25.32. **IDENTIFY:** The sum of the potential changes around the circuit loop is zero. Potential decreases by IR when going through a resistor in the direction of the current and increases by $\mathcal{E}$ when passing through an emf in the direction from the $-$ to $+$ terminal.

SET UP: The current is counterclockwise, because the 16-V battery determines the direction of current flow.

EXECUTE: $+16.0\text{ V}-8.0\text{ V}-I(1.6\,\Omega+5.0\,\Omega+1.4\,\Omega+9.0\,\Omega)=0$

$$I=\frac{16.0\text{ V}-8.0\text{ V}}{1.6\,\Omega+5.0\,\Omega+1.4\,\Omega+9.0\,\Omega}=0.47\text{ A}$$

(b) $V_b+16.0\text{ V}-I(1.6\,\Omega)=V_a$, so $V_a-V_b=V_{ab}=16.0\text{ V}-(1.6\,\Omega)(0.47\text{ A})=15.2\text{ V}$.

(c) $V_c+8.0\text{ V}+I(1.4\,\Omega+5.0\,\Omega)=V_a$ so $V_{ac}=(5.0\,\Omega)(0.47\text{ A})+(1.4\,\Omega)(0.47\text{ A})+8.0\text{ V}=11.0\text{ V}$.

25.58. **IDENTIFY:** Conservation of charge requires that the current is the same in both sections. The voltage drops across each section add, so $R=R_{\text{Cu}}+R_{\text{Ag}}$. The total resistance is the sum of the resistances of each section. $E=\rho J=\frac{\rho I}{A}$, so $E=\frac{IR}{L}$, where R is the resistance of a section and L is its length.

SET UP: For copper, $\rho_{\text{Cu}}=1.72\times10^{-8}\,\Omega\cdot\text{m}$. For silver, $\rho_{\text{Ag}}=1.47\times10^{-8}\,\Omega\cdot\text{m}$.

EXECUTE: **(a)** $I=\frac{V}{R}=\frac{V}{R_{\text{Cu}}+R_{\text{Ag}}}$. $R_{\text{Cu}}=\frac{\rho_{\text{Cu}}L_{\text{Cu}}}{A_{\text{Cu}}}=\frac{(1.72\times10^{-8}\,\Omega\cdot\text{m})(0.8\text{ m})}{(\pi/4)(6.0\times10^{-4}\text{m})^2}=0.049\,\Omega$ and

$R_{\text{Ag}}=\frac{\rho_{\text{Ag}}L_{\text{Ag}}}{A_{\text{Ag}}}=\frac{(1.47\times10^{-8}\,\Omega\cdot\text{m})(1.2\text{ m})}{(\pi/4)(6.0\times10^{-4}\text{ m})^2}=0.062\,\Omega$. This gives $I=\frac{5.0\text{ V}}{0.049\,\Omega+0.062\,\Omega}=45\text{ A}$.

The current in the copper wire is 45 A.

(b) The current in the silver wire is 45 A, the same as that in the copper wire or else charge would build up at their interface.

(c) $E_{\text{Cu}}=J\rho_{\text{Cu}}=\frac{IR_{\text{Cu}}}{L_{\text{Cu}}}=\frac{(45\text{ A})(0.049\,\Omega)}{0.8\text{ m}}=2.76\text{ V/m}$.

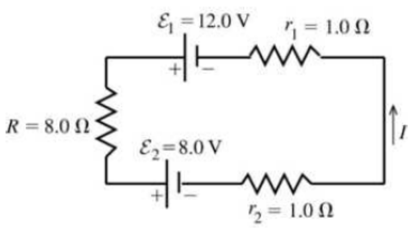
(d) $E_{\text{Ag}}=J\rho_{\text{Ag}}=\frac{IR_{\text{Ag}}}{L_{\text{Ag}}}=\frac{(45\text{ A})(0.062\,\Omega)}{1.2\text{ m}}=2.33\text{ V/m}$.

(e) $V_{\text{Ag}}=IR_{\text{Ag}}=(45\text{ A})(0.062\,\Omega)=2.79\text{ V}$.

EVALUATE: For the copper section, $V_{\text{Cu}}=IR_{\text{Cu}}=2.21\text{ V}$. Note that $V_{\text{Cu}}+V_{\text{Ag}}=5.0\text{ V}$, the voltage applied across the ends of the composite wire.

25.79. **(a) IDENTIFY:** Set the sum of the potential rises and drops around the circuit equal to zero and solve the resulting equation for the current I. Apply Eq. (25.17) to each circuit element to find the power associated with it.

SET UP: The circuit is sketched in Figure 25.79.



EXECUTE: $\mathcal{E}_1-\mathcal{E}_2-I(r_1+r_2+R)=0$

$$I=\frac{\mathcal{E}_1-\mathcal{E}_2}{r_1+r_2+R}$$

$$I=\frac{12.0\text{ V}-8.0\text{ V}}{1.0\,\Omega+1.0\,\Omega+8.0\,\Omega}$$

$I=0.40\text{ A}$

Figure 25.79

(b) $P=I^2R+I^2r_1+I^2r_2=I^2(R+r_1+r_2)=(0.40\text{ A})^2(8.0\,\Omega+1.0\,\Omega+1.0\,\Omega)$

$P=1.6\text{ W}$

(c) Chemical energy is converted to electrical energy in a battery when the current goes through the battery from the negative to the positive terminal, so the electrical energy of the charges increases as the current passes through. This happens in the 12.0-V battery, and the rate of production of electrical energy is $P = \mathcal{E}_1 I = (12.0\text{ V})(0.40\text{ A}) = 4.8\text{ W}$.

(d) Electrical energy is converted to chemical energy in a battery when the current goes through the battery from the positive to the negative terminal, so the electrical energy of the charges decreases as the current passes through. This happens in the 8.0-V battery, and the rate of consumption of electrical energy is $P = \mathcal{E}_2 I = (8.0\text{ V})(0.40\text{ V}) = 3.2\text{ W}$.

(e) EVALUATE: Total rate of production of electrical energy $= 4.8\text{ W}$. Total rate of consumption of electrical energy $= 1.6\text{ W} + 3.2\text{ W} = 4.8\text{ W}$, which equals the rate of production, as it must.

26.14. **IDENTIFY:** Replace the series combinations of resistors by their equivalents. In the resulting parallel network the battery voltage is the voltage across each resistor.

SET UP: The circuit is sketched in Figure 26.14a.

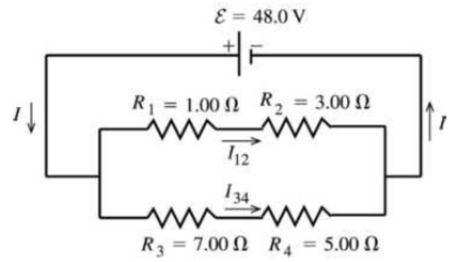


Figure 26.14a

EXECUTE: R_1 and R_2 in series have an equivalent resistance of $R_{12} = R_1 + R_2 = 4.00\ \Omega$. R_3 and R_4 in series have an equivalent resistance of $R_{34} = R_3 + R_4 = 12.0\ \Omega$.

The circuit is equivalent to the circuit sketched in Figure 26.14b.

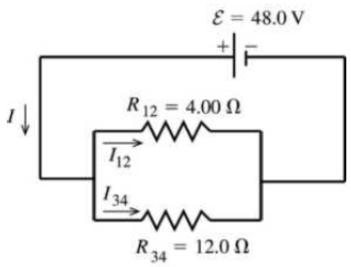


Figure 26.14b

R_{12} and R_{34} in parallel are equivalent to R_{eq} given by $\dfrac{1}{R_{eq}} = \dfrac{1}{R_{12}} + \dfrac{1}{R_{34}} = \dfrac{R_{12} + R_{34}}{R_{12}R_{34}}$

$$R_{eq} = \frac{R_{12}R_{34}}{R_{12} + R_{34}}$$

$$R_{eq} = \frac{(4.00\ \Omega)(12.0\ \Omega)}{4.00\ \Omega + 12.0\ \Omega} = 3.00\ \Omega$$

The voltage across each branch of the parallel combination is ε, so $\varepsilon - I_{12}R_{12} = 0$.

$$I_{12} = \frac{\varepsilon}{R_{12}} = \frac{48.0\text{ V}}{4.00\ \Omega} = 12.0\text{ A}$$

$$\varepsilon - I_{34}R_{34} = 0 \text{ so } I_{34} = \frac{\varepsilon}{R_{34}} = \frac{48.0\text{ V}}{12.0\ \Omega} = 4.0\text{ A}$$

The current is 12.0 A through the $1.00\text{-}\Omega$ and $3.00\text{-}\Omega$ resistors, and it is 4.0 A through the $7.00\text{-}\Omega$ and $5.00\text{-}\Omega$ resistors.

EVALUATE: The current through the battery is $I = I_{12} + I_{34} = 12.0\text{ A} + 4.0\text{ A} = 16.0\text{ A}$, and this is equal to $\varepsilon / R_{eq} = 48.0\text{ V}/3.00\ \Omega = 16.0\text{ A}$.

26.26. **IDENTIFY:** Apply the loop rule and junction rule.

SET UP: The circuit diagram is given in Figure 26.26. The junction rule has been used to find the magnitude and direction of the current in the middle branch of the circuit. There are no remaining unknown currents.

EXECUTE: The loop rule applied to loop (1) gives:

$+20.0\text{V} - (1.00\text{ A})(1.00\,\Omega) + (1.00\text{ A})(4.00\,\Omega) + (1.00\text{ A})(1.00\,\Omega) - \mathcal{E}_1 - (1.00\text{ A})(6.00\,\Omega) = 0$

$\mathcal{E}_1 = 20.0\text{ V} - 1.00\text{ V} + 4.00\text{ V} + 1.00\text{ V} - 6.00\text{ V} = 18.0\text{ V}$. The loop rule applied to loop (2) gives:

$+20.0\text{ V} - (1.00\text{ A})(1.00\,\Omega) - (2.00\text{ A})(1.00\,\Omega) - \mathcal{E}_2 - (2.00\text{ A})(2.00\,\Omega) - (1.00\text{ A})(6.00\,\Omega) = 0$

$\mathcal{E}_2 = 20.0\text{ V} - 1.00\text{ V} - 2.00\text{ V} - 4.00\text{ V} - 6.00\text{ V} = 7.0\text{ V}$. Going from b to a along the lower branch, $V_b + (2.00\text{ A})(2.00\,\Omega) + 7.0\text{ V} + (2.00\text{ A})(1.00\,\Omega) = V_a \cdot V_b - V_a = -13.0\text{ V}$; point b is at 13.0 V lower potential than point a.

EVALUATE: We can also calculate $V_b - V_a$ by going from b to a along the upper branch of the circuit. $V_b - (1.00\text{ A})(6.00\,\Omega) + 20.0\text{ V} - (1.00\text{ A})(1.00\,\Omega) = V_a$ and $V_b - V_a = -13.0\text{ V}$. This agrees with $V_b - V_a$ calculated along a different path between b and a.

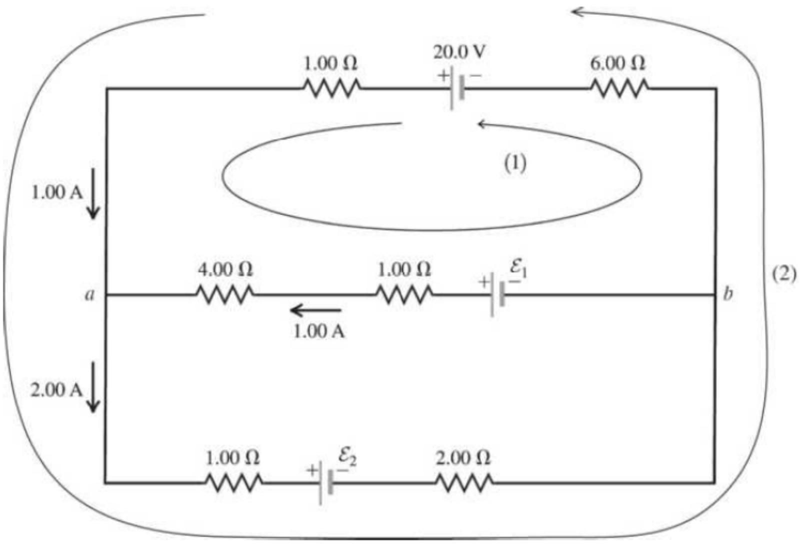


Figure 26.26

26.27. **IDENTIFY:** Apply the junction rule at points a, b, c and d to calculate the unknown currents. Then apply the loop rule to three loops to calculate $\mathcal{E}_1, \mathcal{E}_2$ and R.

(a) SET UP: The circuit is sketched in Figure 26.27.

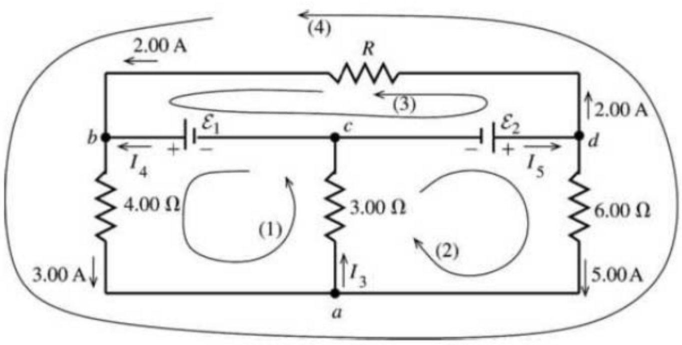


Figure 26.27

EXECUTE: Apply the junction rule to point a: $3.00\text{ A} + 5.00\text{ A} - I_3 = 0$

$I_3 = 8.00\text{ A}$

Apply the junction rule to point b: $2.00\text{ A} + I_4 - 3.00\text{ A} = 0$

$I_4 = 1.00\text{ A}$

Apply the junction rule to point c: $I_3 - I_4 - I_5 = 0$

$I_5 = I_3 - I_4 = 8.00\text{ A} - 1.00\text{ A} = 7.00\text{ A}$

EVALUATE: As a check, apply the junction rule to point d: $I_5 - 2.00\text{ A} - 5.00\text{ A} = 0$

$I_5 = 7.00\text{ A}$

(b) EXECUTE: Apply the loop rule to loop (1): $\varepsilon_1 - (3.00\text{ A})(4.00\ \Omega) - I_3(3.00\ \Omega) = 0$

$\varepsilon_1 = 12.0\text{ V} + (8.00\text{ A})(3.00\ \Omega) = 36.0\text{ V}$

Apply the loop rule to loop (2): $\varepsilon_2 - (5.00\text{ A})(6.00\ \Omega) - I_3(3.00\ \Omega) = 0$

$\varepsilon_2 = 30.0\text{ V} + (8.00\text{ A})(3.00\ \Omega) = 54.0\text{ V}$

(c) Apply the loop rule to loop (3): $-(2.00\text{ A})R - \varepsilon_1 + \varepsilon_2 = 0$

$$R = \frac{\varepsilon_2 - \varepsilon_1}{2.00\text{ A}} = \frac{54.0\text{ V} - 36.0\text{ V}}{2.00\text{ A}} = 9.00\ \Omega$$

EVALUATE: Apply the loop rule to loop (4) as a check of our calculations:

$-(2.00\text{ A})R - (3.00\text{ A})(4.00\ \Omega) + (5.00\text{ A})(6.00\ \Omega) = 0$

$-(2.00\text{ A})(9.00\ \Omega) - 12.0\text{ V} + 30.0\text{ V} = 0$

$-18.0\text{ V} + 18.0\text{ V} = 0$

26.61. **IDENTIFY:** The ohmmeter reads the equivalent resistance between points a and b. Replace series and parallel combinations by their equivalent.

SET UP: For resistors in parallel, $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$. For resistors in series, $R_{eq} = R_1 + R_2$.

EXECUTE: Circuit (a): The $75.0\text{-}\Omega$ and $40.0\text{-}\Omega$ resistors are in parallel and have equivalent resistance $26.09\ \Omega$. The $25.0\text{-}\Omega$ and $50.0\text{-}\Omega$ resistors are in parallel and have an equivalent resistance of $16.67\ \Omega$. The equivalent network is given in Figure 26.61a. $\frac{1}{R_{eq}} = \frac{1}{100.0\ \Omega} + \frac{1}{23.05\ \Omega}$, so $R_{eq} = 18.7\ \Omega$.

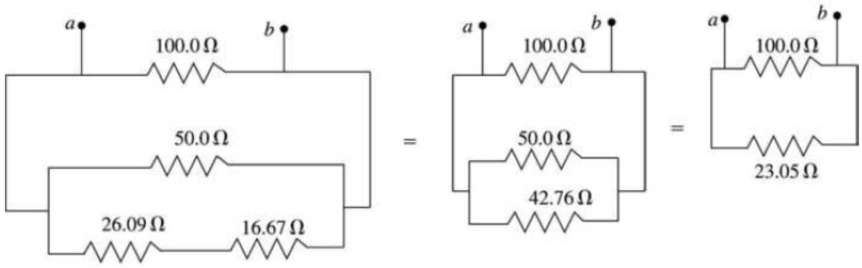


Figure 26.61a

Circuit (b): The $30.0\text{-}\Omega$ and $45.0\text{-}\Omega$ resistors are in parallel and have equivalent resistance $18.0\ \Omega$. The equivalent network is given in Figure 26.61b. $\frac{1}{R_{eq}} = \frac{1}{10.0\ \Omega} + \frac{1}{30.3\ \Omega}$, so $R_{eq} = 7.5\ \Omega$.

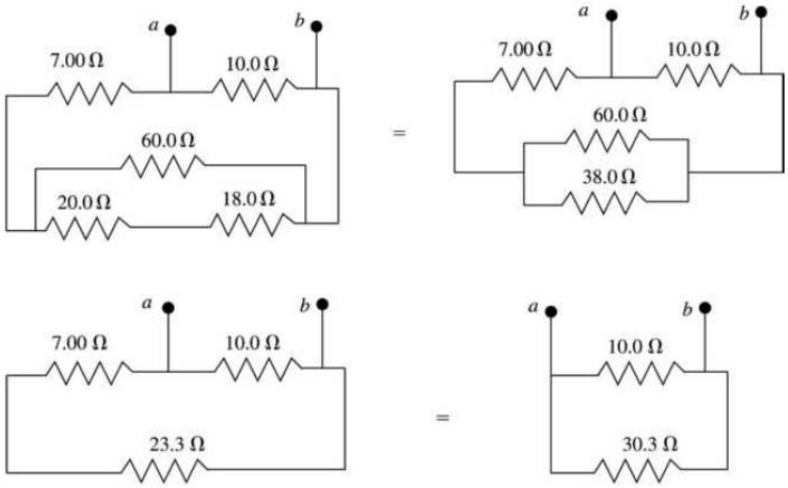


Figure 26.61b

EVALUATE: In circuit (a) the resistance along one path between a and b is $100.0\ \Omega$, but that is not the equivalent resistance between these points. A similar comment can be made about circuit (b).

26.63. **IDENTIFY:** Apply the junction rule to express the currents through the $5.00\text{-}\Omega$ and $8.00\text{-}\Omega$ resistors in terms of I_1, I_2 and I_3. Apply the loop rule to three loops to get three equations in the three unknown currents.

SET UP: The circuit is sketched in Figure 26.63.

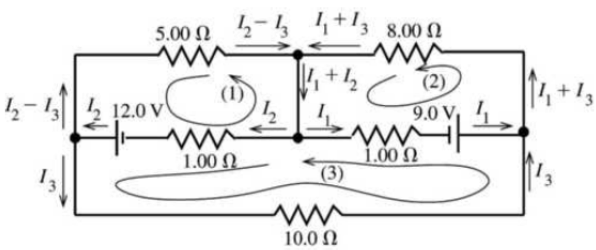


Figure 26.63

The current in each branch has been written in terms of I_1, I_2 and I_3 such that the junction rule is satisfied at each junction point.

EXECUTE: Apply the loop rule to loop (1).

$-12.0\ \text{V} + I_2(1.00\ \Omega) + (I_2 - I_3)(5.00\ \Omega) = 0$

$I_2(6.00\ \Omega) - I_3(5.00\ \Omega) = 12.0\ \text{V}$ eq. (1)

Apply the loop rule to loop (2).

$-I_1(1.00\ \Omega) + 9.00\ \text{V} - (I_1 + I_3)(8.00\ \Omega) = 0$

$I_1(9.00\ \Omega) + I_3(8.00\ \Omega) = 9.00\ \text{V}$ eq. (2)

Apply the loop rule to loop (3).

$-I_3(10.0\ \Omega) - 9.00\ \text{V} + I_1(1.00\ \Omega) - I_2(1.00\ \Omega) + 12.0\ \text{V} = 0$

$-I_1(1.00\ \Omega) + I_2(1.00\ \Omega) + I_3(10.0\ \Omega) = 3.00\ \text{V}$ eq. (3)

Eq. (1) gives $I_2 = 2.00\ \text{A} + \frac{5}{6}I_3$; eq. (2) gives $I_1 = 1.00\ \text{A} - \frac{8}{9}I_3$

Using these results in eq. (3) gives $-(1.00\ \text{A} - \frac{8}{9}I_3)(1.00\ \Omega) + (2.00\ \text{A} + \frac{5}{6}I_3)(1.00\ \Omega) + I_3(10.0\ \Omega) = 3.00\ \text{V}$

$(\frac{16+15+180}{18})I_3 = 2.00\ \text{A};\ I_3 = \frac{18}{211}(2.00\ \text{A}) = 0.171\ \text{A}$

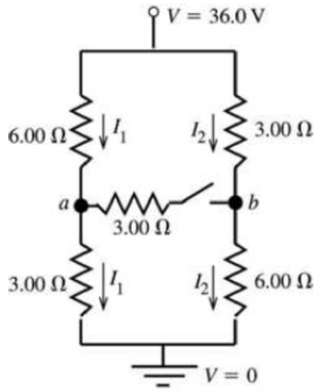
Then $I_2 = 2.00\text{ A} + \frac{5}{6}I_3 = 2.00\text{ A} + \frac{5}{6}(0.171\text{ A}) = 2.14\text{ A}$ and

$I_1 = 1.00\text{ A} - \frac{8}{9}I_3 = 1.00\text{ A} - \frac{8}{9}(0.171\text{ A}) = 0.848\text{ A}.$

EVALUATE: We could check that the loop rule is satisfied for a loop that goes through the $5.00\text{-}\Omega$, $8.00\text{-}\Omega$ and $10.0\text{-}\Omega$ resistors. Going around the loop clockwise:

$-(I_2 - I_3)(5.00\ \Omega) + (I_1 + I_3)(8.00\ \Omega) + I_3(10.0\ \Omega) = -9.85\text{ V} + 8.15\text{ V} + 1.71\text{ V}$, which does equal zero, apart from rounding.

26.77. **(a) IDENTIFY and SET UP:** The circuit is sketched in Figure 26.77a.



With the switch open there is no current through it and there are only the two currents I_1 and I_2 indicated in the sketch.

Figure 26.77a

The potential drop across each parallel branch is 36.0 V. Use this fact to calculate I_1 and I_2. Then travel from point a to point b and keep track of the potential rises and drops in order to calculate V_{ab}.

EXECUTE: $-I_1(6.00\ \Omega + 3.00\ \Omega) + 36.0\text{ V} = 0$

$$I_1 = \frac{36.0\text{ V}}{6.00\ \Omega + 3.00\ \Omega} = 4.00\text{ A}$$

$-I_2(3.00\ \Omega + 6.00\ \Omega) + 36.0\text{ V} = 0$

$$I_2 = \frac{36.0\text{ V}}{3.00\ \Omega + 6.00\ \Omega} = 4.00\text{ A}$$

To calculate $V_{ab} = V_a - V_b$ start at point b and travel to point a, adding up all the potential rises and drops along the way. We can do this by going from b up through the $3.00\text{-}\Omega$ resistor:

$V_b + I_2(3.00\ \Omega) - I_1(6.00\ \Omega) = V_a$

$V_a - V_b = (4.00\text{ A})(3.00\ \Omega) - (4.00\text{ A})(6.00\ \Omega) = 12.0\text{ V} - 24.0\text{ V} = -12.0\text{ V}$

$V_{ab} = -12.0\text{ V}$ (point a is 12.0 V lower in potential than point b)

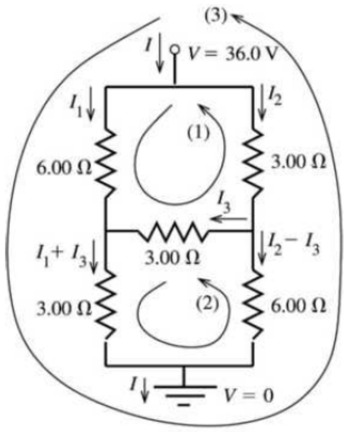
EVALUATE: Alternatively, we can go from point b down through the $6.00\text{-}\Omega$ resistor.

$V_b - I_2(6.00\ \Omega) + I_1(3.00\ \Omega) = V_a$

$V_a - V_b = -(4.00\text{ A})(6.00\ \Omega) + (4.00\text{ A})(3.00\ \Omega) = -24.0\text{ V} + 12.0\text{ V} = -12.0\text{ V}$, which checks.

(b) IDENTIFY: Now there are multiple current paths, as shown in Figure 26.77b. Use the junction rule to write the current in each branch in terms of three unknown currents I_1, I_2 and I_3. Apply the loop rule to three loops to get three equations for the three unknowns. The target variable is I_3, the current through the switch. R_{eq} is calculated from $V = IR_{eq}$, where I is the total current that passes through the network.

SET UP:



The three unknown currents I_1, I_2 and I_3 are labeled on Figure 26.77b.

Figure 26.77b

EXECUTE: Apply the loop rule to loops (1), (2) and (3).

loop (1): $-I_1(6.00\ \Omega) + I_3(3.00\ \Omega) + I_2(3.00\ \Omega) = 0$

$I_2 = 2I_1 - I_3$ eq. (1)

loop (2): $-(I_1 + I_3)(3.00\ \Omega) + (I_2 - I_3)(6.00\ \Omega) - I_3(3.00\ \Omega) = 0$

$6I_2 - 12I_3 - 3I_1 = 0$ so $2I_2 - 4I_3 - I_1 = 0$

Use eq (1) to replace I_2:

$4I_1 - 2I_3 - 4I_3 - I_1 = 0$

$3I_1 = 6I_3$ and $I_1 = 2I_3$ eq. (2)

loop (3) (This loop is completed through the battery [not shown], in the direction from the − to the + terminal.):

$-I_1(6.00\ \Omega) - (I_1 + I_3)(3.00\ \Omega) + 36.0\ \text{V} = 0$

$9I_1 + 3I_3 = 36.0$ A and $3I_1 + I_3 = 12.0$ A eq. (3)

Use eq. (2) in eq. (3) to replace I_1:

$3(2I_3) + I_3 = 12.0$ A

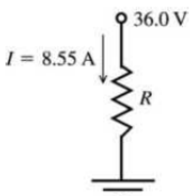
$I_3 = 12.0\ \text{A}/7 = 1.71$ A

$I_1 = 2I_3 = 3.42$ A

$I_2 = 2I_1 - I_3 = 2(3.42\ \text{A}) - 1.71\ \text{A} = 5.13$ A

The current through the switch is $I_3 = 1.71$ A.

(c) From the results in part (a) the current through the battery is $I = I_1 + I_2 = 3.42\ \text{A} + 5.13\ \text{A} = 8.55$ A. The equivalent circuit is a single resistor that produces the same current through the 36.0-V battery, as shown in Figure 26.77c.



$$-IR + 36.0\ \text{V} = 0$$

$$R = \frac{36.0\ \text{V}}{I} = \frac{36.0\ \text{V}}{8.55\ \text{A}} = 4.21\ \Omega$$

Figure 26.77c

EVALUATE: With the switch open (part a), point b is at higher potential than point a, so when the switch is closed the current flows in the direction from b to a. With the switch closed the circuit cannot be simplified using series and parallel combinations but there is still an equivalent resistance that represents the network.

26.91. **IDENTIFY:** Consider one segment of the network attached to the rest of the network.

SET UP: We can re-draw the circuit as shown in Figure 26.91.

EXECUTE: $R_T = 2R_1 + \left(\frac{1}{R_2} + \frac{1}{R_T}\right)^{-1} = 2R_1 + \frac{R_2 R_T}{R_2 + R_T}$. $R_T^2 - 2R_1 R_T - 2R_1 R_2 = 0$.

$R_T = R_1 \pm \sqrt{R_1^2 + 2R_1 R_2}$. $R_T > 0$, so $R_T = R_1 + \sqrt{R_1^2 + 2R_1 R_2}$.

EVALUATE: Even though there are an infinite number of resistors, the equivalent resistance of the network is finite.

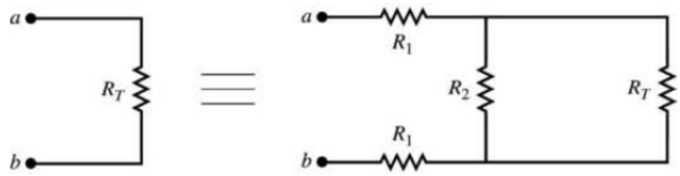


Figure 26.91

26.92. **IDENTIFY:** Assume a voltage V applied between points a and b and consider the currents that flow along each path between a and b.

SET UP: The currents are shown in Figure 26.92.

EXECUTE: Let current I enter at a and exit at b. At a there are three equivalent branches, so current is $I/3$ in each. At the next junction point there are two equivalent branches so each gets current $I/6$. Then at b there are three equivalent branches with current $I/3$ in each. The voltage drop from a to b then is $V = \left(\frac{I}{3}\right)R + \left(\frac{I}{6}\right)R + \left(\frac{I}{3}\right)R = \frac{5}{6}IR$. This must be the same as $V = IR_{eq}$, so $R_{eq} = \frac{5}{6}R$.

EVALUATE: The equivalent resistance is less than R, even though there are 12 resistors in the network.

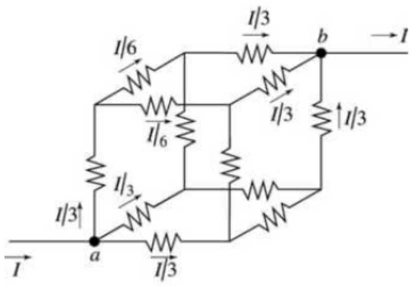


Figure 26.92