \frac{1}{s_i} = -\frac{2}{r}$$

$$\Rightarrow s_i = (-)\cdot\frac{s_o \cdot r}{2s_o + r} = -38\,cm$$

From $$m = \frac{y_i}{y_o} = -\frac{s_i}{s_o} = (-)\cdot\frac{(-38\,cm)}{1.9\,cm}$$

$$= +20$$

$y_i = 20 \times y_o = 2\,cm$. It is not inverted.

3-(c) The distance between the eye and the image is $d = 4\,cm + 38\,cm = 42\,cm$. Thus

$$\alpha = \frac{y_i}{d} = \frac{2\,cm}{42\,cm} = 4.76 \times 10^{-2}\ rad.$$

4-(a) The total intensity from the air gap is given by

$$I(t, \theta_t) = I_1 + I_2 + 2\sqrt{I_1 \cdot I_2} \cdot \cos(\phi_2 - \phi_1)$$

$$= 2I_1\left(1 - \cos\left(\frac{2\pi}{\lambda_0} \cdot (2n_{air} \cdot t \cdot \cos\theta_t)\right)\right)$$

with $t = 5\,\mu m$, $\lambda_0 = 0.5\,\mu m$, $n_{air} \simeq 1$.

Thus

$$I(t, \theta_t) = 2I_1\left(1 - \cos\left(2\pi \cdot 20 \cdot \cos\theta_t\right)\right)$$

when $\theta_t = 0°$, $\cos\theta_t = 1$,

$$I(t, \theta_t = 0) = 2I_1\left(1 - \cos(40\pi)\right) = 0$$

or when $\theta_t = 0°$, $2n_{air} \cdot t = 20\lambda_0$,

$$I(t, \theta_t = 0) = 2I_1\left(1 - \cos\left(\frac{2\pi}{\lambda_0} \cdot 20\lambda_0\right)\right) = 0$$

4-(b) When θ_t increases from $0°$, $\cos\theta_t$ decreases.
When θ_t is such that

$$2 n_{air} \cdot t \cdot \cos\theta_t = 19\tfrac{1}{2}\lambda_0$$

or equivalently, when

$$\frac{2\pi}{\lambda_0}\left(2 n_{air} \cdot t \cdot \cos\theta_t\right) = 39\pi$$

$$I(t; \theta_t) = 2I_1\left(1-\cos(39\pi)\right) = 4I_1$$

From

$$\frac{2\pi}{\lambda_0}\left(2 n_{air} \cdot t \cdot \cos\theta_t\right) = 39\pi$$

$$\Rightarrow \quad 40\pi \cdot \cos\theta_t = 39\pi$$

$$\cos\theta_t = \frac{39}{40}$$

$$\Rightarrow \theta_t = 12.8°$$

4-(c) . At normal incident, $\theta_t = 0$, the reflected intensity from the air gap appears brightest for a given wavelength λ. if

$$\phi_2 - \phi_1 = 2m\pi$$

or

$$\frac{2\pi}{\lambda_0}\left(2n_{air} \cdot t\right) = (2m-1)\pi$$

$$\Rightarrow \lambda_0 = \frac{4 \cdot t}{2m-1} = \frac{20\,\mu m}{2m-1}$$

They are, between 0.4 μm and 0.7 μm these wavelengths appear bright:

0.690 μm; 0.645 μm; 0.606 μm;

0.571 μm; 0.541 μm; 0.513 μm;

0.488 μm; 0.465 μm, 0.444 μm

0.426 μm; 0.408 μm