21.4. **IDENTIFY:** Use the mass m of the ring and the atomic mass M of gold to calculate the number of gold atoms. Each atom has 79 protons and an equal number of electrons.

SET UP: $N_A = 6.02 \times 10^{23}$ atoms/mol. A proton has charge $+e$.

EXECUTE: The mass of gold is 17.7 g and the atomic weight of gold is 197 g/mol. So the number of atoms is

$N_A n = (6.02 \times 10^{23} \text{ atoms/mol})\left(\frac{17.7 \text{ g}}{197 \text{ g/mol}}\right) = 5.41 \times 10^{22}$ atoms. The number of protons is

$n_p = (79 \text{ protons/atom})(5.41 \times 10^{22} \text{ atoms}) = 4.27 \times 10^{24}$ protons.

$Q = (n_p)(1.60 \times 10^{-19} \text{ C/proton}) = 6.83 \times 10^5$ C.

(b) The number of electrons is $n_e = n_p = 4.27 \times 10^{24}$.

EVALUATE: The total amount of positive charge in the ring is very large, but there is an equal amount of negative charge.

21.9. **IDENTIFY:** Apply Coulomb's law.

SET UP: Consider the force on one of the spheres.

(a) EXECUTE: $q_1 = q_2 = q$

$$F = \frac{1}{4\pi\epsilon_0}\frac{|q_1 q_2|}{r^2} = \frac{q^2}{4\pi\epsilon_0 r^2} \text{ so } q = r\sqrt{\frac{F}{(1/4\pi\epsilon_0)}} = 0.150 \text{ m}\sqrt{\frac{0.220 \text{ N}}{8.988 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2}} = 7.42 \times 10^{-7} \text{ C (on each)}$$

(b) $q_2 = 4q_1$

$$F = \frac{1}{4\pi\epsilon_0}\frac{|q_1 q_2|}{r^2} = \frac{4q_1^2}{4\pi\epsilon_0 r^2} \text{ so } q_1 = r\sqrt{\frac{F}{4(1/4\pi\epsilon_0)}} = \tfrac{1}{2}r\sqrt{\frac{F}{(1/4\pi\epsilon_0)}} = \tfrac{1}{2}(7.42 \times 10^{-7} \text{ C}) = 3.71 \times 10^{-7} \text{ C.}$$

And then $q_2 = 4q_1 = 1.48 \times 10^{-6}$ C.

EVALUATE: The force on one sphere is the same magnitude as the force on the other sphere, whether the spheres have equal charges or not.

21.16. **IDENTIFY:** Apply Coulomb's law and find the vector sum of the two forces on Q.

SET UP: The force that q_1 exerts on Q is repulsive, as in Example 21.4, but now the force that q_2 exerts is attractive.

EXECUTE: The x-components cancel. We only need the y-components, and each charge contributes equally. $F_{1y} = F_{2y} = -\frac{1}{4\pi\epsilon_0}\frac{(2.0 \times 10^{-6} \text{ C})(4.0 \times 10^{-6} \text{ C})}{(0.500 \text{ m})^2}\sin\alpha = -0.173$ N (since $\sin\alpha = 0.600$).

Therefore, the total force is $2F = 0.35$ N, in the $-y$-direction.

EVALUATE: If q_1 is $-2.0\ \mu$C and q_2 is $+2.0\ \mu$C, then the net force is in the $+y$-direction.

21.21. **IDENTIFY:** Apply Coulomb's law to calculate the force each of the two charges exerts on the third charge. Add these forces as vectors.

SET UP: The three charges are placed as shown in Figure 21.21a.

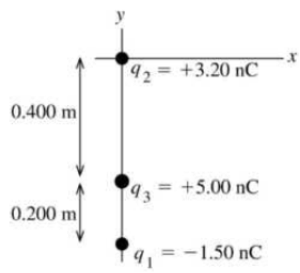
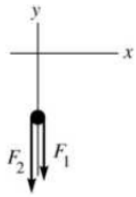


Figure 21.21a

EXECUTE: Like charges repel and unlike attract, so the free-body diagram for q_3 is as shown in Figure 21.21b.



$$F_1 = \frac{1}{4\pi\epsilon_0}\frac{|q_1 q_3|}{r_{13}^2}$$

$$F_2 = \frac{1}{4\pi\epsilon_0}\frac{|q_2 q_3|}{r_{23}^2}$$

Figure 21.21b

$$F_1 = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(1.50\times10^{-9}\ \text{C})(5.00\times10^{-9}\ \text{C})}{(0.200\ \text{m})^2} = 1.685\times10^{-6}\ \text{N}$$

$$F_2 = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(3.20\times10^{-9}\ \text{C})(5.00\times10^{-9}\ \text{C})}{(0.400\ \text{m})^2} = 8.988\times10^{-7}\ \text{N}$$

The resultant force is $\vec{R} = \vec{F}_1 + \vec{F}_2$.

$R_x = 0$.

$R_y = -(F_1 + F_2) = -(1.685\times10^{-6}\ \text{N} + 8.988\times10^{-7}\ \text{N}) = -2.58\times10^{-6}\ \text{N}$.

The resultant force has magnitude $2.58\times10^{-6}\ \text{N}$ and is in the $-y$-direction.

EVALUATE: The force between q_1 and q_3 is attractive and the force between q_2 and q_3 is replusive.

21.22. **IDENTIFY:** Apply $F = k\frac{|qq'|}{r^2}$ to each pair of charges. The net force is the vector sum of the forces due to q_1 and q_2.

SET UP: Like charges repel and unlike charges attract. The charges and their forces on q_3 are shown in Figure 21.22.

EXECUTE: $F_1 = k\frac{|q_1 q_3|}{r_1^2} = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(4.00\times10^{-9}\ \text{C})(6.00\times10^{-9}\ \text{C})}{(0.200\ \text{m})^2} = 5.394\times10^{-6}\ \text{N}$.

$F_2 = k\frac{|q_2 q_3|}{r_2^2} = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(5.00\times10^{-9}\ \text{C})(6.00\times10^{-9}\ \text{C})}{(0.300\ \text{m})^2} = 2.997\times10^{-6}\ \text{N}$.

$F_x = F_{1x} + F_{2x} = +F_1 - F_2 = 2.40\times10^{-6}\ \text{N}$. The net force has magnitude $2.40\times10^{-6}\ \text{N}$ and is in the $+x$-direction.

EVALUATE: Each force is attractive, but the forces are in opposite directions because of the placement of the charges. Since the forces are in opposite directions, the net force is obtained by subtracting their magnitudes.

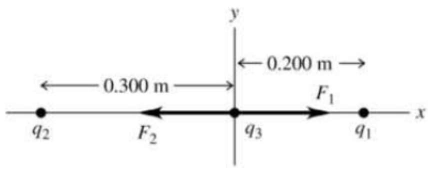


Figure 21.22

21.28. **IDENTIFY:** Use constant acceleration equations to calculate the upward acceleration a and then apply $\vec{F} = q\vec{E}$ to calculate the electric field.

SET UP: Let $+y$ be upward. An electron has charge $q = -e$.

EXECUTE: **(a)** $v_{0y} = 0$ and $a_y = a$, so $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $y - y_0 = \frac{1}{2}at^2$. Then

$$a = \frac{2(y - y_0)}{t^2} = \frac{2(4.50\ \text{m})}{(3.00\times 10^{-6}\ \text{s})^2} = 1.00\times 10^{12}\ \text{m/s}^2.$$

$$E = \frac{F}{q} = \frac{ma}{q} = \frac{(9.11\times 10^{-31}\ \text{kg})(1.00\times 10^{12}\ \text{m/s}^2)}{1.60\times 10^{-19}\ \text{C}} = 5.69\ \text{N/C}$$

The force is up, so the electric field must be *downward* since the electron has negative charge.

(b) The electron's acceleration is $\sim 10^{11} g$, so gravity must be negligibly small compared to the electrical force.

EVALUATE: Since the electric field is uniform, the force it exerts is constant and the electron moves with constant acceleration.

21.50. **IDENTIFY:** Apply Eq. (21.7) to calculate the field due to each charge and then calculate the vector sum of those fields.

SET UP: The fields due to q_1 and to q_2 are sketched in Figure 21.50.

EXECUTE: $\vec{E}_2 = \dfrac{1}{4\pi\epsilon_0}\dfrac{(6.00\times 10^{-9}\ \text{C})}{(0.6\ \text{m})^2}(-\hat{i}) = -150\hat{i}$ N/C.

$$\vec{E}_1 = \frac{1}{4\pi\epsilon_0}(4.00\times 10^{-9}\ \text{C})\left(\frac{1}{(1.00\ \text{m})^2}(0.600)\hat{i} + \frac{1}{(1.00\ \text{m})^2}(0.800)\hat{j}\right) = (21.6\hat{i} + 28.8\hat{j})\text{N/C}.$$

$\vec{E} = \vec{E}_1 + \vec{E}_2 = (-128.4\ \text{N/C})\hat{i} + (28.8\ \text{N/C})\hat{j}$. $E = \sqrt{(128.4\ \text{N/C})^2 + (28.8\ \text{N/C})^2} = 131.6\ \text{N/C}$ at

$\theta = \tan^{-1}\left(\dfrac{28.8}{128.4}\right) = 12.6^\circ$ above the $-x$-axis and therefore 167.4° counterclockwise from the $+x$-axis.

EVALUATE: $\vec{E}_1$ is directed toward q_1 because q_1 is negative and $\vec{E}_2$ is directed away from q_2 because q_2 is positive.

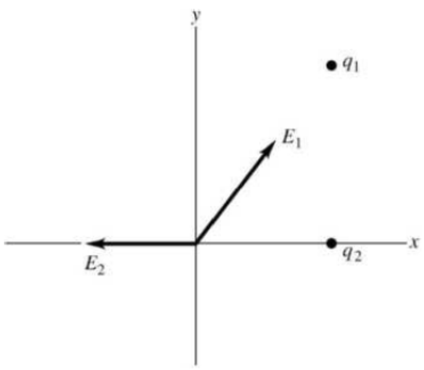


Figure 21.50

21.68. **IDENTIFY:** Apply $\sum F_x = 0$ and $\sum F_y = 0$ to one of the spheres.

SET UP: The free-body diagram is sketched in Figure 21.68. F_e is the repulsive Coulomb force between the spheres. For small θ, $\sin\theta \approx \tan\theta$.

EXECUTE: $\sum F_x = T\sin\theta - F_e = 0$ and $\sum F_y = T\cos\theta - mg = 0$. So $\dfrac{mg\sin\theta}{\cos\theta} = F_e = \dfrac{kq^2}{d^2}$. But

$\tan\theta \approx \sin\theta = \dfrac{d}{2L}$, so $d^3 = \dfrac{2kq^2L}{mg}$ and $d = \left(\dfrac{q^2L}{2\pi\epsilon_0 mg}\right)^{1/3}$.

EVALUATE: d increases when q increases.

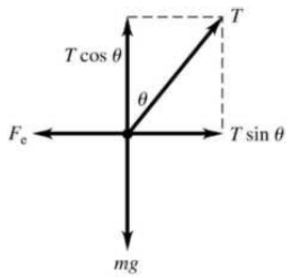


Figure 21.68

21.76. **IDENTIFY:** For the acceleration (and hence the force) on Q to be upward, as indicated, the forces due to q_1 and q_2 must have equal strengths, so q_1 and q_2 must have equal magnitudes. Furthermore, for the force to be upward, q_1 must be positive and q_2 must be negative.

SET UP: Since we know the acceleration of Q, Newton's second law gives us the magnitude of the force on it. We can then add the force components using $F = F_{Qq_1}\cos\theta + F_{Qq_2}\cos\theta = 2F_{Qq_1}\cos\theta$. The electrical force on Q is given by Coulomb's law, $F_{Qq_1} = \dfrac{1}{4\pi\epsilon_0}\dfrac{Qq_1}{r^2}$ (for q_1) and likewise for q_2.

EXECUTE: First find the net force: $F = ma =$ (0.00500 kg)(324 m/s^2) = 1.62 N. Now add the force components, calling θ the angle between the line connecting q_1 and q_2 and the line connecting q_1 and Q.

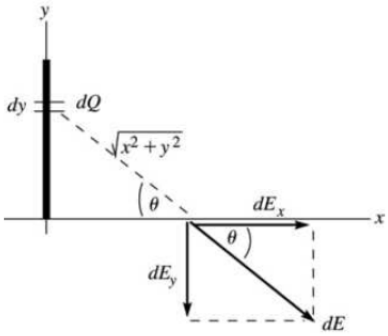
$F = F_{Qq_1}\cos\theta + F_{Qq_2}\cos\theta = 2F_{Qq_1}\cos\theta$ and $F_{Qq_1} = \dfrac{F}{2\cos\theta} = \dfrac{1.62\text{ N}}{2\left(\dfrac{2.25\text{ cm}}{3.00\text{ cm}}\right)} = 1.08\text{ N}$. Now find the charges by solving for q_1 in Coulomb's law and use the fact that q_1 and q_2 have equal magnitudes but opposite signs. $F_{Qq_1} = \dfrac{1}{4\pi\epsilon_0}\dfrac{|Q|q_1}{r^2}$ and $q_1 = \dfrac{r^2 F_{Qq_1}}{\dfrac{1}{4\pi\epsilon_0}|Q|} = \dfrac{(0.0300\text{ m})^2(1.08\text{ N})}{(9.00\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(1.75\times10^{-6}\text{ C})} = 6.17\times10^{-8}\text{ C}$.

$q_2 = -q_1 = -6.17\times10^{-8}$ C.

EVALUATE: Simple reasoning allows us first to conclude that q_1 and q_2 must have equal magnitudes but opposite signs, which makes the equations much easier to set up than if we had tried to solve the problem in the general case. As Q accelerates and hence moves upward, the magnitude of the acceleration vector will change in a complicated way.

21.90. **IDENTIFY:** Use Eq. (21.7) to calculate the electric field due to a small slice of the line of charge and integrate as in Example 21.10. Use Eq. (21.3) to calculate $\vec{F}$.

SET UP: The electric field due to an infinitesimal segment of the line of charge is sketched in Figure 21.90.



$$\sin\theta = \frac{y}{\sqrt{x^2+y^2}}$$

$$\cos\theta = \frac{x}{\sqrt{x^2+y^2}}$$

Figure 21.90

Slice the charge distribution up into small pieces of length dy. The charge dQ in each slice is $dQ = Q(dy/a)$. The electric field this produces at a distance x along the x-axis is dE. Calculate the components of $d\vec{E}$ and then integrate over the charge distribution to find the components of the total field.

EXECUTE: $dE = \dfrac{1}{4\pi\epsilon_0}\left(\dfrac{dQ}{x^2+y^2}\right) = \dfrac{Q}{4\pi\epsilon_0 a}\left(\dfrac{dy}{x^2+y^2}\right)$

$$dE_x = dE\cos\theta = \frac{Qx}{4\pi\epsilon_0 a}\left(\frac{dy}{(x^2+y^2)^{3/2}}\right)$$

$$dE_y = -dE\sin\theta = -\frac{Q}{4\pi\epsilon_0 a}\left(\frac{ydy}{(x^2+y^2)^{3/2}}\right)$$

$$E_x = \int dE_x = -\frac{Qx}{4\pi\epsilon_0 a}\int_0^a \frac{dy}{(x^2+y^2)^{3/2}} = \frac{Qx}{4\pi\epsilon_0 a}\left[\frac{1}{x^2}\frac{y}{\sqrt{x^2+y^2}}\right]_0^a = \frac{Q}{4\pi\epsilon_0 x}\frac{1}{\sqrt{x^2+a^2}}$$

$$E_y = \int dEy = -\frac{Q}{4\pi\epsilon_0 a}\int_0^a \frac{ydy}{(x^2+y^2)^{3/2}} = -\frac{Q}{4\pi\epsilon_0 a}\left[-\frac{1}{\sqrt{x^2+y^2}}\right]_0^a = -\frac{Q}{4\pi\epsilon_0 a}\left(\frac{1}{x}-\frac{1}{\sqrt{x^2+a^2}}\right)$$

(b) $\vec{F} = q_0\vec{E}$

$$F_x = -qE_x = \frac{-qQ}{4\pi\epsilon_0 x}\frac{1}{\sqrt{x^2+a^2}};\; F_y = -qE_y = \frac{qQ}{4\pi\epsilon_0 a}\left(\frac{1}{x}-\frac{1}{\sqrt{x^2+a^2}}\right)$$

(c) For $x \gg a$, $\dfrac{1}{\sqrt{x^2+a^2}} = \dfrac{1}{x}\left(1+\dfrac{a^2}{x^2}\right)^{-1/2} = \dfrac{1}{x}\left(1-\dfrac{a^2}{2x^2}\right) = \dfrac{1}{x}-\dfrac{a^2}{2x^3}$

$$F_x \approx -\frac{qQ}{4\pi\epsilon_0 x^2},\; F_y \approx \frac{qQ}{4\pi\epsilon_0 a}\left(\frac{1}{x}-\frac{1}{x}+\frac{a^2}{2x^3}\right) = \frac{qQa}{8\pi\epsilon_0 x^3}$$

EVALUATE: For $x \gg a$, $F_y \ll F_x$ and $F \approx |F_x| = \dfrac{qQ}{4\pi\epsilon_0 x^2}$ and $\vec{F}$ is in the $-x$-direction. For $x \gg a$ the charge distribution Q acts like a point charge.

21.104. IDENTIFY: Apply Eq. (21.11) for the electric field of a disk. The hole can be described by adding a disk of charge density $-\sigma$ and radius R_1 to a solid disk of charge density $+\sigma$ and radius R_2.

SET UP: The area of the annulus is $\pi(R_2^2 - R_1^2)\sigma$. The electric field of a disk, Eq. (21.11) is

$$E = \frac{\sigma}{2\epsilon_0}\left[1 - 1/\sqrt{(R/x)^2 + 1}\right].$$

EXECUTE: **(a)** $Q = A\sigma = \pi(R_2^2 - R_1^2)\sigma$

(b) $\vec{E}(x) = \frac{\sigma}{2\epsilon_0}\left(\left[1 - 1/\sqrt{(R_2/x)^2 + 1}\right] - \left[1 - 1/\sqrt{(R_1/x)^2 + 1}\right]\right)\frac{|x|}{x}\hat{i}.$

$\vec{E}(x) = \frac{\sigma}{2\epsilon_0}\left(1/\sqrt{(R_1/x)^2 + 1} - 1/\sqrt{(R_2/x)^2 + 1}\right)\frac{|x|}{x}\hat{i}$. The electric field is in the $+x$-direction at points above the disk and in the $-x$-direction at points below the disk, and the factor $\frac{|x|}{x}\hat{i}$ specifies these directions.

(c) Note that $1/\sqrt{(R_1/x)^2 + 1} = \frac{|x|}{R_1}(1 + (x/R_1)^2)^{-1/2} \approx \frac{|x|}{R_1}$. This gives

$\vec{E}(x) = \frac{\sigma}{2\epsilon_0}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)\frac{|x|^2}{x}\hat{i} = \frac{\sigma}{2\epsilon_0}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)x\hat{i}$. Sufficiently close means that $(x/R_1)^2 \ll 1$.

(d) $F_x = -qE_x = -\frac{q\sigma}{2\epsilon_0}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)x$. The force is in the form of Hooke's law: $F_x = -kx$, with

$k = \frac{q\sigma}{2\epsilon_0}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$. $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{q\sigma}{2\epsilon_0 m}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}$.

EVALUATE: The frequency is independent of the initial position of the particle, so long as this position is sufficiently close to the center of the annulus for $(x/R_1)^2$ to be small.