24.3. **IDENTIFY and SET UP:** It is a parallel-plate air capacitor, so we can apply the equations of Section 24.1.

EXECUTE: **(a)** $C = \dfrac{Q}{V_{ab}}$ so $V_{ab} = \dfrac{Q}{C} = \dfrac{0.148\times10^{-6}\ \text{C}}{245\times10^{-12}\ \text{F}} = 604\ \text{V}$

(b) $C = \dfrac{\epsilon_0 A}{d}$ so $A = \dfrac{Cd}{\epsilon_0} = \dfrac{(245\times10^{-12}\ \text{F})(0.328\times10^{-3}\ \text{m})}{8.854\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2} = 9.08\times10^{-3}\ \text{m}^2 = 90.8\ \text{cm}^2$

(c) $V_{ab} = Ed$ so $E = \dfrac{V_{ab}}{d} = \dfrac{604\ \text{V}}{0.328\times10^{-3}\ \text{m}} = 1.84\times10^{6}\ \text{V/m}$

(d) $E = \dfrac{\sigma}{\epsilon_0}$ so $\sigma = E\epsilon_0 = (1.84\times10^{6}\ \text{V/m})(8.854\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2) = 1.63\times10^{-5}\ \text{C/m}^2$

EVALUATE: We could also calculate σ directly as Q/A. $\sigma = \dfrac{Q}{A} = \dfrac{0.148\times10^{-6}\ \text{C}}{9.08\times10^{-3}\ \text{m}^2} = 1.63\times10^{-5}\ \text{C/m}^2$, which checks.

24.5. **IDENTIFY:** $C = \dfrac{Q}{V_{ab}}$. $C = \dfrac{\epsilon_0 A}{d}$.

SET UP: When the capacitor is connected to the battery, $V_{ab} = 12.0\ \text{V}$.

EXECUTE: **(a)** $Q = CV_{ab} = (10.0\times10^{-6}\ \text{F})(12.0\ \text{V}) = 1.20\times10^{-4}\ \text{C} = 120\ \mu\text{C}$

(b) When d is doubled C is halved, so Q is halved. $Q = 60\ \mu\text{C}$.

(c) If r is doubled, A increases by a factor of 4. C increases by a factor of 4 and Q increases by a factor of 4. $Q = 480\ \mu\text{C}$.

EVALUATE: When the plates are moved apart, less charge on the plates is required to produce the same potential difference. With the separation of the plates constant, the electric field must remain constant to produce the same potential difference. The electric field depends on the surface charge density, σ. To produce the same σ, more charge is required when the area increases.

24.12. **IDENTIFY and SET UP:** Use the expression for C/L derived in Example 24.4. Then use Eq. (24.1) to calculate Q.

EXECUTE: **(a)** From Example 24.4, $\dfrac{C}{L} = \dfrac{2\pi\epsilon_0}{\ln(r_b/r_a)}$

$$\frac{C}{L} = \frac{2\pi(8.854\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)}{\ln(3.5\ \text{mm}/1.5\ \text{mm})} = 6.57\times10^{-11}\ \text{F/m} = 66\ \text{pF/m}$$

(b) $C = (6.57\times10^{-11}\ \text{F/m})(2.8\ \text{m}) = 1.84\times10^{-10}\ \text{F}$.

$Q = CV = (1.84\times10^{-10}\ \text{F})(350\times10^{-3}\ \text{V}) = 6.4\times10^{-11}\ \text{C} = 64\ \text{pC}$

The conductor at higher potential has the positive charge, so there is +64 pC on the inner conductor and −64 pC on the outer conductor.

EVALUATE: C depends only on the dimensions of the capacitor. Q and V are proportional.

24.14. **IDENTIFY:** Apply the results of Example 24.3. $C = Q/V$.

SET UP: $r_a = 15.0\ \text{cm}$. Solve for r_b.

EXECUTE: **(a)** For two concentric spherical shells, the capacitance is $C = \dfrac{1}{k}\left(\dfrac{r_a r_b}{r_b - r_a}\right)$. $kCr_b - kCr_a = r_a r_b$

and $r_b = \dfrac{kCr_a}{kC - r_a} = \dfrac{k(116\times10^{-12}\ \text{F})(0.150\ \text{m})}{k(116\times10^{-12}\ \text{F}) - 0.150\ \text{m}} = 0.175\ \text{m}$.

(b) $V = 220\ \text{V}$ and $Q = CV = (116\times10^{-12}\ \text{F})(220\ \text{V}) = 2.55\times10^{-8}\ \text{C}$.

EVALUATE: A parallel-plate capacitor with $A = 4\pi r_a r_b = 0.33\ \text{m}^2$ and $d = r_b - r_a = 2.5\times10^{-2}\ \text{m}$ has $C = \dfrac{\epsilon_0 A}{d} = 117\ \text{pF}$, in excellent agreement with the value of C for the spherical capacitor.

24.25. **IDENTIFY and SET UP:** The energy density is given by Eq. (24.11): $u = \frac{1}{2}\epsilon_0 E^2$. Use $V = Ed$ to solve for E.

EXECUTE: Calculate E: $E = \dfrac{V}{d} = \dfrac{400\ \text{V}}{5.00\times10^{-3}\ \text{m}} = 8.00\times10^{4}\ \text{V/m}$.

Then $u = \frac{1}{2}\epsilon_0 E^2 = \frac{1}{2}(8.854\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(8.00\times10^4\ \text{V/m})^2 = 0.0283\ \text{J/m}^3$.

EVALUATE: E is smaller than the value in Example 24.8 by about a factor of 6 so u is smaller by about a factor of $6^2 = 36$.

24.34. **IDENTIFY:** Apply Eq. (24.11).

SET UP: Example 24.3 shows that $E = \dfrac{Q}{4\pi\epsilon_0 r^2}$ between the conducting shells and that

$\dfrac{Q}{4\pi\epsilon_0} = \left(\dfrac{r_a r_b}{r_b - r_a}\right)V_{ab}$.

EXECUTE: $E = \left(\dfrac{r_a r_b}{r_b - r_a}\right)\dfrac{V_{ab}}{r^2} = \left(\dfrac{[0.125\ \text{m}][0.148\ \text{m}]}{0.148\ \text{m} - 0.125\ \text{m}}\right)\dfrac{120\ \text{V}}{r^2} = \dfrac{96.5\ \text{V}\cdot\text{m}}{r^2}$

(a) For $r = 0.126\ \text{m}$, $E = 6.08\times10^3\ \text{V/m}$. $u = \frac{1}{2}\epsilon_0 E^2 = 1.64\times10^{-4}\ \text{J/m}^3$.

(b) For $r = 0.147\ \text{m}$, $E = 4.47\times10^3\ \text{V/m}$. $u = \frac{1}{2}\epsilon_0 E^2 = 8.85\times10^{-5}\ \text{J/m}^3$.

EVALUATE: **(c)** No, the results of parts (a) and (b) show that the energy density is not uniform in the region between the plates. E decreases as r increases, so u decreases also.

24.47. **IDENTIFY:** $P = E/t$, where E is the total light energy output. The energy stored in the capacitor is $U = \frac{1}{2}CV^2$.

SET UP: $E = 0.95U$

EXECUTE: **(a)** The power output is $2.70\times10^5\ \text{W}$, and 95% of the original energy is converted, so

$E = Pt = (2.70\times10^5\ \text{W})(1.48\times10^{-3}\ \text{s}) = 400\ \text{J}$. $U = \dfrac{400\ \text{J}}{0.95} = 421\ \text{J}$.

(b) $U = \frac{1}{2}CV^2$ so $C = \dfrac{2U}{V^2} = \dfrac{2(421\ \text{J})}{(125\ \text{V})^2} = 0.054\ \text{F}$.

EVALUATE: For a given V, the stored energy increases linearly with C.

24.70. **IDENTIFY:** The electric field energy density is $u = \frac{1}{2}\epsilon_0 E^2$. $U = \dfrac{Q^2}{2C}$.

SET UP: For this charge distribution, $E = 0$ for $r < r_a$, $E = \dfrac{\lambda}{2\pi\epsilon_0 r}$ for $r_a < r < r_b$ and $E = 0$ for $r > r_b$.

Example 24.4 shows that $\dfrac{U}{L} = \dfrac{2\pi\epsilon_0}{\ln(r_b/r_a)}$ for a cylindrical capacitor.

EXECUTE: **(a)** $u = \frac{1}{2}\epsilon_0 E^2 = \frac{1}{2}\epsilon_0\left(\dfrac{\lambda}{2\pi\epsilon_0 r}\right)^2 = \dfrac{\lambda^2}{8\pi^2\epsilon_0 r^2}$

(b) $U = \int u\,dV = 2\pi L\int u r\,dr = \dfrac{L\lambda^2}{4\pi\epsilon_0}\displaystyle\int_{r_a}^{r_b}\dfrac{dr}{r}$ and $\dfrac{U}{L} = \dfrac{\lambda^2}{4\pi\epsilon_0}\ln(r_b/r_a)$.

(c) Using Eq. (24.9), $U = \dfrac{Q^2}{2C} = \dfrac{Q^2}{4\pi\epsilon_0 L}\ln(r_b/r_a) = \dfrac{\lambda^2 L}{4\pi\epsilon_0}\ln(r_b/r_a)$. This agrees with the result of part (b).

EVALUATE: We could have used the results of part (b) and $U = \dfrac{Q^2}{2C}$ to calculate C/L and would obtain the same result as in Example 24.4.