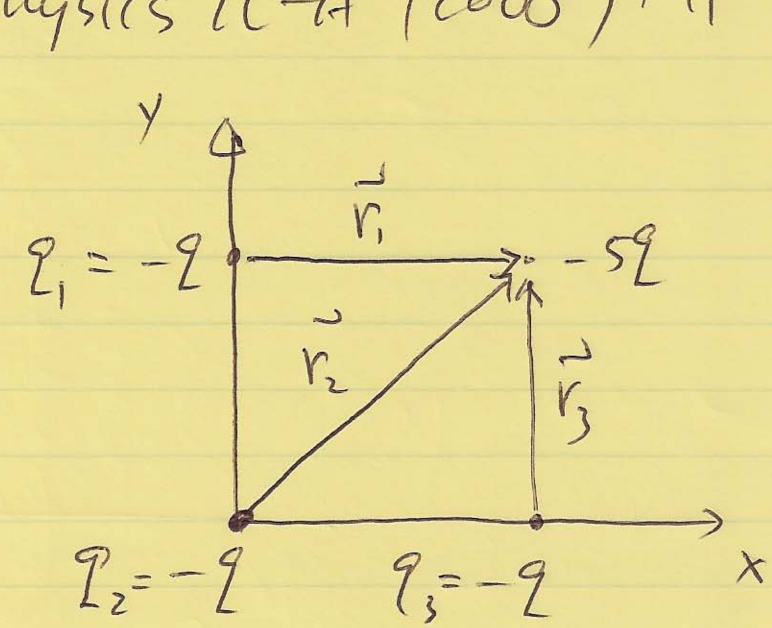
Solutions to Physics 9C-A (2008) MT#1

1-(a)



$$\vec{F}_{on-5q} = \vec{F}_{q_1, on-5q} + \vec{F}_{q_2, on-5q} + \vec{F}_{q_3, on-5q}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{(-q)(-5q)}{L^2}\hat{i} + \frac{1}{4\pi\varepsilon_0}\frac{(-q)(-5q)}{(\sqrt{2}L)^2}\cdot\frac{\hat{i}+\hat{j}}{\sqrt{2}}$$

$$+ \frac{1}{4\pi\varepsilon_0}\cdot\frac{(-q)(-5q)}{L^2}\cdot\hat{j}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{5q^2}{L^2}\left(1+\frac{1}{2\sqrt{2}}\right)\hat{i} + \frac{1}{4\pi\varepsilon_0}\frac{5q^2}{L^2}\left(1+\frac{1}{2\sqrt{2}}\right)\hat{j}$$

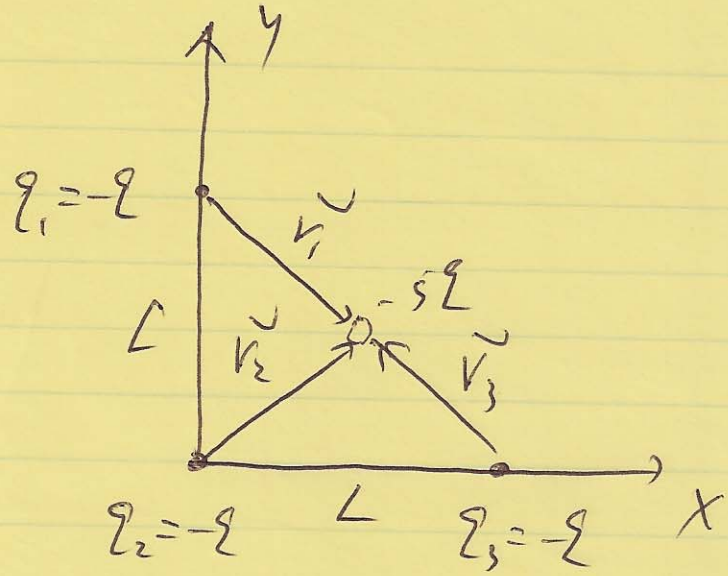
$$= \frac{1}{4\pi\varepsilon_0}\frac{5q^2}{L^2}\left(\sqrt{2}+\frac{1}{2}\right)\frac{\hat{i}+\hat{j}}{\sqrt{2}}$$

$$\therefore \left|\vec{F}_{on-5q}\right| = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{L^2}\left(5\sqrt{2}+\frac{5}{2}\right).$$

$\vec{F}_{on-5q}$ is along $\frac{\hat{i}+\hat{j}}{\sqrt{2}}$ direction.

1-(b)

The work is the sum of the works done by the electric forces from the three charges.



$$W_E = W_E^{(1)} + W_E^{(2)} + W_E^{(3)}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{(-q)(-5q)}{|\vec{r}_1|} + \frac{1}{4\pi\varepsilon_0}\cdot\frac{(-q)(-5q)}{|\vec{r}_2|}$$

$$+ \frac{1}{4\pi\varepsilon_0}\frac{(-q)(-5q)}{|\vec{r}_3|}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{15q^2}{(\sqrt{2}/2)\cdot L}$$

※

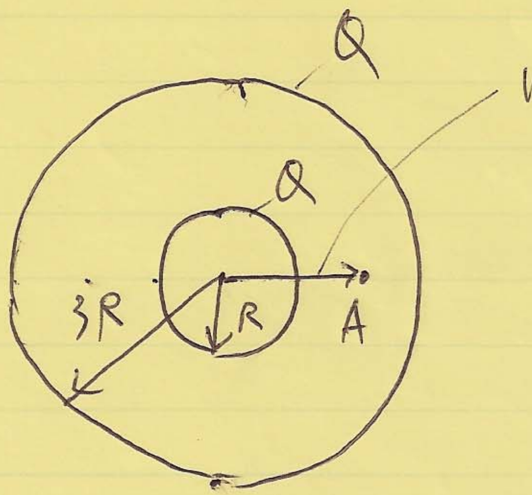
$$W_E^{(n)} = \frac{1}{4\pi\varepsilon_0}\frac{(-q)(-5q)}{|\vec{r}_n|}\Bigg|_{+\infty}^{|\vec{r}_n|} = \frac{1}{4\pi\varepsilon_0}\frac{5q^2}{|\vec{r}_n|}$$

※

2-(a) Inside two spherical shells of uniformly distributed charges, the electric field is zero:

$$\vec{E}\left(0<|\vec{r}|<R\right)=0$$

2-(b) By definition, the electric potential $V(\vec{r})$ relative to its value at the infinity (set to zero) is



$$V(\vec{r})-V(+\infty)$$

$$=\int_{|\vec{r}|}^{+\infty}\vec{E}\cdot d\vec{l}=\int_{r}^{3R}\vec{E}\left(R<|\vec{r}|<3R\right)\cdot d\vec{l}$$

$$+\int_{3R}^{+\infty}\vec{E}\left(|\vec{r}|>3R\right)\cdot d\vec{l}$$

$$=\frac{1}{4\pi\varepsilon_0}Q\left(\frac{1}{r}-\frac{1}{3R}\right)+\frac{1}{4\pi\varepsilon_0}(2Q)\left(\frac{1}{3R}-\frac{1}{+\infty}\right)$$

Thus,

$$V(\vec{r};\ R<|\vec{r}|<3R) = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r} + \frac{1}{3R}\right)$$

\#\#

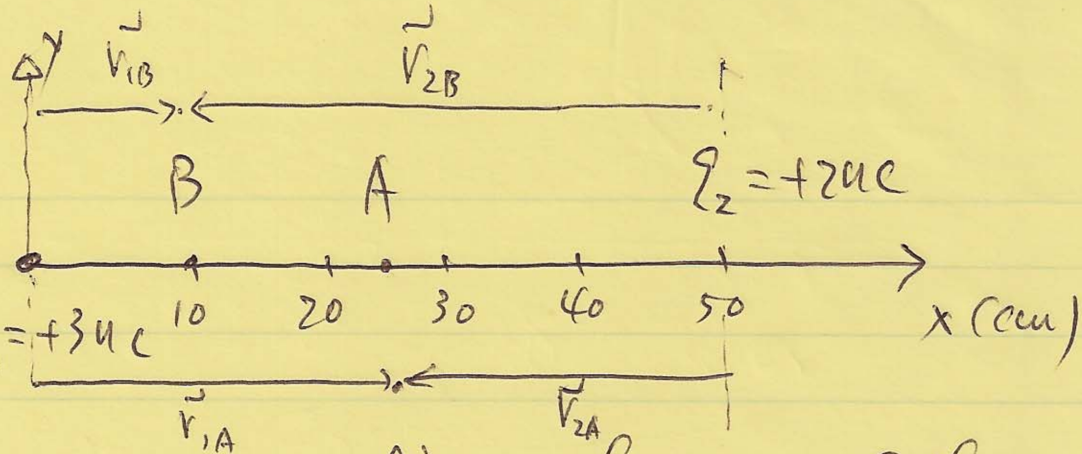
2-(c) $$V(\vec{r};\ |\vec{r}| = 4R) = \frac{1}{4\pi\varepsilon_0}\frac{(2Q)}{4R}$$

referenced to infinity. Thus, the electric potential energy of a positive charge q placed at $r = |\vec{r}| = 4R$ is simply

$$U_q(\vec{r}) = q\,V(\vec{r};\ |\vec{r}| = 4R) = \frac{1}{4\pi\varepsilon_0}\frac{(2Q)\cdot q}{4R}$$

\#\#

3.



By energy conservation, the gain of the kinetic energy from zero for the electron equals to the change in ~~its~~ its potential energy when it moves from A to B:

$$\frac{m_e}{2}V^2 = \frac{1}{4\pi\varepsilon_0} q_1(-e)\left(\frac{1}{|\vec{r}_{1A}|} - \frac{1}{|\vec{V}_{1B}|}\right) + \frac{1}{4\pi\varepsilon_0} q_2(-e)\left(\frac{1}{|\vec{V}_{2A}|} - \frac{1}{|\vec{r}_{2B}|}\right)$$

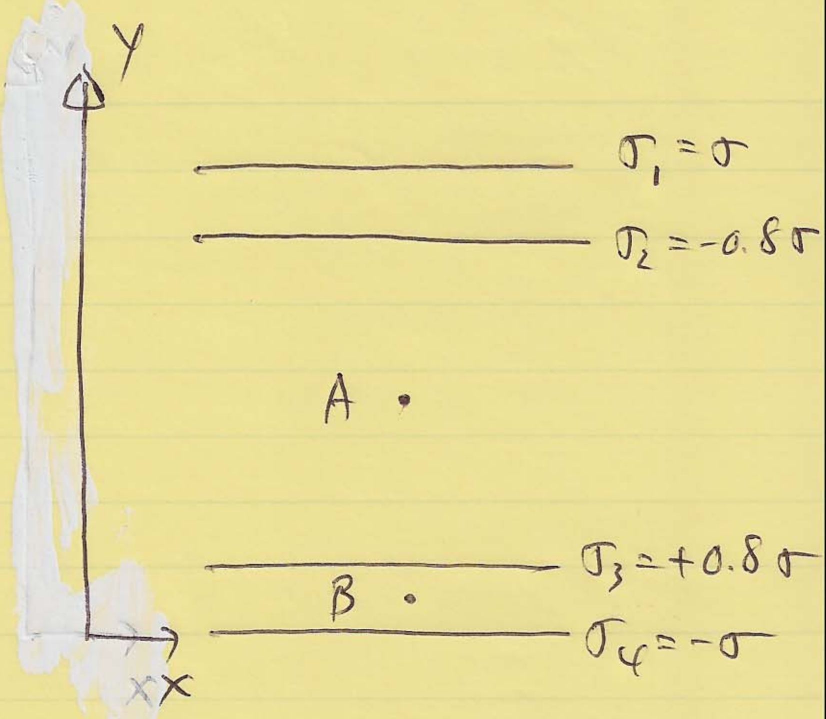
$$\therefore V = \sqrt{\frac{2}{4\pi\varepsilon_0 \cdot m_e}\left[q_1(-e)\left(\frac{1}{|V_{1A}|} - \frac{1}{|V_{1B}|}\right) + q_2(-e)\left(\frac{1}{|\vec{V}_{2A}|} - \frac{1}{|\vec{V}_{2B}|}\right)\right]}$$

$$= \sqrt{\frac{2\times 9\times 10^{9}(Nm^2/c^2)\times 1.6\times 10^{-19}c\times 10^{-9}c}{9.1\times 10^{-31}Kg}\left[3\left(\frac{1}{0.1} - \frac{1}{0.25}\right) + 2\left(\frac{1}{0.4} - \frac{1}{0.25}\right)\right]}$$

$$\cong 7\times 10^{6}\ m/sec.$$

*

4-(a)



$$\vec{E}_A = \vec{E}_A^{(1)} + \vec{E}_A^{(2)} + \vec{E}_A^{(3)} + \vec{E}_A^{(4)}$$

$$= \frac{\sigma_1}{2\varepsilon_0}(-\hat{j}) + \frac{\sigma_2}{2\varepsilon_0}(-\hat{j})$$

$$+ \frac{\sigma_3}{2\varepsilon_0}(+\hat{j}) + \frac{\sigma_4}{2\varepsilon_0}(+\hat{j})$$

$$= \frac{\sigma}{2\varepsilon_0}\hat{j}\left(-1+0.8+0.8-1\right)$$

$$= \frac{\sigma}{\varepsilon_0}(0.2)(-\hat{j})$$

4-(b) $\vec{E}_B = \vec{E}_B^{(1)} + \vec{E}_B^{(2)} + \vec{E}_B^{(3)} + \vec{E}_B^{(4)}$

$$= \frac{\sigma_1}{2\varepsilon_0}(-\hat{j}) + \frac{\sigma_2}{2\varepsilon_0}(-\hat{j}) + \frac{\sigma_3}{2\varepsilon_0}(-\hat{j}) + \frac{\sigma_4}{2\varepsilon_0}(+\hat{j})$$

$$= \frac{\sigma}{2\varepsilon_0}\hat{j}\left(-1+0.8-0.8-1\right)$$

$$= \frac{\sigma}{\varepsilon_0}(-\hat{j})$$