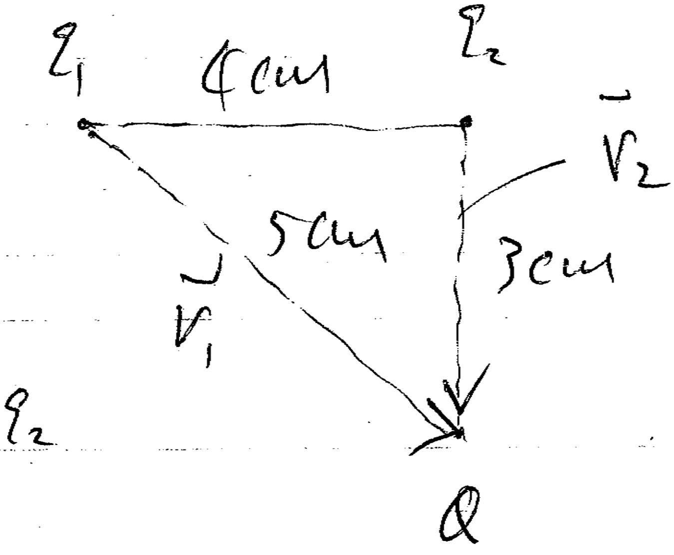
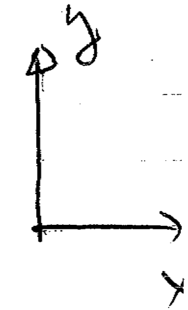
Solutions to PC-A MT#1 (2016)

1 (a) the total force on Q is the sum of the forces exerted by q_1 and q_2 on Q:





+2 $$\vec{F}_{on\,Q} = \vec{F}_{q_1 \, on\, Q} + \vec{F}_{q_2\, on\, Q}$$

+2 $$\vec{F}_{q_1\, on\, Q} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 \cdot Q}{|\vec{r}_1|^2}\hat{r}_1$$

$$= \frac{1}{4\pi\varepsilon_0}\cdot\frac{(-10^{-9}C)(-4\times10^{-9}C)}{(0.05\,m)^2}\cdot\frac{0.04\,m\,\hat{i} - 0.03\,m\,\hat{j}}{0.05\,m}$$

+2 $$= \left(9\times10^{9}\,N\cdot m^2/C^2\right)\cdot\frac{4\times10^{-18}C^2}{2.5\times10^{-3}\,m^2}\left(0.8\,\hat{i} - 0.6\,\hat{j}\right)$$

+1 $$\vec{F}_{q_2\, on\, Q} = \frac{1}{4\pi\varepsilon_0}\frac{q_2 \cdot Q}{|\vec{r}_2|^2}\hat{r}_2$$

+1 $$= \frac{1}{4\pi\varepsilon_0}\frac{(25\times10^{-9}C)(-4\times10^{-9}C)}{(0.03\,m)^2}\cdot(-\hat{j})$$

Thus

+1 $$F_{on q,x} = \left(9\times10^{9}\,N\cdot m^2/c^2\right)\frac{4\times10^{-18}\,c^2}{2.5\times10^{-3}\,m^2}(0.8)$$

$$= 1.15\times10^{-5}\,N$$

+1 $$F_{on q,y} = \left(9\times10^{9}\,N\cdot m^2/c^2\right)\left(\frac{100\times10^{-18}\,c^2}{0.9\times10^{-3}\,m^2}\right)$$

$$- \left(9\times10^{9}\,N\cdot m^2/c^2\right)\frac{4\times10^{-18}\,c^2\times0.6}{2.5\times10^{-3}\,m^2}$$

$$= 9.9\times10^{-4}\,N$$

1-(b) Referenced to infinity the electric potential energy of Q is the sum of the potential energies due to the electric fields produced by q_1 and q_2 referenced to infinity:

+2 $$U_Q = U_Q(q_1, \vec{r}_1) + U_Q(q_2, \vec{r}_2)$$

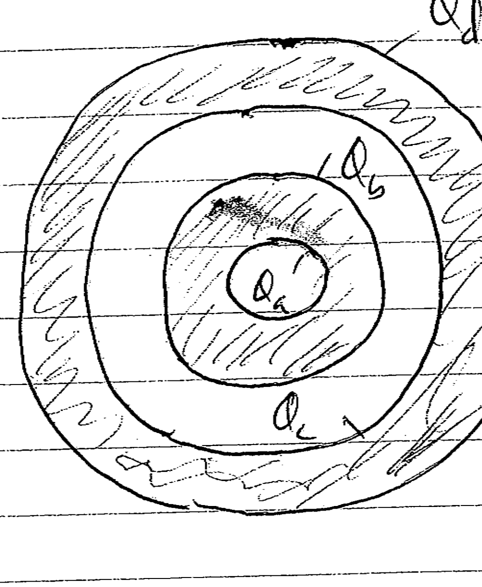
+4 $$= \frac{1}{4\pi\varepsilon_0}\frac{q_1 \cdot Q}{|\vec{r}_1|} + \frac{1}{4\pi\varepsilon_0}\frac{q_2 \cdot Q}{|\vec{r}_2|}$$

$$= \left(9\times10^{9}\ N.m^2/c^2\right)\left(-4\times10^{-9}c\right)\cdot$$

+2 $$\left(\frac{-10^{-9}c}{0.05m} + \frac{25\times10^{-9}c}{0.03m}\right)$$

+2 $$= -2.9\times10^{-5}J.$$

2-(a) Because of spherical symmetry and the fact that the net electric charge on a conductor can only reside on the surface, we can designate the charges on the four surfaces as Q_a, Q_b, Q_c, and Q_d, and they are all uniformly distributed on their respective surfaces. In addition by charge conservation



+2

$$Q_a + Q_b = +4q \qquad (1)$$

$$Q_c + Q_d = -4q \qquad (2)$$

+2

Since the electric field inside the small shell ($a<r<b$) is only contributed by Q_a and it is zero (inside a conductor)

+1

$$\vec{E}(\vec{r}, a<|\vec{r}|<b) = \frac{1}{4\pi\epsilon_0}\frac{Q_a}{|\vec{r}|^2}\hat{r} = 0 \qquad (3)$$

+2

$$\therefore Q_a = 0; \quad Q_b = +4q. \qquad (4)$$

+1 Since the electric field inside the large shell is also zero, and is contributed by $Q_a = 0$, $Q_b = +4\xi$, and Q_c as follows

$$\vec{E}(\vec{r}, c < |\vec{r}| < d) = \frac{1}{4\pi\varepsilon_0} \frac{Q_a + Q_b + Q_c}{|\vec{r}|^2} \hat{r} = 0$$

Thus

+2
$$Q_c = -Q_b = -4\xi$$

$$Q_d = 0$$

$$\therefore \; Q_a = 0 \; ; \; Q_b = +4\xi ; \; Q_c = -4\xi ; \; Q_d = 0$$

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2-(b) $\vec{E}(\vec{r}, |\vec{r}| < a) = 0$ as the field point is inside all four shells of charges.

+3 $\vec{E}(\vec{r}, |\vec{r}| < a) = \frac{K}{r^2} \hat{r} \cdot Q_{\text{inside a spherical surface of radius } r < a}$

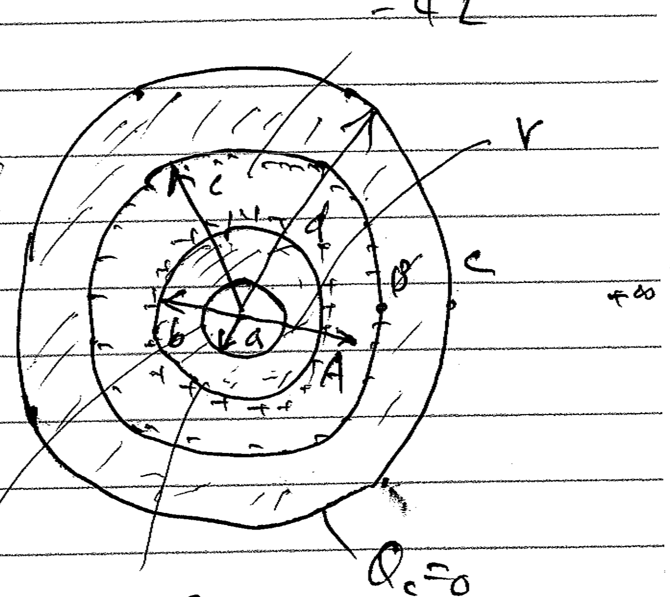
+2 $= 0.$

$\vec{E}(\vec{r}, |\vec{r}| > d) = 0$ since $Q_a + Q_b + Q_c + Q_d = 0$

+3 $\vec{E}(\vec{r}, |\vec{r}| > d) = \frac{K}{r^2} \hat{r} (Q_a + Q_b + Q_c + Q_d) = 0$

+2

2-(c) By definition, the potential at A with $b < r < c$ referenced to infinity is given by the line integral of the electric field from A to infinity



$V_A(r) - V(+\infty)$

+2 $$= \int_A^{+\infty} \vec{E} \cdot d\vec{l} = \int_A^B \vec{E}\Big|_{\text{between } A \,\&\, B} \cdot d\vec{l} + \int_B^C \vec{E}\Big|_{\text{between } B \,\&\, C} \cdot d\vec{l}$$

+3 $$+ \int_C^{+\infty} \vec{E}\Big|_{\text{between } C \,\&\, \text{infinity}} \cdot d\vec{l}$$

But, $\vec{E}\Big|_{\text{between } B \,\&\, C} = 0$; $\vec{E}\Big|_{\text{between } C \,\&\, \text{infinity}} = 0$,

+3 $$\vec{E}\Big|_{\text{between } A \,\&\, B} = \frac{K(4q)}{r'^2}\hat{r}',$$

Thus the last two terms yield zero, and the first term is simply

+2 $$V_A(r) - V(+\infty) = \int_A^B \vec{E} \cdot d\vec{l} = 4Kq\left(\frac{1}{r} - \frac{1}{c}\right)$$

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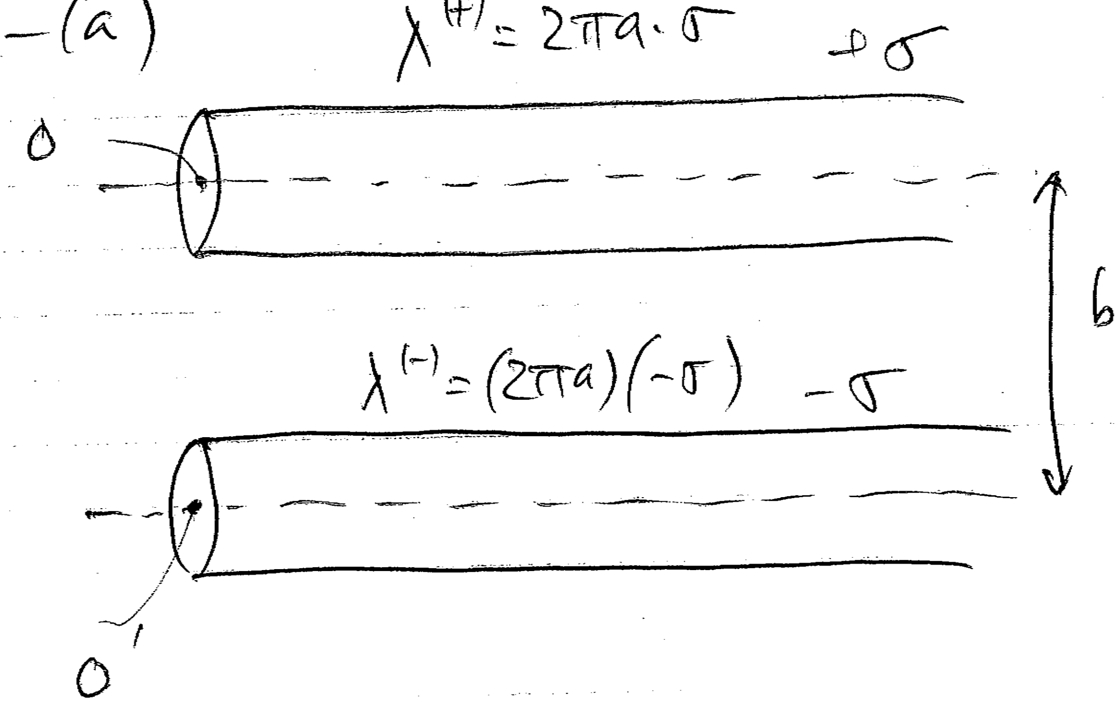
3-(a)



+2 The electric field at point O is the sum of the electric fields produced by both cylindrical shells of charges

+2 But the contribution from the cylindrical shell with $+\sigma$ at O is zero. And thus the net electric field at O is only contributed by the shell with $-\sigma$.

+2 ~~[illegible]~~ as if by an infinite line charge at a distance b away with line charge density

+2 $$\lambda^{(-)} = (-\sigma)\cdot(2\pi a)$$

Thus

+2 $$\vec{E}(0) = \frac{\lambda^{(-)}}{2\pi\varepsilon_0 \cdot b}\,\hat{\rho}' = \frac{-\sigma \cdot a}{\varepsilon_0 \cdot b}\cdot\hat{\rho}'$$

$\hat{\rho}'$ is a unit vector pointing from the axis of $-\sigma$

to the axis of $+\sigma$ and perpendicular to both cylindrical axes.

3-(b) The electric potential difference is the sum of the contributions from

+3 two cylindrical distributions of charges. Thus

$$+2 \quad V_A - V_B = \int_A^B \vec{E} \cdot d\vec{l}$$

$$= \int_A^B \vec{E}(-\sigma) \cdot d\vec{l} \;+2\; + \int_A^B \vec{E}(+\sigma) \cdot d\vec{l} \;+2$$

$$= \frac{(-\sigma) \cdot 2\pi a}{2\pi\varepsilon_0} \ln \frac{a}{b-a} \;+2\; + \frac{(+\sigma) \cdot 2\pi a}{2\pi\varepsilon_0} \ln \frac{b-a}{a} \;+2$$

+2

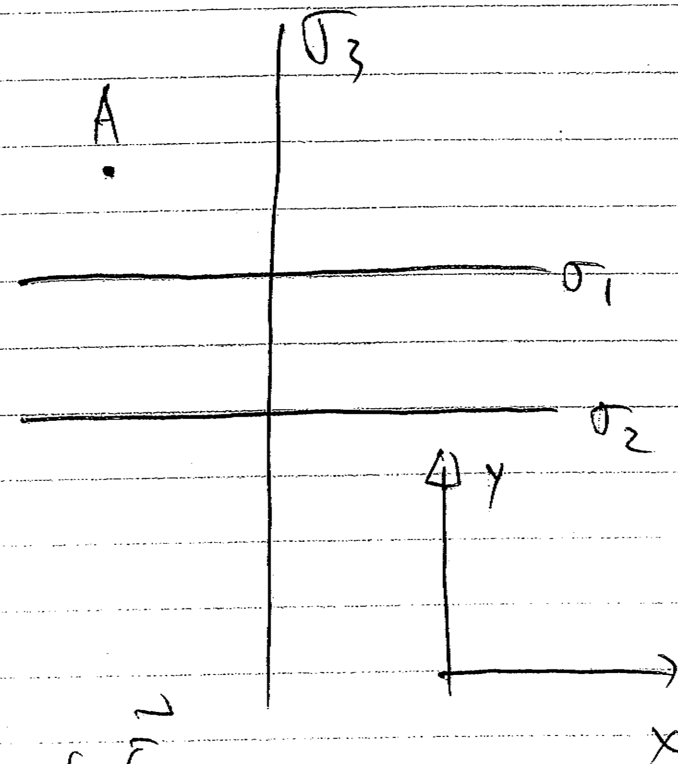
$$= \left(\frac{\sigma}{\pi\varepsilon_0} \ln \frac{b-a}{a}\right) \cdot 2\pi a$$

$$= \frac{2a\sigma}{\varepsilon_0} \ln \frac{b-a}{a}$$

✱

4-(a) The total electric field at point A is the vector sum of the electric fields produced by three sheets of charges at point A.

+3



+2 $\vec{E}_A = \vec{E}_{A,\sigma_1} + \vec{E}_{A,\sigma_2} + \vec{E}_{A,\sigma_3}$

+2 +2 +2

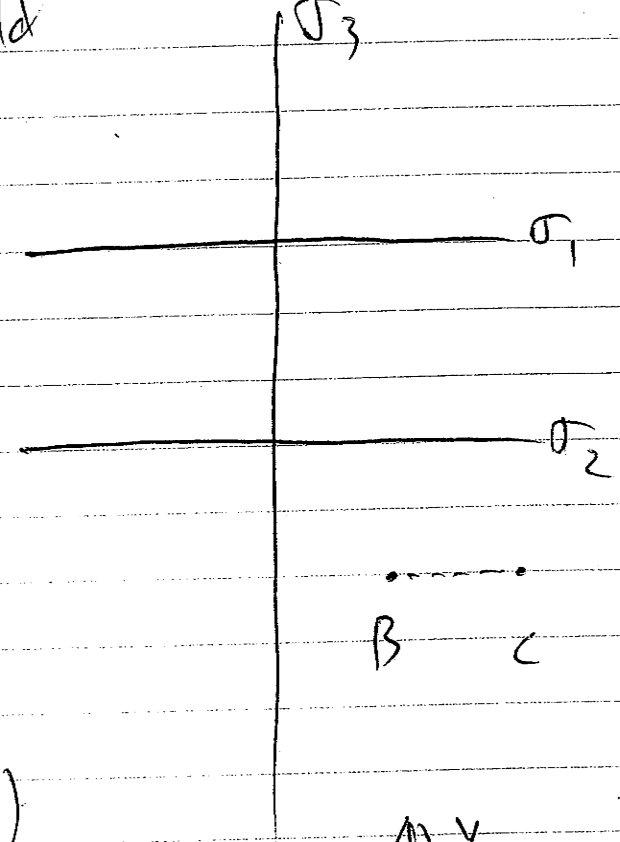
$$= \frac{\sigma_1}{2\varepsilon_0}\hat{j} + \frac{\sigma_2}{2\varepsilon_0}\hat{j} + \frac{\sigma_3}{2\varepsilon_0}(-\hat{i})$$

+2 $$= -\hat{i}\left(\frac{-4\times10^{-6}\,C/m^2}{2\times8.85\times10^{-12}\,C^2/N{\cdot}m^2}\right)$$

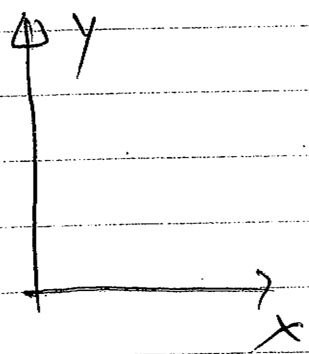
+2 $$+\hat{j}\,\frac{2\times2\times10^{-6}\,C/m^2}{2\times8.85\times10^{-12}\,C^2/N{\cdot}m^2}$$

+ $$= +2.26\times10^5\,N/C\,\hat{i} + 2.26\times10^5\,N/C\,\hat{j}$$

4- (a) The electric field in the region of points B and C is given by the sum of the electric fields produced by the three sheets of charges



$$+2 \left\{ \begin{aligned} \vec{E}(BC) &= \vec{E}_{\sigma_1}(BC) \\ &+ \vec{E}_{\sigma_2}(BC) + \vec{E}_{\sigma_3}(BC) \end{aligned} \right.$$



$$+2 \left\{ = \frac{\sigma_3}{2\varepsilon_0}\,\hat{i} + \frac{\sigma_1 + \sigma_2}{2\varepsilon_0}\left(-\hat{j}\right) \right.$$

$\times 10^4$ Volts

$$+3 \left\{ V_B - V_C = \int_B^C \vec{E}_{BC} \cdot d\vec{l} \right.$$

Choosing $d\vec{l} = \hat{i}\,dx$, then $\vec{E}_{BC} \cdot d\vec{l} = \frac{\sigma_3}{2\varepsilon_0}\,dx$

$$+3 \left\{ V_B - V_C = \frac{\sigma_3}{2\varepsilon_0}\,\overline{BC} = \frac{-4 \times 10^{-6}\ C/m^2}{2 \times 8.85 \times 10^{-12}\ C^2/N \cdot m^2} \cdot (0.1\,m) = -2.25 \right.$$