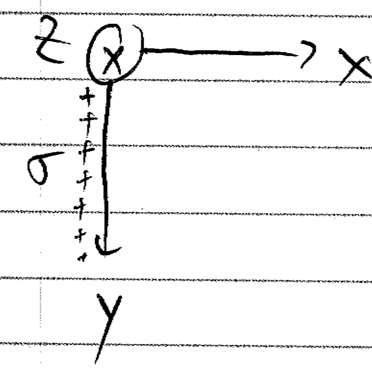
Solution to Physics 9C-A Final Exam (2010)

1-(a) The electric force on the small sphere comes from the positively charged sheet σ. The sheet produces a uniform electric field where the sphere reside

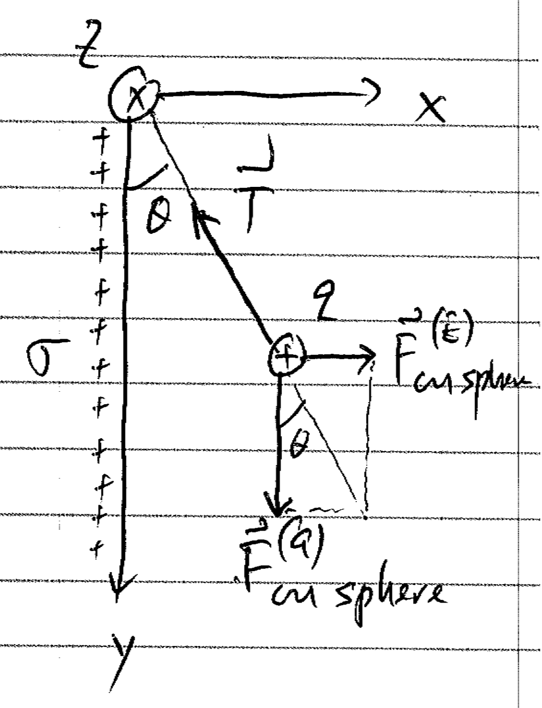


$$\vec{E}_\sigma = \frac{\sigma}{2\epsilon_0}\hat{x}$$

Thus

$$\vec{F}^{(E)}_{on\,q} = \vec{F}^{(E)}_{on\ sphere} = q\vec{E}_\sigma = \frac{q\sigma}{2\epsilon_0}\hat{x}$$

1-(b) The sphere also experiences a gravitational force due to its mass



$$\vec{F}^{(G)}_{on\ sphere} = mg\,(+\hat{y}),$$

and the tension along the fiber $\vec{T}$. In equilibrium,

$$\vec{T} + \vec{F}^{(E)}_{on\ sphere} + \vec{F}^{(G)}_{on\ sphere} = 0$$

$$\therefore\ T\cos\theta = |\vec{F}^{(G)}_{on\ sphere}| = mg$$

$$T\sin\theta = |\vec{F}^{(E)}_{on\ sphere}| = \frac{q\sigma}{2\epsilon_0} \quad \Rightarrow \tan\theta = \frac{q\sigma}{2mg\epsilon_0}$$

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2-(a) The charges on all four surfaces should uniformly distributed over the respective surfaces. Let the charges be Q_a, Q_b, Q_c, Q_d. From charge conservation

$$Q_a + Q_b = +2q$$

$$Q_c + Q_d = +4q$$

Now, since the electric field in the region (inside the small conducting shell) $a<r<b$ is effectively only contributed by Q_a

$$\vec{E}(\vec{r}, a<|\vec{r}|<b) = \frac{1}{4\pi\epsilon_0} \frac{Q_a}{|\vec{r}|^2} \hat{r}$$

and must be zero, we arrive at

$$Q_a = 0$$

Thus $Q_b = +2q$ #

Now since the electric field in the region $c<r<d$ (inside the large conducting shell) is effectively only contributed by $Q_b = +2q$ and Q_c, and must be zero,

$$\vec{E}(\vec{r}, c<|\vec{r}|<d) = \frac{1}{4\pi\epsilon_0} \frac{\hat{r}}{|\vec{r}|^2} (Q_b + Q_c) = 0$$

$\therefore Q_c = -Q_b = -2q; \quad Q_d = 4q - Q_c = +6q$

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2-(b) For $r<a$, $\vec{E}(\vec{r})=0$.

For $b<r<c$, the electric field is effectively contributed by $Q_b=+2\xi$ only,

$$\vec{E}(\vec{r},\ b<r<c)=\frac{1}{4\pi\varepsilon_0}\frac{2\xi}{|\vec{r}|^2}\hat{r}$$

2-(c) The electric potential at $r=d$ relative to the infinity equals to the line integration of the electric field from $r=d$ to $r=+\infty$. Since the electric field in the region of $d\leq r<+\infty$ is effectively contributed only by $Q_d=+6\xi$, and

$$\vec{E}(\vec{r},\ d\leq r<+\infty)=\frac{1}{4\pi\varepsilon_0}\frac{\hat{r}}{|\vec{r}|^2}(+6\xi)$$

$$V(r=d)-V(r=+\infty)=\int_d^{+\infty}\vec{E}\cdot d\vec{l}=\frac{1}{4\pi\varepsilon_0}\frac{6\xi}{d}$$

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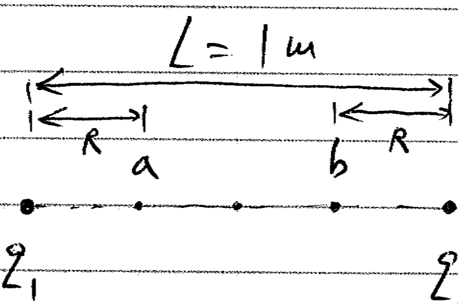
3-(a) The electric potential at point a relative to infinity equals to the line integration of the total electric field produced by the two charged spheres from a to infinity. We can choose the path of the line integration always outside the two charged spheres. In this case, the electric fields produced by these two charged spheres are the same as by two point charges $q_1 = +7.5\,\mu C$ placed at the center of the positively charged sphere, and $q_2 = -7.5\,\mu C$ placed at the center of the negatively charged sphere.

As a result



$$V_a = V_a^{(+)} + V_a^{(-)}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q_1}{R} + \frac{1}{4\pi\varepsilon_0}\frac{q_2}{L-R}$$

R = 0.25 m

$$= \left(9\times10^{9}\,N\cdot m^2/c^2\right)\left(+7.5\times10^{-6}\,c\right)\left(\frac{1}{0.25m} - \frac{1}{0.75m}\right)$$

$$= 1.8\times10^{5}\,V.$$

3-(b) By symmetry or similar calculation,

V_b (relative to infinity)

$= V_b^{(+)} + V_b^{(-)}$

$= -V_a$ (relative to infinity)

$= -1.8 \times 10^5\ V.$

As a result,

$V_a - V_b = 3.6 \times 10^5\ V$ #

4-(a) The 3 outer C_1 capacitors are in series, and thus their equivalence capacitance is

$$C'_{cd} = \frac{1}{\frac{1}{C_1}+\frac{1}{C_1}+\frac{1}{C_1}} = \frac{C_1}{3} = 2.3\,\mu F$$

The net capacitance between c and d is that of C_2 and C'_{cd} in parallel

$$C_{cd} = C_2 + C'_{cd} = 4.6\,\mu c + 2.3\,\mu c = 6.9\,\mu F.$$

The equivalent capacitance along the segment of ecdf is given by

$$C'_{ef} = \frac{1}{\frac{1}{C_1}+\frac{1}{C_{cd}}+\frac{1}{C_1}} = \frac{C_1}{3} = 2.3\,\mu F$$

The net capacitance between e and f is that of C_2 (between e and f) and C'_{ef} in parallel

$$C_{ef} = C_2 + C'_{ef} = 6.9\,\mu F$$

Finally, the network capacitance between ~~and~~ a and b is

$$C_{ab} = \frac{1}{\frac{1}{C_1}+\frac{1}{C_{ef}}+\frac{1}{C_1}} = \frac{C_1}{3} = 2.3\,\mu F$$

4- (b) The charges on C_1 between a and e and C_1 between ~~[illegible]~~ f and b are the same as that on the network capacitor C_{ab}:

$$Q_{ab} = (420V) \cdot C_{ab} = 966\,\mu C$$

Thus

$$Q\big|_{on\ C_1 (a \to e)} = 966\,\mu C$$

$$Q\big|_{on\ C_1 (f \to b)} = 966\,\mu C$$

And

$$Q\big|_{on\ C_{ef}} = 966\,\mu C$$

This means that the potential difference between e and f

$$V_{ef} = \frac{Q\big|_{on\ C_{ef}}}{C_{ef}} = \frac{966\,\mu C}{6.9\,\mu F} = 140\,V$$

Thus the charge on C_2 between e and f is

$$Q\big|_{on\ C_2 (e \to f)} = V_{ef} \cdot C_2 = (140V)(4.6\,\mu F)$$

$$= 644\,\mu C$$ *

4-(c) Since $V_{ef} = 140V$, and $C_1 = C_{cd} = 6.9\mu F$, V_{ef} is evenly distributed over C_1, C_{cd}, and C_1, namely, $V_{ec} = V_{cd} = V_{df} = V_{ef}/3$.

Thus $V_{cd} = V_{on\ C_{cd}} = \frac{1}{3} V_{ef} = 46.7V$ #

5-(a) Immediately after the switch is closed, the bar is ~~[illegible]~~ a still resistor with $R = 6\Omega$. This makes the equivalent resistance between a and b

$$R_{ab} = \frac{1}{\frac{1}{R} + \frac{1}{R_1}} = 3\Omega$$

Thus the total current through R_{ab} is the same as that through the battery

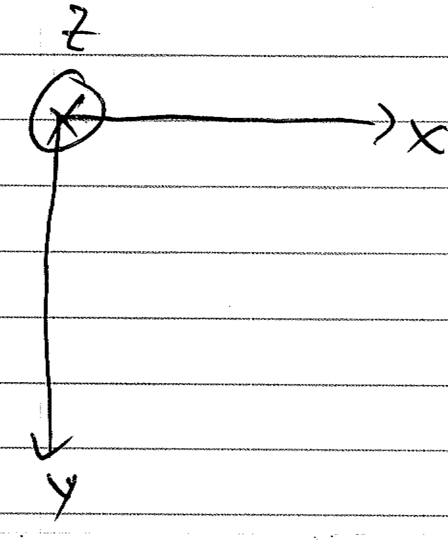
$$I_{ab} = \frac{\varepsilon}{R_2 + R_{ab}} = \frac{36V}{9\Omega} = 4A,$$

and goes from a to b (top to bottom).

The current through the bar is one half of I_{ab}, and also flows from top to bottom

$$I_{bar} = \frac{I_{ab}}{2} = 2A. \qquad \vec{B} = 2T\,\hat{z}$$

5-(b) Since I_{bar} flows from the top to the bottom along the bar of L,



$$\vec{F}^{(M)}_{on\ bar} = I_{bar}(L\,\hat{y}) \times \vec{B}$$

$$= (2A)(1.5m)(2T)(\hat{y} \times \hat{z})$$

(Continue ..)

$$\vec{F}^{(M)}_{\text{on bar}} = 6.0\,N\,\hat{x} \quad \text{(to the right)}$$

5-(C) Under $\vec{F}^{(M)}$, the bar is accelerated towards the right ($\hat{x}$), and gains a velocity $\vec{v} = v\,\hat{x}$. As a result of ~~its gaining speed~~ moving with a velocity $\vec{v}$ in the same magnetic field $\vec{B} = B\,\hat{z}$, an motion emf is produced inside the bar that points upward (from bottom to top) with a ~~magni~~ magnitude $\mathcal{E}_{\text{induce}} = BLv$. This emf reduces the current that flows through the bar.

A terminal velocity is reached $\vec{v}_t = v_t\,\hat{x}$ when the motion emf BLv_t equals to the potential drop between a and b such that no more current flows through the bar (and the acceleration stops as well). In this case, the current produced by $\mathcal{E} = 36V$, only flows through R_1 and R_2,

$$I = \frac{\mathcal{E}}{R_1 + R_2} = \frac{36V}{12\Omega} = 3A$$

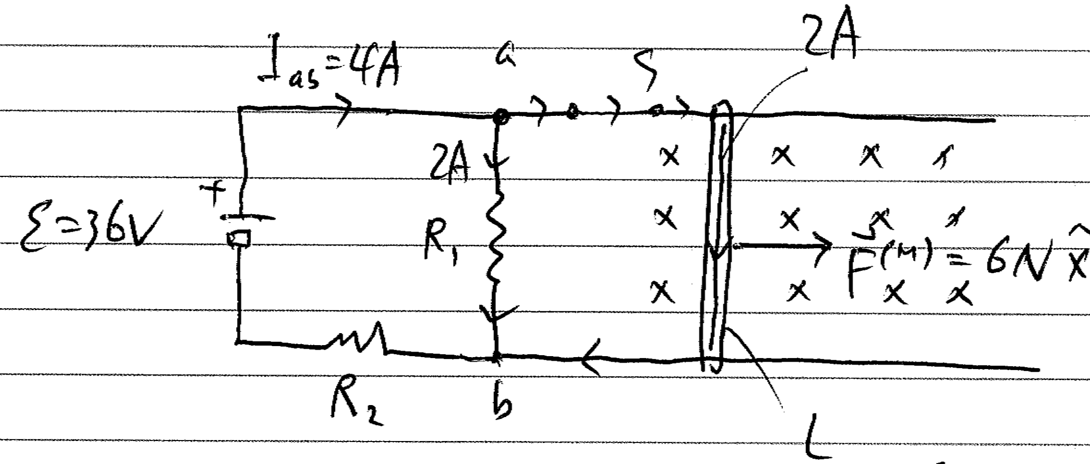
Thus the potential difference between a and b is

$$V_{ab} = I \cdot R_1 = 18V$$

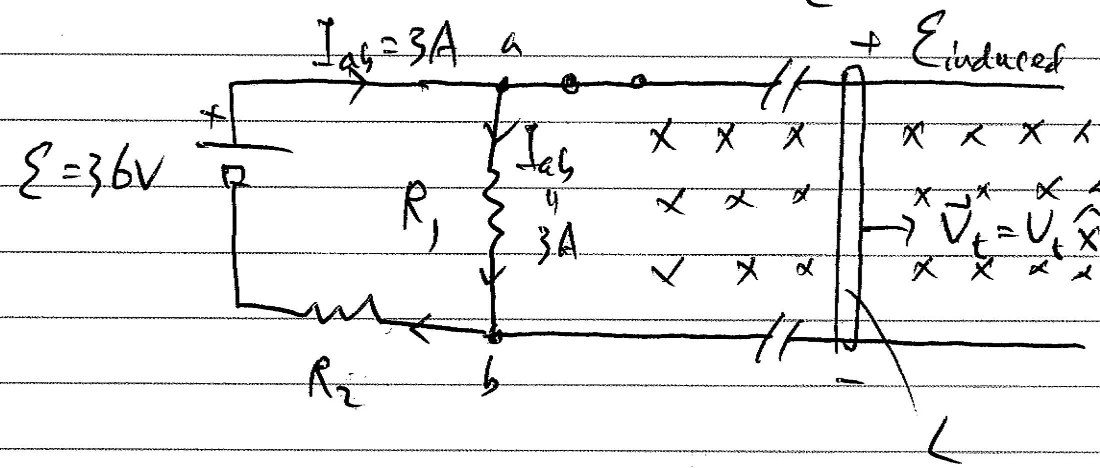
and it equals to the "terminal" motion emf BLV_t

$$\therefore V_t = \frac{V_{ab}}{BL} = \frac{18V}{(2T)(1.5m)} = 6\, m/sec$$

$t=0$:

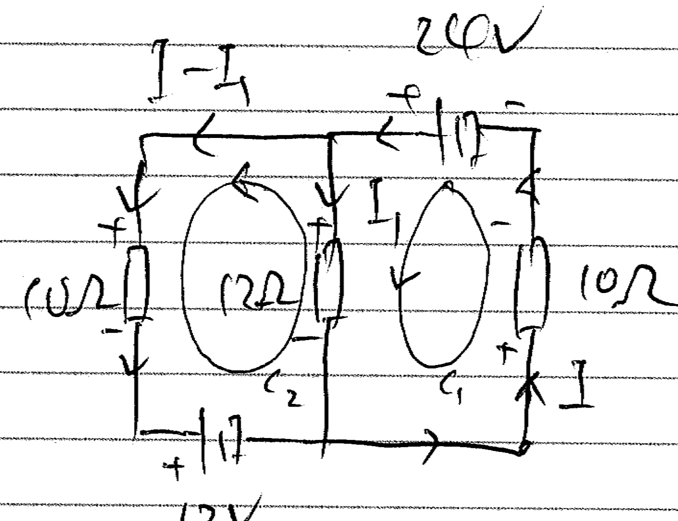


$t=+\infty$:



$$\mathcal{E}_{induced} = BLV_t = V_a - V_b = I_{ab} \cdot R_1$$

6-(a). Assign the currents as shown in the figure.



Along loop#1 (C_1).

① .. $10I - 24 + 12I_1 = 0$

①' .. ∴ $5I + 6I_1 = 12$

Along loop#2 (C_2).

② .. $-12I_1 + (I - I_1)10 + 12 = 0$

②' ∴ $5I - 11I_1 = -6$

①' - ②':

$17I_1 = 18$ ∴ $I_1 = \frac{18}{17}$ A (as assigned)

$11 \times$ ①' $+ 6 \times$ ②':

$85I = 96$ ∴ $I = \frac{96}{85}$ A (as assigned)

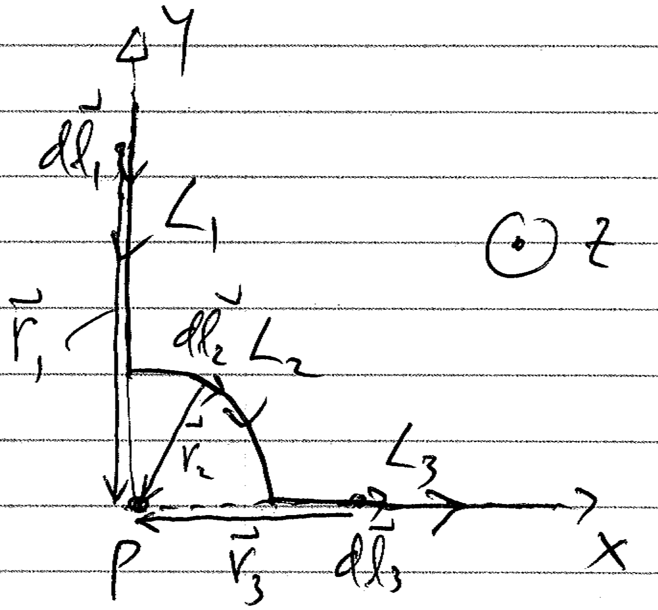
$I - I_1 = \frac{6}{85}$ A (as assigned)

The current through 12v battery is $\frac{6}{85}$ A in the direction as assigned to $I - I_1$ ☆

6 - (b) The power dissipation is

$$P(12\Omega) = I_1^2 \cdot (12\Omega) = 13.45 \text{ Watt}.$$

7. The magnetic field produced by the section L_1 is given by



$$\vec{B}_1 = \int_{(L_1)} \frac{\mu_0}{4\pi} \frac{I d\vec{l}_1 \times \hat{r}_1}{|\vec{r}_1|^2}$$

But $d\vec{l}_1 = |dy|(-\hat{y})$, and $\hat{r}_1 = (-\hat{y})$, thus $d\vec{l}_1 \times \hat{r}_1 = 0$, therefore

$$\vec{B}_1 = 0.$$

The magnetic field produced by the section L_3 is given by

$$\vec{B}_3 = \int_{(L_3)} \frac{\mu_0}{4\pi} \frac{I d\vec{l}_3 \times \hat{r}_3}{|\vec{r}_3|}$$

But $d\vec{l}_3 = dx\,\hat{x}$, $\hat{r}_3 = -\hat{x}$, thus $d\vec{l}_3 \times \hat{r}_3 = 0$, therefore

$$\vec{B}_3 = 0.$$

The magnetic field produced by the section L_2 is given by

$$\vec{B}_2 = \int_{(L_2)} \frac{\mu_0}{4\pi} \cdot \frac{I\, d\vec{l}_2 \times \hat{r}_2}{|\vec{r}_2|^2}$$

$$= \frac{\mu_0 I}{4\pi R^2} \int_{(L_2)} d\vec{l}_2 \times \hat{r}_2$$

$$= \frac{\mu_0 I}{4\pi R^2} \int_{(L_2)} dl_2 (-\hat{k})$$

$$= \frac{\mu_0 I}{4\pi R^2} (-\hat{k}) \cdot \frac{2\pi R}{4}$$

$$= -\frac{\mu_0 I}{8R} \hat{k}$$

※

8-(a) Let $\vec{B} = -B\hat{k}$.

Orientation #1: $\vec{m}_1 = NIA\hat{z}$, $\vec{\tau}_1 = \vec{m}_1 \times \vec{B} = 0$

Orientation #2: $\vec{m}_2 = NIA\hat{y}$, $\vec{\tau}_2 = \vec{m}_2 \times \vec{B} = -NIAB\hat{x}$

Orientation #3: $\vec{m}_3 = NIA(-\hat{z})$, $\vec{\tau}_3 = \vec{m}_3 \times \vec{B} = 0$

Orientation #4: $\vec{m}_4 = -NIA\hat{y}$, $\vec{\tau}_4 = \vec{m}_4 \times \vec{B} = NIAB \cdot \hat{x}$

8-(b) With $\vec{B} = -B\hat{k}$, $U^{(m)} = -\vec{m} \cdot \vec{B}$.

Orientation #1, $U_1^{(m)} = -\vec{m}_1 \cdot \vec{B} = NIAB$

Orientation #2, $U_2^{(m)} = -\vec{m}_2 \cdot \vec{B} = 0$

Orientation #3, $U_3^{(m)} = -\vec{m}_3 \cdot \vec{B} = -NIAB$

Orientation #4: $U_4^{(m)} = -\vec{m}_4 \cdot \vec{B} = 0$

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