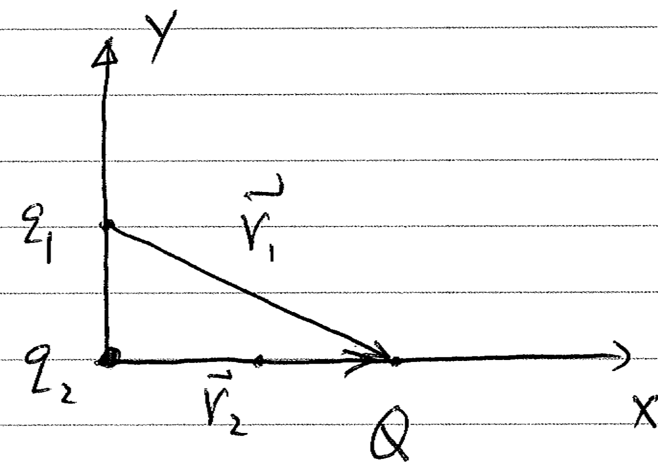
Solutions to 9C-A MT # 1 (W2010)

1-(a) The net electric force on Q is a linear superposition of the electric forces produced by q_1 and q_2 on Q

$$\vec{F}_{on\,Q} = \vec{F}_{q_1\,on\,Q} + \vec{F}_{q_2\,on\,Q}$$

In the coordinate frame as shown to the right, we can specify each force in terms of unit vectors $\hat{x}$ and $\hat{y}$:



$$\vec{F}_{q_1\,on\,Q} = \left(\frac{1}{4\pi\varepsilon_0}\right)\cdot\frac{q_1\cdot Q}{|\vec{r}_1|^2}\,\hat{r}_1$$

$$= \frac{q_1\cdot Q}{4\pi\varepsilon_0}\cdot\frac{\vec{r}_1}{|\vec{r}_1|^3}$$

$$= \left(9.0\times10^{9}\,N\cdot m^2/C^2\right)\left(-4\times10^{-9}C\right)\left(10\times10^{-9}C\right)\cdot\frac{(0.2m)\,\hat{x}-(0.1m)\,\hat{y}}{\left((0.2m)^2+(0.1m)^2\right)^{3/2}}$$

$$= \left(6.44\times10^{-6}\,N\,\hat{x} - 3.22\times10^{-6}\,N\,\hat{y}\right)(-)$$

$$\vec{F}_{q_2 \text{ on } Q} = \frac{1}{4\pi\varepsilon_0}\frac{q_2 \cdot Q}{|\vec{r}_2|^2}\hat{r}_2$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q_2 Q}{|\vec{r}_2|^3}\vec{r}_2$$

$$= \left(9.0\times10^{9}\,Nm^2/C^2\right)\left(4\times10^{-9}\,C\right)\left(10\times10^{-9}\,C\right) \cdot \frac{\hat{x}}{(0.2m)^2}$$

$$= 9\times10^{-6}\,N\cdot\hat{x}$$

Thus the total force on Q is

$$\vec{F}_{\text{on } Q} = 2.56\times10^{-6}\,N\,\hat{x} + 3.22\times10^{-6}\,N\,\hat{y}$$

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1-(b) The total work done on Q equals to the sum of the potential energies of Q in the electric fields of q_1 and q_2 referenced to that of Q at infinity

$$W_E(\text{on } Q) = \frac{1}{4\pi\varepsilon_0}\frac{q_1 Q}{|\vec{r}_1|} + \frac{1}{4\pi\varepsilon_0}\frac{q_2 Q}{|\vec{r}_2|}$$

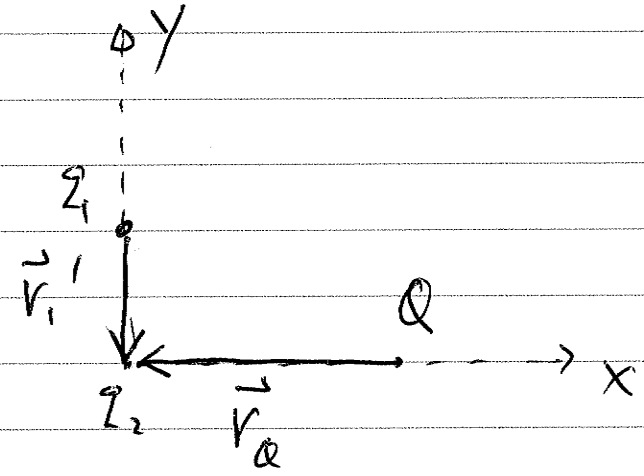
$$= \left(9.0\times10^{9}\,N\cdot m^2/C^2\right)\left(-4\times10^{-9}C\right)\left(10\times10^{-9}C\right)\cdot\frac{1}{\left((0.2m)^2+(0.1m)^2\right)^{1/2}}$$

$$+\left(9.0\times10^{9}\,N\cdot m^2/C^2\right)\left(4\times10^{-9}C\right)\left(10\times10^{-9}C\right)\cdot\frac{1}{(0.2m)}$$

$$= +1.9\times10^{-7}\,N\cdot m$$

$$= 1.9\times10^{-7}\,J$$

1-(c) The total electric field at q_2 is the vector sum of the electric fields at q_2 produced by q_1 and Q.



$$\vec{E}(\text{at } q_2) = \vec{E}_{q_1}(\text{at } q_2) + \vec{E}_Q(\text{at } q_2)$$

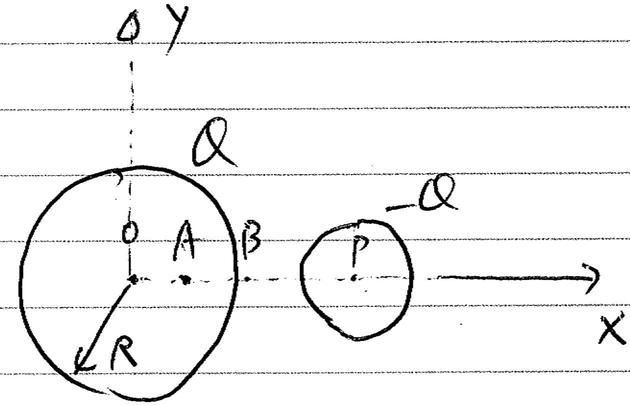
$$= \frac{1}{4\pi\varepsilon_0}\frac{q_1}{|\vec{r}_1{}'|^2}\hat{r}_1{}' + \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{|\vec{r}_Q|^2}\hat{r}_Q$$

$$= (9.0\times10^9\,Nm^2/c^2)(-4\times10^{-9}C)\cdot\frac{(-\hat{y})}{(0.1m)^2}$$

$$+ (9.0\times10^9\,Nm^2/c^2)(10\times10^{-9}c)\cdot\frac{(-\hat{x})}{(0.2m)^2}$$

$$= -2.25\times10^3\,N/c\,\hat{x} + 3.6\times10^3\,N/c\cdot\hat{y}$$

2-(a) The total electric field at A is the vector sum of the electric fields produced by the charge Q on the large shell (R) and the charge $-Q$ on the small shell ($R/2$)



$$\vec{E}_A = \vec{E}_A^{(Q)} + \vec{E}_A^{(-Q)}$$

But $\vec{E}_A^{(Q)} = 0$, so

$$\vec{E}_A = \vec{E}_A^{(-Q)} = \frac{1}{4\pi\varepsilon_0} \frac{-Q}{(3R/2)^2} (-\hat{x})$$

$$= \frac{1}{4\pi\varepsilon_0} \frac{4Q}{9R^2} \hat{x}$$

Similarly,

$$\vec{E}_B = \vec{E}_B^{(Q)} + \vec{E}_B^{(-Q)} = \frac{1}{4\pi\varepsilon_0} \frac{Q}{R^2} \hat{x} + \frac{1}{4\pi\varepsilon_0} \frac{-Q}{R^2} (-\hat{x})$$

$$= \frac{1}{4\pi\varepsilon_0} \frac{2Q}{R^2} \hat{x}$$

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1-(b) The electric potential at B is the sum of the electric potentials at B produced by Q on the large shell and $-Q$ on the small shell:

$$V_B = V_B^{(Q)} + V_B^{(-Q)}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{Q}{R} + \frac{1}{4\pi\varepsilon_0}\frac{-Q}{R}$$

$$= 0$$

1-(c) The electric potential at P is the sum of the electric potentials at P produced by Q on the large shell and $-Q$ on the small shell:

$$V_P = V_P^{(Q)} + V_P^{(-Q)}$$

$$V_P^{(Q)} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{(2R)}$$

$$V_P^{(-Q)} = V^{(-Q)}\Big|_{\text{on the outer surface of the small shell}}$$ since $\vec{E}^{(-Q)}$ inside $= 0$

$$= \frac{1}{4\pi\varepsilon_0}\frac{-Q}{R}$$

Thus,

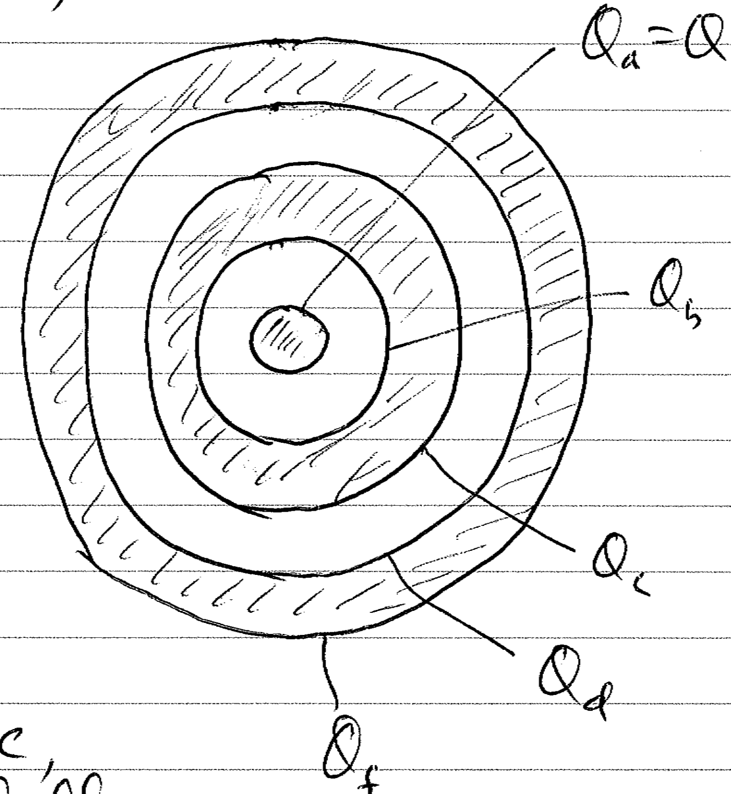
$$V_p = V_p^{(Q)} + V_p^{(-Q)}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{Q}{(2R)} + \frac{1}{4\pi\varepsilon}\frac{-Q}{R}$$

$$= -\frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{2R}$$

3-(a) By spherical symmetry and the property of conductors, the charges on each conductor reside on the surface and distribute uniformly.

Let the charges on the surfaces with radii a, b, c, d, f be $Q_a = Q$, Q_b, Q_c, Q_d and Q_f.



Since Q_c, Q_d and Q_f produce no net electric field in the region $b < r < c$, and the electric field in this region, contributed only by $Q_a = Q$ and Q_b, should be zero, we need

$$\vec{E}(b < |\vec{r}| < c) = \frac{1}{4\pi\varepsilon_0} \frac{\hat{r}}{|\vec{r}|^2} (Q_a + Q_b) = 0$$

Thus $Q_b = -Q_a = -Q$. By charge conservation, $Q_c = -Q_b = Q$.

Similarly, since Q_f produces no net electric field in the region $d < r < f$, and the net electric fields, contributed by Q_a, Q_b, Q_c, and Q_d, add up to zero in this region, we require

$$\vec{E}(d<r<f) = \frac{1}{4\pi\varepsilon_0}\frac{\hat{r}}{|\vec{r}|^2}(Q_a + Q_b + Q_c + Q_d)$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{\hat{r}}{|\vec{r}|^2}(Q - Q + Q + Q_d) = 0$$

Thus $Q_d = -Q$. By charge conservation on the outer shell, $Q_f = -Q_d - Q = 0$

3-(b) Since the electric fields from Q_b, Q_c, Q_d and Q_f are zero in this region, while the electric field from $Q_a = Q$ is

$$\vec{E}(\vec{r}) = \vec{E}_a(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{|\vec{r}|^2}\hat{r}$$

the potential difference

$$V_a - V_b = \frac{1}{4\pi\varepsilon_0}\frac{Q}{a} - \frac{1}{4\pi\varepsilon_0}\frac{Q}{b}$$

$$= \int_a^b \vec{E}_a(\vec{r})\cdot d\vec{\ell}$$

3-(c) Since Q_d and Q_f produce no electric field inside, they do not contribute to the potential difference between any two points inside.

The electric field in the region between c and d, produced by Q_a, Q_b, Q_c, is given by

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{\hat{r}}{|\vec{r}|^2}\left(Q_a + Q_b + Q_c\right)$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{\hat{r}}{|\vec{r}|^2} Q$$

Thus the potential difference between the first shell and the second (larger) shell is given by

$$V_c - V_d = \frac{1}{4\pi\varepsilon_0}\frac{Q}{c} - \frac{1}{4\pi\varepsilon_0}\frac{Q}{d}$$

$$= \int_c^d \vec{E}(\vec{r})\cdot d\vec{l}$$

3-(d) The electric field outside the second conducting shell is the superposition of the electric fields produced by all five "thin" shells of charges.

$$\vec{E}(\vec{r})\Big|_{|\vec{r}|>f} = \frac{1}{4\pi\varepsilon_0}\frac{\hat{r}}{|\vec{r}|^2}\left(Q_a+Q_b+Q_c+Q_d+Q_f\right)$$

$$= \hat{0}$$