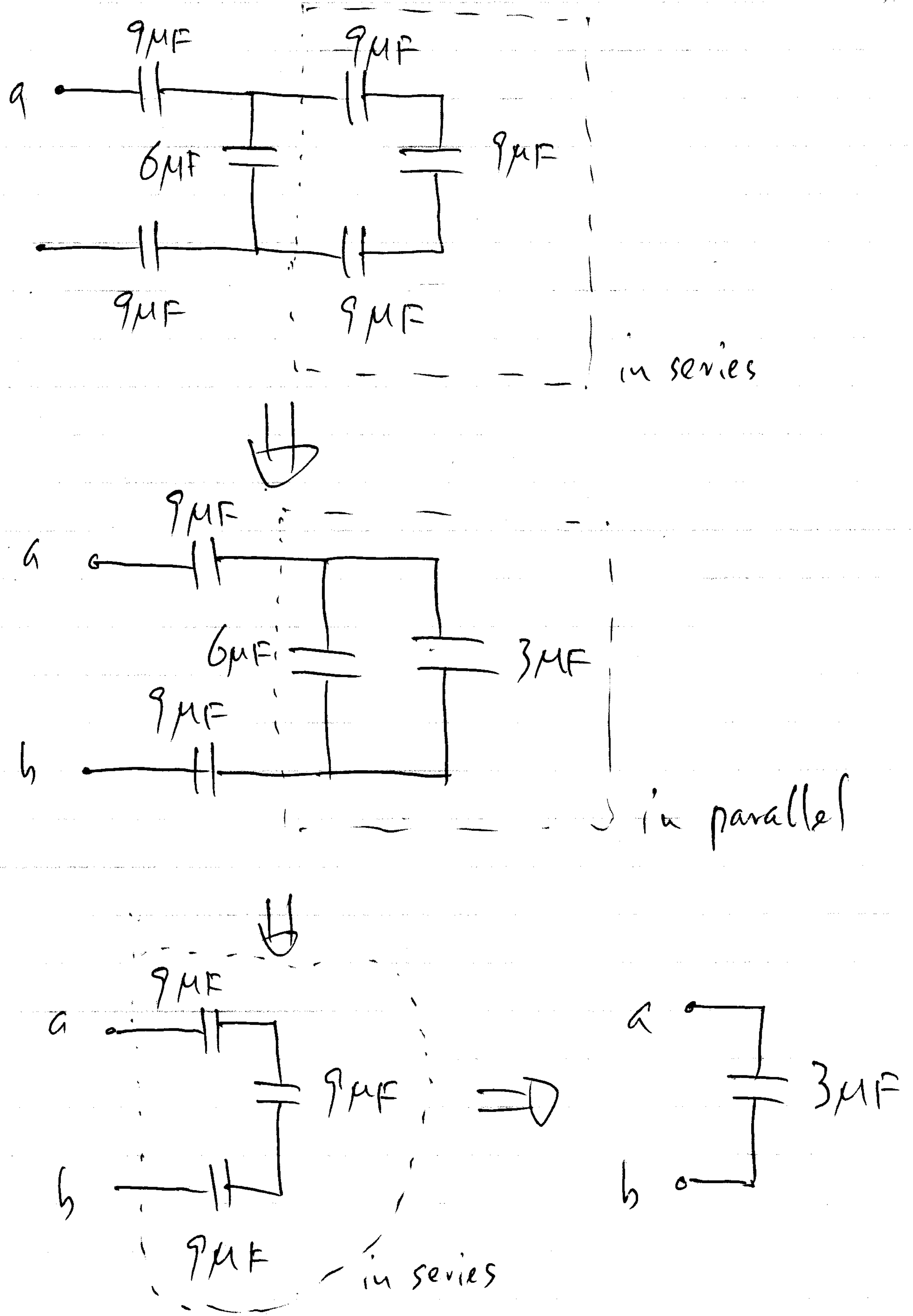
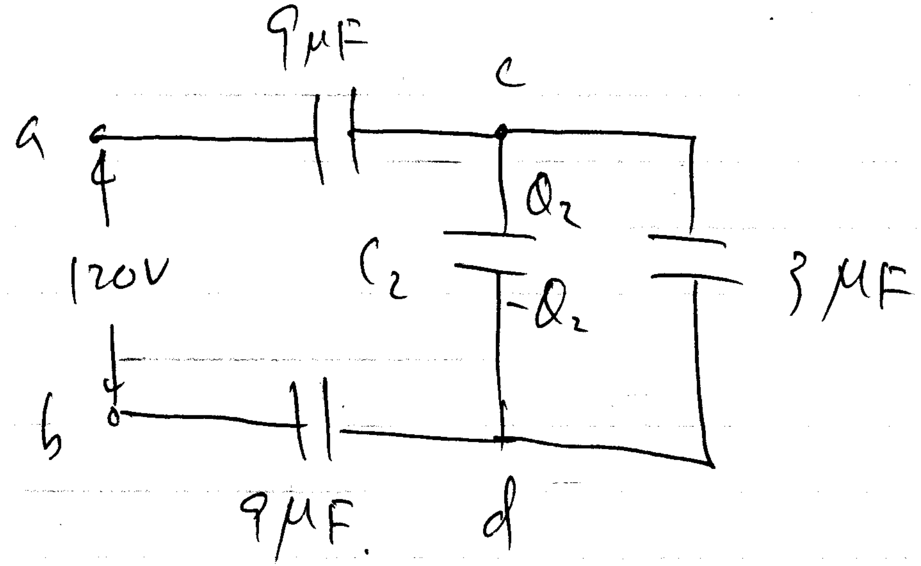
Solutions to 9C-A MT#2 (W-2008)

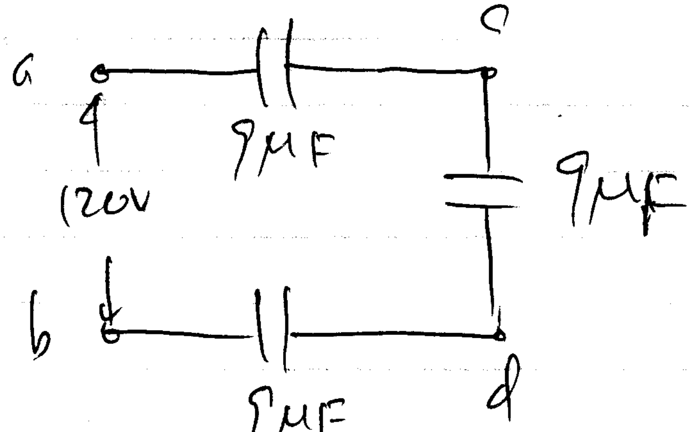
1-(a)



1-(b)



To find the charge Q_2 on C_2, we need to know the potential drop across C_2, i.e., $V_c - V_d \equiv V_{cd}$. Since



(see part 1-(a))

we have $V_{cd} = \frac{1}{3}(V_{ab}) = 40V$.

from $Q_2 = C_2 \cdot V_{cd}$ we have

$$Q_2 = (6\mu F)(40 \text{ volts}) = 240\mu C$$

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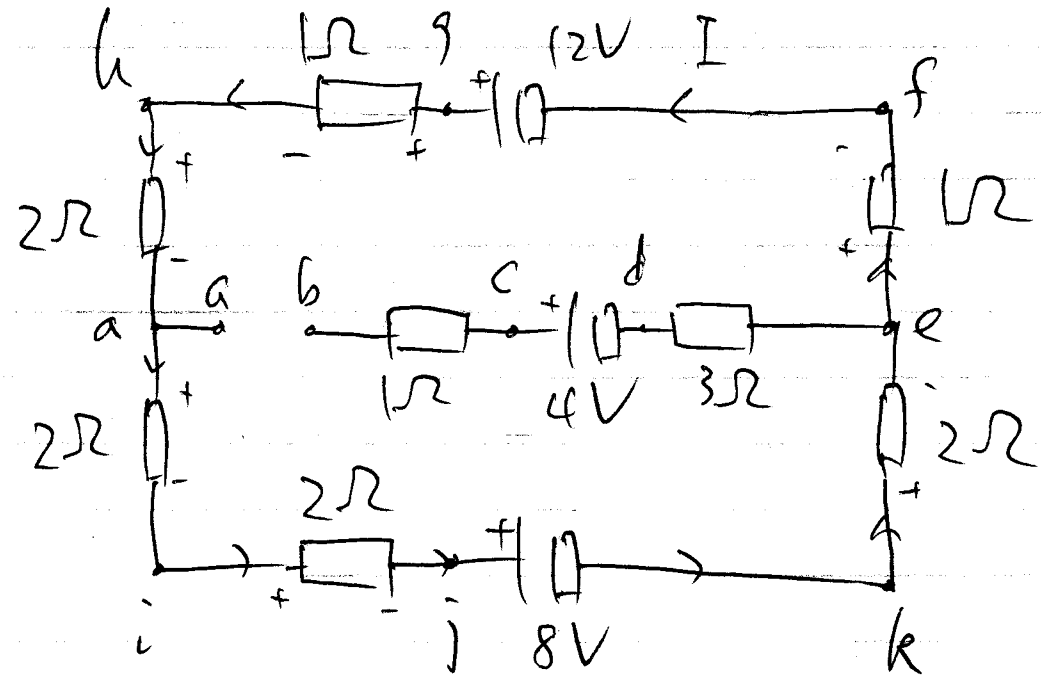
2-(a)



The potential difference between a and b is the difference between $V_{ae} - V_{be}$:

$$V_{ab} = V_{ae} - V_{be} = (V_a - V_e) - (V_b - V_e) \cdots (1)$$

since a and b are disconnected, there is no current flowing from b to e, as a result,

$$V_{be} = V_b - V_e = (V_b - V_c) + (V_c - V_d) + (V_d - V_e)$$

$$= V_c - V_d = 4 \text{ volts.} \cdots (2)$$

So, we only need to find $V_a - V_e$ to obtain V_{ab} from (1). To do so, we need to find the current that flows through the outer loop as assigned.

Starting at point a and counter-clockwise along the outer loop, we apply the loop rule:

$$2I + 2I + 8 + 2I + I - 12 + I + 2I = 0$$

$$\therefore \quad 10I = 4\,V \qquad I = \frac{2}{5}\,A$$

Then

$$V_a - V_e = 2I + 2I + 8 + 2I$$

$$= 6I + 8 \text{ volts}$$

$$= 10\tfrac{2}{5} \text{ volts.}$$

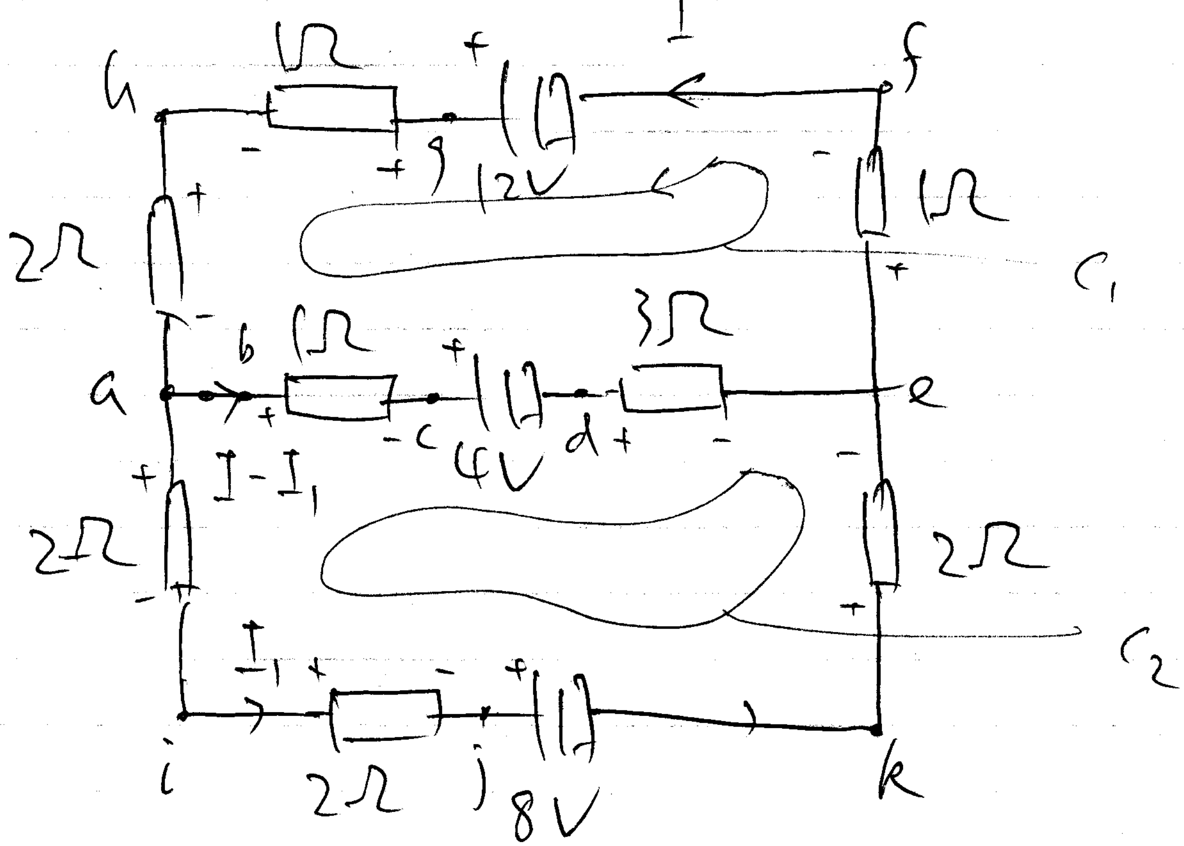
Thus

$$V_{ab} = V_a - V_b = (V_a - V_e) - (V_b - V_e)$$

$$= 10\tfrac{2}{5} \text{ volts} - 4 \text{ volts}$$

$$= 6\tfrac{2}{5} \text{ volts}$$

2-(b) When a and b are connected, there will be a current flowing between a and e now.



Assign I and I_1 as two unknowns, and apply the junction rule at a and e.

Along Loop C_1 and counter-clockwise from a:

$$4(I-I_1)+4+4I-12=0 \quad \cdots \text{①}$$

or

$$2I-I_1=2 \quad \cdots\cdots \text{①}'$$

Along Loop C_2 counter-clockwise from a:

$$6I_1 + 8 - 4(I - I_1) - 4 = 0 \quad \cdots \quad (2)$$

or,

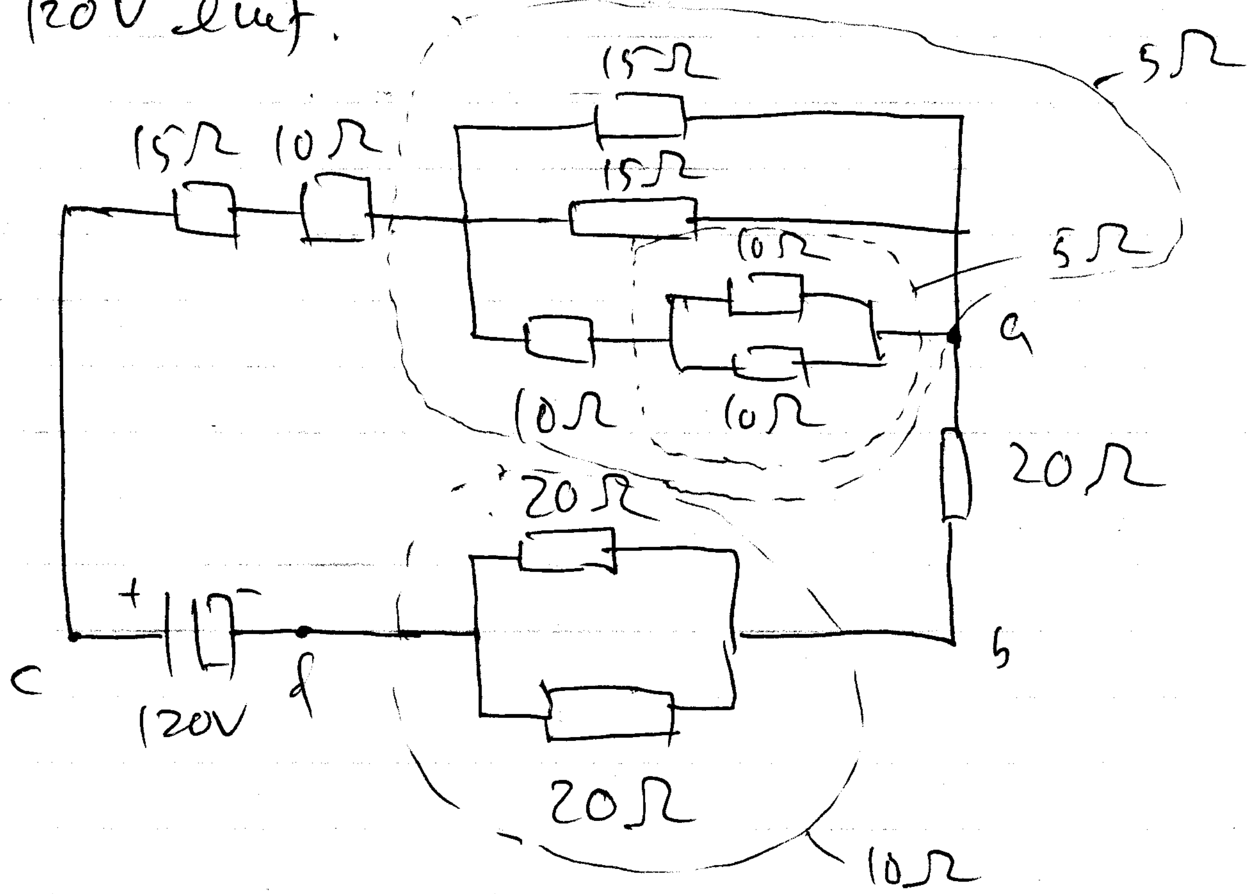
$$2I - 5I_1 = 2 \quad \cdots\cdots \quad (2')$$

Thus $I_1 = 0$

$I = 1A$

The current through the 12-V battery is $I = 1A$ ##

3. We need to find the current through the $20\,\Omega$ resistor between a and b. To do that we need to determine the equivalent resistance between the two ends of the 120V emf.



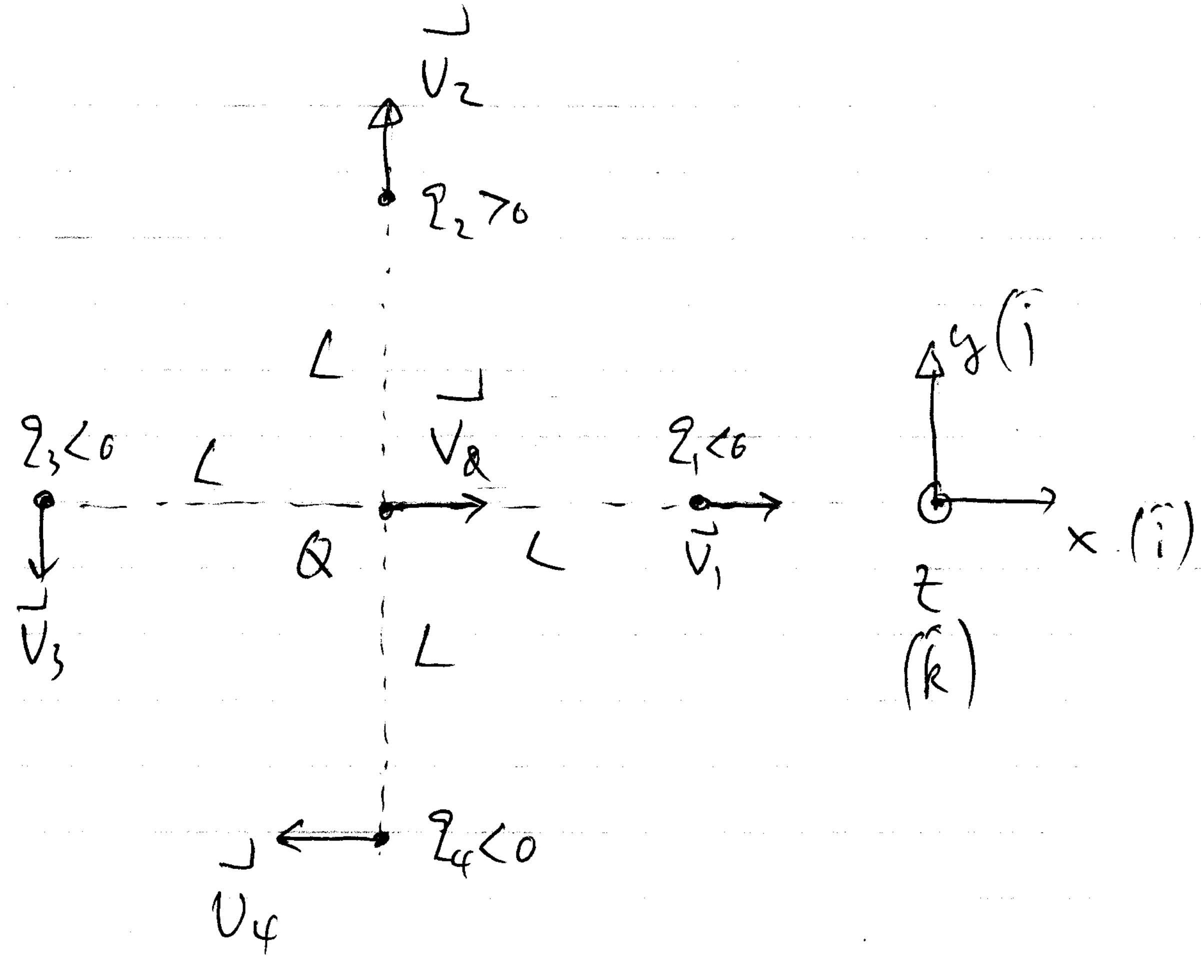
The total resistance between c and d is

$$R_{cd} = 15 + 10 + 5 + 20 + 10 = 60\,\Omega$$

$$\therefore\ I = \frac{120\,V}{R_{cd}} = \frac{120\,V}{60\,\Omega} = 2A$$

$$\therefore\ P_{20\Omega}\,(\text{between a \& b}) = I^2R = (2A)^2(20\,\Omega) = 80\,\text{Watt}$$

4-(a)



$$\vec{F}^{(M)}_{Q \text{ on } q_1} = \frac{\mu_0}{4\pi} \cdot \frac{q_1 Q\, \vec{v}_1 \times \left(\vec{V}_Q \times (\hat{i})\right)}{L^2}$$

$$= \frac{\mu_0}{4\pi} \frac{q_1 Q v_1 V_Q}{L^2} \cdot \left(\hat{i} \times (\hat{i} \times \hat{i})\right)$$

$$= 0$$

$$\vec{F}^{(M)}_{Q \text{ on } q_2} = \frac{\mu_0}{4\pi} \frac{q_2 Q}{L^2} V_2 V_Q \left[\hat{j} \times (\hat{i} \times \hat{j})\right]$$

$$= -\frac{\mu_0}{4\pi} \frac{q_2 Q}{L^2} V_2 V_Q\, \hat{i}$$

$$\vec{F}^{(M)}_{Q \text{ on } q_3} = \frac{\mu_0}{4\pi} \frac{q_3 Q v_3 v_Q}{L^2} \left[(-\hat{j}) \times (\hat{i} \times (-\hat{i})) \right]$$

$$= 0$$

$$\vec{F}^{(M)}_{Q \text{ on } q_4} = \frac{\mu_0}{4\pi} \cdot \frac{q_4 Q \cdot v_4 v_Q}{L^2} \left[(-\hat{i}) \times \underbrace{(\hat{i} \times (-\hat{j}))}_{-\hat{k}} \right]$$

$$= \frac{\mu_0}{4\pi} \cdot \frac{q_4 Q \cdot v_4 v_Q}{L^2} \left((-\hat{i}) \times (-\hat{k}) \right)$$

$$= -\frac{\mu_0}{4\pi} \cdot \frac{q_4 Q \cdot v_4 \cdot v_Q}{L^2} \hat{j}$$

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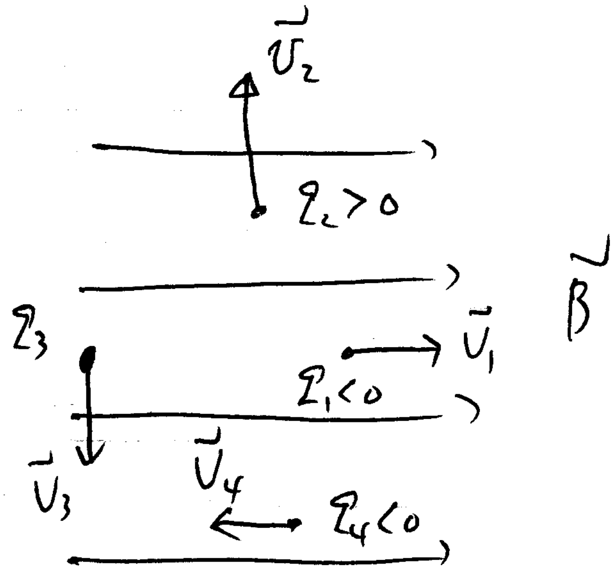
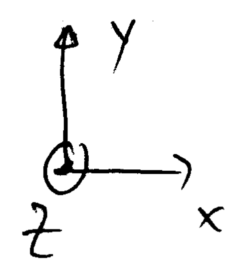
4-(b)



$$\vec{F}^{(M)}_{\text{on } q_1} = q_1 \vec{v}_1 \times \vec{B}$$
$$= q_1 v_1 B \cdot (\hat{i} \times \hat{i})$$
$$= 0$$



$$\vec{F}^{(M)}_{\text{on } q_2} = q_2 \vec{v}_2 \times \vec{B}$$
$$= q_2 v_2 B \ (\hat{j} \times \hat{i})$$
$$= - q_2 v_2 B \hat{k}$$

$$\vec{F}^{(M)}_{\text{on } q_3} = q_3 \vec{v}_3 \times \vec{B}$$
$$= q_3 v_3 B \ (-\hat{j} \times \hat{i})$$
$$= q_3 v_3 B \ \hat{k}$$

$$\vec{F}^{(M)}_{\text{on } q_4} = q_4 \vec{v}_4 \times \vec{B}$$
$$= q_4 v_4 B \ (-\hat{i} \times \hat{i})$$
$$= 0$$

5-(a) $\vec{F}_{\text{on the wire}} = \vec{F}_{\text{on } ab} + \vec{F}_{\text{on } bc} + \vec{F}_{\text{on } cd} + \vec{F}_{\text{on } de} + \vec{F}_{\text{on } ef}$

$$\vec{F}_{\text{on } ab} = I\,\vec{ab} \times \vec{B} = 0$$

$$\vec{F}_{\text{on } bc} = I\,\vec{Cb} \times \vec{B}$$
$$= (6A)(0.5T)(0.5m)(-\hat{i} + \hat{j}) \times \hat{i}$$
$$= -1.5N\,\hat{k}$$

$$\vec{F}_{\text{on } cd} = I\,\vec{cd} \times \vec{B}$$
$$= (6A)(0.5T)(0.5m)(-\hat{k} \times \hat{i})$$
$$= -1.5N\,\hat{j}$$

$$\vec{F}_{\text{on } de} = I\,\vec{de} \times \vec{B} = (6A)(0.5T)(0.5m)(\hat{i} + (-\hat{j})) \times \hat{i}$$
$$= 1.5N\,\hat{k}$$

$$\vec{F}_{\text{on } ef} = I\,\vec{ef} \times \vec{B} = (6A)(0.5T)(0.5m)(-\hat{i}) \times (\hat{i}) = 0$$

Thus the net force on the wire is

$$\vec{F}_{\text{on the wire}} = -1.5\,N\,\hat{j}$$

5-(b) $$\vec{F}_{\text{on } af} = I\,\vec{af} \times \vec{B}$$

$$= (0.5T)(0.5m)(6A)(\hat{k} \times \hat{i})$$

$$= +1.5\,N\,\hat{j}$$

$$\vec{F}_{\text{on } af} = -\vec{F}_{\text{on } abcdef}$$

✠