27.39. **IDENTIFY:** Apply $F = IlB\sin\phi$.

SET UP: Label the three segments in the field as a, b, and c. Let x be the length of segment a. Segment b has length 0.300 m and segment c has length $0.600\text{ m} - x$. Figure 27.39a shows the direction of the force on each segment. For each segment, $\phi = 90°$. The total force on the wire is the vector sum of the forces on each segment.

EXECUTE: $F_a = IlB = (4.50\text{ A})x(0.240\text{ T})$. $F_c = (4.50\text{ A})(0.600\text{ m} - x)(0.240\text{ T})$. Since $\vec{F}_a$ and $\vec{F}_c$ are in the same direction their vector sum has magnitude

$F_{ac} = F_a + F_c = (4.50\text{ A})(0.600\text{ m})(0.240\text{ T}) = 0.648\text{ N}$ and is directed toward the bottom of the page in Figure 27.39a. $F_b = (4.50\text{ A})(0.300\text{ m})(0.240\text{ T}) = 0.324\text{ N}$ and is directed to the right. The vector addition diagram for $\vec{F}_{ac}$ and $\vec{F}_b$ is given in Figure 27.39b.

$F = \sqrt{F_{ac}^2 + F_b^2} = \sqrt{(0.648\text{ N})^2 + (0.324\text{ N})^2} = 0.724\text{ N}$. $\tan\theta = \dfrac{F_{ac}}{F_b} = \dfrac{0.648\text{ N}}{0.324\text{ N}}$ and $\theta = 63.4°$. The net force has magnitude 0.724 N and its direction is specified by $\theta = 63.4°$ in Figure 27.39b.

EVALUATE: All three current segments are perpendicular to the magnetic field, so $\phi = 90°$ for each in the force equation. The direction of the force on a segment depends on the direction of the current for that segment.

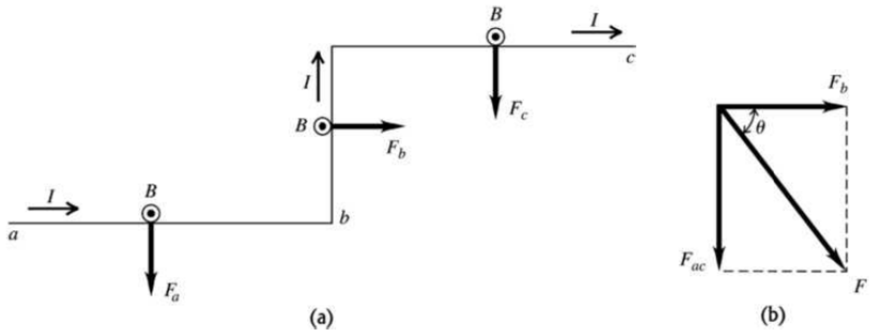


Figure 27.39

27.46. **IDENTIFY:** $\tau = IAB\sin\phi$, where ϕ is the angle between $\vec{B}$ and the normal to the loop.

SET UP: The coil as viewed along the axis of rotation is shown in Figure 27.46a for its original position and in Figure 27.46b after it has rotated $30.0°$.

EXECUTE: **(a)** The forces on each side of the coil are shown in Figure 27.46a. $\vec{F}_1 + \vec{F}_2 = 0$ and $\vec{F}_3 + \vec{F}_4 = 0$. The net force on the coil is zero. $\phi = 0°$ and $\sin\phi = 0$, so $\tau = 0$. The forces on the coil produce no torque.

(b) The net force is still zero. $\phi = 30.0°$ and the net torque is

$\tau = (1)(1.40\text{ A})(0.220\text{ m})(0.350\text{ m})(1.50\text{ T})\sin 30.0° = 0.0808\text{ N}\cdot\text{m}$. The net torque is clockwise in Figure 27.46b and is directed so as to increase the angle ϕ.

EVALUATE: For any current loop in a uniform magnetic field the net force on the loop is zero. The torque on the loop depends on the orientation of the plane of the loop relative to the magnetic field direction.

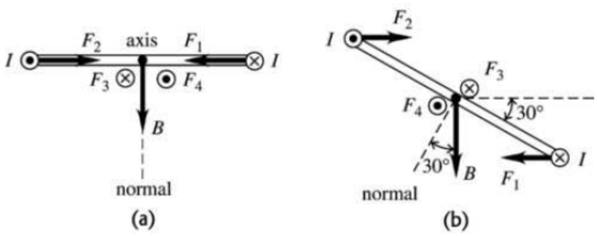


Figure 27.46

27.48. **IDENTIFY:** $\vec{\tau} = \vec{\mu} \times \vec{B}$ and $U = -\mu B\cos\phi$, where $\mu = NIB$. $\tau = \mu B\sin\phi$.

SET UP: ϕ is the angle between $\vec{B}$ and the normal to the plane of the loop.

EXECUTE: **(a)** $\phi = 90°$. $\tau = NIAB\sin(90°) = NIAB$, direction $\hat{k} \times \hat{j} = -\hat{i}$. $U = -\mu B\cos\phi = 0$.

(b) $\phi = 0$. $\tau = NIAB\sin(0) = 0$, no direction. $U = -\mu B\cos\phi = -NIAB$.

(c) $\phi = 90°$. $\tau = NIAB\sin(90°) = NIAB$, direction $-\hat{k} \times \hat{j} = \hat{i}$. $U = -\mu B\cos\phi = 0$.

(d) $\phi = 180°$: $\tau = NIAB\sin(180°) = 0$, no direction, $U = -\mu B\cos(180°) = NIAB$.

EVALUATE: When τ is maximum, $U = 0$. When $|U|$ is maximum, $\tau = 0$.

27.70. **IDENTIFY:** The current in the bar is downward, so the magnetic force on it is vertically upwards. The net force on the bar is equal to the magnetic force minus the gravitational force, so Newton's second law gives the acceleration. The bar is in parallel with the 10.0-Ω resistor, so we must use circuit analysis to find the initial current through it.

SET UP: First find the current. The equivalent resistance across the battery is $30.0\ \Omega$, so the total current is $4.00\ \text{A}$, half of which goes through the bar. Applying Newton's second law to the bar gives $\Sigma F = ma = F_B - mg = ILB - mg$.

EXECUTE: Equivalent resistance of the $10.0\text{-}\Omega$ resistor and the bar is $5.0\ \Omega$. Current through the $25.0\text{-}\Omega$ resistor is $I_{tot} = \dfrac{120.0\ \text{V}}{30.0\ \Omega} = 4.00\ \text{A}$. The current in the bar is 2.00 A, toward the bottom of the page. The force $\vec{F}_I$ that the magnetic field exerts on the bar has magnitude $F_I = IlB$ and is directed to the right. $a = \dfrac{F_I}{m} = \dfrac{IlB}{m} = \dfrac{(2.00\ \text{A})(1.50\ \text{m})(1.60\ \text{T})}{(2.60\ \text{N})/(9.80\text{m/s}^2)} = 18.1\ \text{m/s}^2$. $\vec{a}$ is directed to the right.

EVALUATE: Once the bar has acquired a non-zero speed there will be an induced emf (Chapter 29) and the current and acceleration will start to decrease.

27.80. **IDENTIFY:** Apply $\vec{F} = I\vec{l} \times \vec{B}$ to calculate the force on each side of the loop.

SET UP: The net force is the vector sum of the forces on each side of the loop.

EXECUTE: **(a)** $F_{PQ} = (5.00\ \text{A})(0.600\ \text{m})(3.00\ \text{T})\sin(0°) = 0\ \text{N}$.

$F_{RP} = (5.00\ \text{A})(0.800\ \text{m})(3.00\ \text{T})\sin(90°) = 12.0\ \text{N}$, into the page.

$F_{QR} = (5.00\ \text{A})(1.00\ \text{m})(3.00\ \text{T})(0.800/1.00) = 12.0\ \text{N}$, out of the page.

(b) The net force on the triangular loop of wire is zero.

(c) For calculating torque on a straight wire we can assume that the force on a wire is applied at the wire's center. Also, note that we are finding the torque with respect to the *PR*-axis (not about a point), and consequently the lever arm will be the distance from the wire's center to the *x*-axis. $\tau = rF\sin\phi$ gives $\tau_{PQ} = r(0\ \text{N}) = 0$, $\tau_{RP} = (0\ \text{m})F\sin\phi = 0$ and $\tau_{QR} = (0.300\ \text{m})(12.0\ \text{N})\sin(90°) = 3.60\ \text{N}\cdot\text{m}$. The net torque is $3.60\ \text{N}\cdot\text{m}$.

(d) According to Eq. (27.28), $\tau = NIAB\sin\phi = (1)(5.00\ \text{A})\left(\tfrac{1}{2}\right)(0.600\ \text{m})(0.800\ \text{m})(3.00\ \text{T})\sin(90°) = 3.60\ \text{N}\cdot\text{m}$, which agrees with part (c).

(e) Since F_{QR} is out of the page and since this is the force that produces the net torque, the point Q will be rotated out of the plane of the figure.

EVALUATE: In the expression $\tau = NIAB\sin\phi$, ϕ is the angle between the plane of the loop and the direction of $\vec{B}$. In this problem, $\phi = 90°$.

28.2. **IDENTIFY:** A moving charge creates a magnetic field as well as an electric field.

SET UP: The magnetic field caused by a moving charge is $B=\frac{\mu_0}{4\pi}\frac{qv\sin\phi}{r^2}$, and its electric field is $E=\frac{1}{4\pi\epsilon_0}\frac{e}{r^2}$ since $q=e$.

EXECUTE: Substitute the appropriate numbers into the above equations.

$$B=\frac{\mu_0}{4\pi}\frac{qv\sin\phi}{r^2}=\frac{4\pi\times10^{-7}\ \mathrm{T\cdot m/A}}{4\pi}\frac{(1.60\times10^{-19}\,\mathrm{C})(2.2\times10^{6}\,\mathrm{m/s})\sin 90^\circ}{(5.3\times10^{-11}\,\mathrm{m})^2}=13\ \mathrm{T},\ \text{out of the page.}$$

$$E=\frac{1}{4\pi\epsilon_0}\frac{e}{r^2}=\frac{(9.00\times10^{9}\,\mathrm{N\cdot m^2/C^2})(1.60\times10^{-19}\,\mathrm{C})}{(5.3\times10^{-11}\,\mathrm{m})^2}=5.1\times10^{11}\,\mathrm{N/C},\ \text{toward the electron.}$$

EVALUATE: There are enormous fields within the atom!

28.27. **IDENTIFY:** The net magnetic field at the center of the square is the vector sum of the fields due to each wire.

SET UP: For each wire, $B=\frac{\mu_0 I}{2\pi r}$ and the direction of $\vec{B}$ is given by the right-hand rule that is illustrated in Figure 28.6 in the textbook.

EXECUTE: **(a)** and **(b)** $B=0$ since the magnetic fields due to currents at opposite corners of the square cancel.

(c) The fields due to each wire are sketched in Figure 28.27.

$$B=B_a\cos45^\circ+B_b\cos45^\circ+B_c\cos45^\circ+B_d\cos45^\circ=4B_a\cos45^\circ=4\left(\frac{\mu_0 I}{2\pi r}\right)\cos45^\circ.$$

$r=\sqrt{(10\ \mathrm{cm})^2+(10\ \mathrm{cm})^2}=10\sqrt{2}\ \mathrm{cm}=0.10\sqrt{2}\ \mathrm{m}$, so

$$B=4\frac{(4\pi\times10^{-7}\ \mathrm{T\cdot m/A})(100\ \mathrm{A})}{2\pi(0.10\sqrt{2}\ \mathrm{m})}\cos 45^\circ=4.0\times10^{-4}\ \mathrm{T},\ \text{to the left.}$$

EVALUATE: In part (c), if all four currents are reversed in direction, the net field at the center of the square would be to the right.

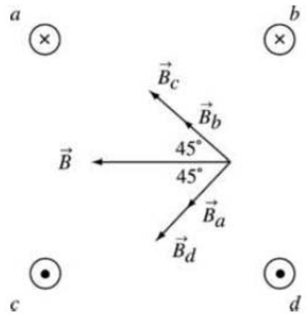
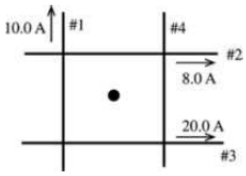


Figure 28.27

28.28. **IDENTIFY:** Use Eq. (28.9) and the right-hand rule to determine the field due to each wire. Set the sum of the four fields equal to zero and use that equation to solve for the field and the current of the fourth wire.

SET UP: The three known currents are shown in Figure 28.28.



$\vec{B}_1\otimes, \vec{B}_2\otimes, \vec{B}_3\odot$

$B=\frac{\mu_0 I}{2\pi r}$; $r=0.200$ m for each wire

Figure 28.28

Execute: Let $\odot$ be the positive z-direction. $I_1 = 10.0\text{ A}, I_2 = 8.0\text{ A}, I_3 = 20.0\text{ A}$. Then

$B_1 = 1.00\times10^{-5}\text{ T}$, $B_2 = 0.80\times10^{-5}\text{ T}$, and $B_3 = 2.00\times10^{-5}\text{ T}$.

$B_{1z} = -1.00\times10^{-5}\text{ T}, B_{2z} = -0.80\times10^{-5}\text{ T}, B_{3z} = +2.00\times10^{-5}\text{ T}$

$B_{1z} + B_{2z} + B_{3z} + B_{4z} = 0$

$B_{4z} = -(B_{1z} + B_{2z} + B_{3z}) = -2.0\times10^{-6}\text{ T}$

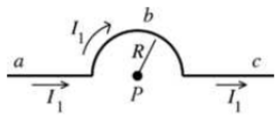
To give $\vec{B}_4$ in the $\otimes$ direction the current in wire 4 must be toward the bottom of the page.

$$B_4 = \frac{\mu_0 I}{2\pi r} \text{ so } I_4 = \frac{rB_4}{(\mu_0/2\pi)} = \frac{(0.200\text{ m})(2.0\times10^{-6}\text{ T})}{(2\times10^{-7}\text{ T}\cdot\text{m/A})} = 2.0\text{ A}$$

Evaluate: The fields of wires #2 and #3 are in opposite directions and their net field is the same as due to a current $20.0\text{ A} - 8.0\text{ A} = 12.0\text{ A}$ in one wire. The field of wire #4 must be in the same direction as that of wire #1, and $10.0\text{ A} + I_4 = 12.0\text{ A}$.

28.37. **Identify:** Calculate the magnetic field vector produced by each wire and add these fields to get the total field.

Set Up: First consider the field at P produced by the current I_1 in the upper semicircle of wire. See Figure 28.37a.

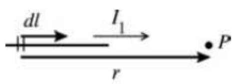


Consider the three parts of this wire
a: long straight section
b: semicircle
c: long, straight section

Figure 28.37a

Apply the Biot-Savart law $d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{l}\times\hat{r}}{r^2} = \frac{\mu_0}{4\pi}\frac{Id\vec{l}\times\vec{r}}{r^3}$ to each piece.

Execute: part a See Figure 28.37b.

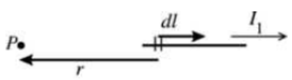


$d\vec{l}\times\vec{r} = 0$,
so $dB = 0$

Figure 28.37b

The same is true for all the infinitesimal segments that make up this piece of the wire, so $B = 0$ for this piece.

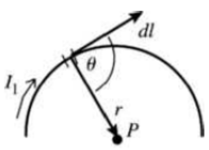
part c See Figure 28.37c.



$d\vec{l}\times\vec{r} = 0$,
so $dB = 0$ and $B = 0$ for this piece.

Figure 28.37c

part b See Figure 28.37d.



$d\vec{l}\times\vec{r}$ is directed into the paper for all infinitesimal segments that make up this semicircular piece, so $\vec{B}$ is directed into the paper and $B = \int dB$ (the vector sum of the $d\vec{B}$ is obtained by adding their magnitudes since they are in the same direction).

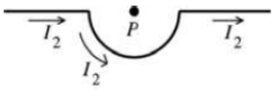
Figure 28.37d

$\left|d\vec{l} \times \vec{r}\right| = r dl \sin\theta$. The angle θ between $d\vec{l}$ and $\vec{r}$ is 90° and $r = R$, the radius of the semicircle. Thus $\left|d\vec{l} \times \vec{r}\right| = R\,dl$

$$dB = \frac{\mu_0}{4\pi}\frac{I\left|d\vec{l} \times \vec{r}\right|}{r^3} = \frac{\mu_0 I_1}{4\pi}\frac{R}{R^3}dl = \left(\frac{\mu_0 I_1}{4\pi R^2}\right)dl$$

$$B = \int dB = \left(\frac{\mu_0 I_1}{4\pi R^2}\right)\int dl = \left(\frac{\mu_0 I_1}{4\pi R^2}\right)(\pi R) = \frac{\mu_0 I_1}{4R}$$

(We used that $\int dl$ is equal to πR, the length of wire in the semicircle.) We have shown that the two straight sections make zero contribution to $\vec{B}$, so $B_1 = \mu_0 I_1/4R$ and is directed into the page.



For current in the direction shown in Figure 28.37e, a similar analysis gives $B_2 = \mu_0 I_2/4R$, out of the paper.

Figure 28.37e

$\vec{B}_1$ and $\vec{B}_2$ are in opposite directions, so the magnitude of the net field at P is $B = |B_1 - B_2| = \dfrac{\mu_0|I_1 - I_2|}{4R}$.

EVALUATE: When $I_1 = I_2$, $B = 0$.

28.64. **IDENTIFY:** The net magnetic field is the vector sum of the fields due to each wire.

SET UP: $B = \dfrac{\mu_0 I}{2\pi r}$. The direction of $\vec{B}$ is given by the right-hand rule.

EXECUTE: **(a)** The currents are the same so points where the two fields are equal in magnitude are equidistant from the two wires. The net field is zero along the dashed line shown in Figure 28.64a.
(b) For the magnitudes of the two fields to be the same at a point, the point must be 3 times closer to the wire with the smaller current. The net field is zero along the dashed line shown in Figure 28.64b.
(c) As in (a), the points are equidistant from both wires. The net field is zero along the dashed line shown in Figure 28.64c.

EVALUATE: The lines of zero net field consist of points at which the fields of the two wires have opposite directions and equal magnitudes.

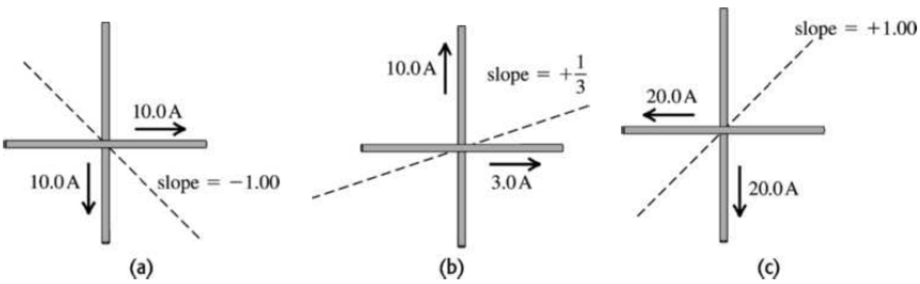


Figure 28.64

28.74. **IDENTIFY:** Apply $d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{l} \times \hat{r}}{r^2}$.

SET UP: The two straight segments produce zero field at P. The field at the center of a circular loop of radius R is $B = \frac{\mu_0 I}{2R}$, so the field at the center of curvature of a semicircular loop is $B = \frac{\mu_0 I}{4R}$.

EXECUTE: The semicircular loop of radius a produces field out of the page at P and the semicircular loop of radius b produces field into the page. Therefore, $B = B_a - B_b = \frac{1}{2}\left(\frac{\mu_0 I}{2}\right)\left(\frac{1}{a} - \frac{1}{b}\right) = \frac{\mu_0 I}{4a}\left(1 - \frac{a}{b}\right)$, out of page.

EVALUATE: If $a = b$, $B = 0$.

28.78. **IDENTIFY:** Apply $d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{l} \times \hat{r}}{r^2}$.

SET UP: The horizontal wire yields zero magnetic field since $d\vec{l} \times \vec{r} = 0$. The vertical current provides the magnetic field of half of an infinite wire. (The contributions from all infinitesimal pieces of the wire point in the same direction, so there is no vector addition or components to worry about.)

EXECUTE: $B = \frac{1}{2}\left(\frac{\mu_0 I}{2\pi R}\right) = \frac{\mu_0 I}{4\pi R}$ and is directed out of the page.

EVALUATE: In the equation preceding Eq. (28.8) the limits on the integration are 0 to a rather than $-a$ to a and this introduces a factor of $\frac{1}{2}$ into the expression for B.