24.20. **IDENTIFY:** For capacitors in parallel the voltages are the same and the charges add. For capacitors in series, the charges are the same and the voltages add. $C = Q/V$.

SET UP: C_1 and C_2 are in parallel and C_3 is in series with the parallel combination of C_1 and C_2.

EXECUTE: **(a)** C_1 and C_2 are in parallel and so have the same potential across them:

$V_1 = V_2 = \dfrac{Q_2}{C_2} = \dfrac{40.0\times10^{-6}\text{ C}}{3.00\times10^{-6}\text{ F}} = 13.33\text{ V}$. Therefore, $Q_1 = V_1C_1 = (13.33\text{ V})(6.00\times10^{-6}\text{ F}) = 80.0\times10^{-6}\text{ C}$.

Since C_3 is in series with the parallel combination of C_1 and C_2, its charge must be equal to their combined charge: $Q_3 = 40.0\times10^{-6}\text{ C} + 80.0\times10^{-6}\text{ C} = 120.0\times10^{-6}\text{ C}$.

(b) The total capacitance is found from $\dfrac{1}{C_{\text{tot}}} = \dfrac{1}{C_{12}} + \dfrac{1}{C_3} = \dfrac{1}{9.00\times10^{-6}\text{ F}} + \dfrac{1}{5.00\times10^{-6}\text{ F}}$ and

$C_{\text{tot}} = 3.21\,\mu\text{F}$. $V_{ab} = \dfrac{Q_{\text{tot}}}{C_{\text{tot}}} = \dfrac{120.0\times10^{-6}\text{ C}}{3.21\times10^{-6}\text{ F}} = 37.4\text{ V}$.

EVALUATE: $V_3 = \dfrac{Q_3}{C_3} = \dfrac{120.0\times10^{-6}\text{ C}}{5.00\times10^{-6}\text{ F}} = 24.0\text{ V}$. $V_{ab} = V_1 + V_3$.

24.29. **IDENTIFY:** Use the rules for series and for parallel capacitors to express the voltage for each capacitor in terms of the applied voltage. Express U, Q and E in terms of the capacitor voltage.

SET UP: Let the applied voltage be V. Let each capacitor have capacitance C. $U = \frac{1}{2}CV^2$ for a single capacitor with voltage V.

EXECUTE: **(a) series**

Voltage across each capacitor is $V/2$. The total energy stored is $U_s = 2\left(\frac{1}{2}C[V/2]^2\right) = \frac{1}{4}CV^2$.

parallel

Voltage across each capacitor is V. The total energy stored is $U_p = 2\left(\frac{1}{2}CV^2\right) = CV^2$.

$U_p = 4U_s$

(b) $Q = CV$ for a single capacitor with voltage V. $Q_s = 2(C[V/2]) = CV$; $Q_p = 2(CV) = 2CV$; $Q_p = 2Q_s$

(c) $E = V/d$ for a capacitor with voltage V. $E_s = V/2d$; $E_p = V/d$; $E_p = 2E_s$

EVALUATE: The parallel combination stores more energy and more charge since the voltage for each capacitor is larger for parallel. More energy stored and larger voltage for parallel means larger electric field in the parallel case.

24.36. **IDENTIFY:** $V = Ed$ and $C = Q/V$. With the dielectric present, $C = KC_0$.

SET UP: $V = Ed$ holds both with and without the dielectric.

EXECUTE: **(a)** $V = Ed = (3.00\times10^4\text{ V/m})(1.50\times10^{-3}\text{ m}) = 45.0\text{ V}$.

$Q = C_0V = (5.00\times10^{-12}\text{ F})(45.0\text{ V}) = 2.25\times10^{-10}\text{ C}$.

(b) With the dielectric, $C = KC_0 = (2.70)(5.00\text{ pF}) = 13.5\text{ pF}$. V is still 45.0 V, so

$Q = CV = (13.5\times10^{-12}\text{ F})(45.0\text{ V}) = 6.08\times10^{-10}\text{ C}$.

EVALUATE: The presence of the dielectric increases the amount of charge that can be stored for a given potential difference and electric field between the plates. Q increases by a factor of K.

24.42. **IDENTIFY:** $C = Q/V$. $C = KC_0$. $V = Ed$.

SET UP: Table 24.1 gives $K = 3.1$ for mylar.

EXECUTE: **(a)** $\Delta Q = Q - Q_0 = (K-1)Q_0 = (K-1)C_0V_0 = (2.1)(2.5\times10^{-7}\text{ F})(12\text{ V}) = 6.3\times10^{-6}\text{ C}$.

(b) $\sigma_i = \sigma(1 - 1/K)$ so $Q_i = Q(1 - 1/K) = (9.3\times10^{-6}\text{ C})(1 - 1/3.1) = 6.3\times10^{-6}\text{ C}$.

(c) The addition of the mylar doesn't affect the electric field since the induced charge cancels the additional charge drawn to the plates.

EVALUATE: $E = V/d$ and V is constant so E doesn't change when the dielectric is inserted.

24.54. **IDENTIFY:** Initially the capacitors are connected in parallel to the source and we can calculate the charges Q_1 and Q_2 on each. After they are reconnected to each other the total charge is $Q = Q_2 - Q_1$.

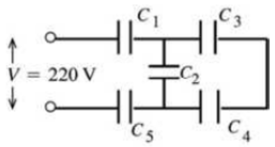
$U = \frac{1}{2}CV^2 = \frac{Q^2}{2C}$.

SET UP: After they are reconnected, the charges add and the voltages are the same, so $C_{eq} = C_1 + C_2$, as for capacitors in parallel.

EXECUTE: Originally $Q_1 = C_1V_1 = (9.0\ \mu\text{F})\ (36\ \text{V}) = 3.24\times10^{-4}\ \text{C}$ and $Q_2 = C_2V_2 = (4.0\ \mu\text{F})(36\ \text{V}) = 1.44\times10^{-4}\ \text{C}$. $C_{eq} = C_1 + C_2 = 13.0\ \mu\text{F}$. The original energy stored is $U = \frac{1}{2}C_{eq}V^2 = \frac{1}{2}(13.0\times10^{-6}\ \text{F})(36\ \text{V})^2 = 8.42\times10^{-3}\ \text{J}$. Disconnect and flip the capacitors, so now the total charge is $Q = Q_2 - Q_1 = 1.8\times10^{-4}\ \text{C}$ and the equivalent capacitance is still the same, $C_{eq} = 13.0\ \mu\text{F}$.

24.57. **(a) IDENTIFY:** Replace series and parallel combinations of capacitors by their equivalents.

SET UP: The network is sketched in Figure 24.57a.

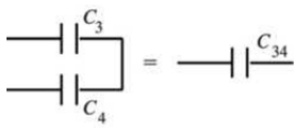


$C_1 = C_5 = 8.4\ \mu\text{F}$

$C_2 = C_3 = C_4 = 4.2\ \mu\text{F}$

Figure 24.57a

EXECUTE: Simplify the circuit by replacing the capacitor combinations by their equivalents: C_3 and C_4 are in series and can be replaced by C_{34} (Figure 24.57b):



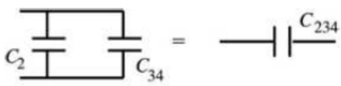
$$\frac{1}{C_{34}} = \frac{1}{C_3} + \frac{1}{C_4}$$

$$\frac{1}{C_{34}} = \frac{C_3 + C_4}{C_3C_4}$$

Figure 24.57b

$$C_{34} = \frac{C_3C_4}{C_3 + C_4} = \frac{(4.2\ \mu\text{F})(4.2\ \mu\text{F})}{4.2\ \mu\text{F} + 4.2\ \mu\text{F}} = 2.1\ \mu\text{F}$$

C_2 and C_{34} are in parallel and can be replaced by their equivalent (Figure 24.57c):



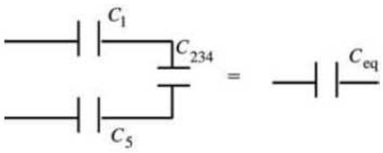
$$C_{234} = C_2 + C_{34}$$

$$C_{234} = 4.2\ \mu\text{F} + 2.1\ \mu\text{F}$$

$$C_{234} = 6.3\ \mu\text{F}$$

Figure 24.57c

C_1, C_5 and C_{234} are in series and can be replaced by C_{eq} (Figure 24.57d):



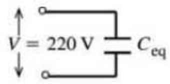
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_5} + \frac{1}{C_{234}}$$

$$\frac{1}{C_{eq}} = \frac{2}{8.4\ \mu\text{F}} + \frac{1}{6.3\ \mu\text{F}}$$

$$C_{eq} = 2.5\ \mu\text{F}$$

Figure 24.57d

EVALUATE: For capacitors in series the equivalent capacitor is smaller than any of those in series. For capacitors in parallel the equivalent capacitance is larger than any of those in parallel.
(b) IDENTIFY and SET UP: In each equivalent network apply the rules for Q and V for capacitors in series and parallel; start with the simplest network and work back to the original circuit.
EXECUTE: The equivalent circuit is drawn in Figure 24.57e.



$$Q_{eq} = C_{eq}V$$

$$Q_{eq} = (2.5\ \mu\text{F})(220\ \text{V}) = 550\ \mu\text{C}$$

Figure 24.57e

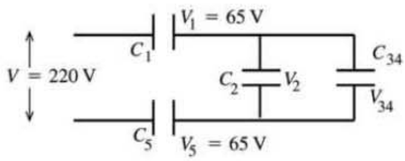
$Q_1 = Q_5 = Q_{234} = 550\ \mu\text{C}$ (capacitors in series have same charge)

$$V_1 = \frac{Q_1}{C_1} = \frac{550\ \mu\text{C}}{8.4\ \mu\text{F}} = 65\ \text{V}$$

$$V_5 = \frac{Q_5}{C_5} = \frac{550\ \mu\text{C}}{8.4\ \mu\text{F}} = 65\ \text{V}$$

$$V_{234} = \frac{Q_{234}}{C_{234}} = \frac{550\ \mu\text{C}}{6.3\ \mu\text{F}} = 87\ \text{V}$$

Now draw the network as in Figure 24.57f.



$V_2 = V_{34} = V_{234} = 87\ \text{V}$

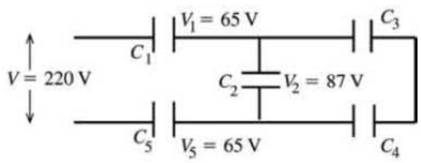
capacitors in parallel have the same potential

Figure 24.57f

$Q_2 = C_2V_2 = (4.2\ \mu\text{F})(87\ \text{V}) = 370\ \mu\text{C}$

$Q_{34} = C_{34}V_{34} = (2.1\ \mu\text{F})(87\ \text{V}) = 180\ \mu\text{C}$

Finally, consider the original circuit (Figure 24.57g).



$Q_3 = Q_4 = Q_{34} = 180\ \mu\text{C}$

capacitors in series have the same charge

Figure 24.57g

$$V_3 = \frac{Q_3}{C_3} = \frac{180\ \mu\text{C}}{4.2\ \mu\text{F}} = 43\ \text{V}$$

$$V_4 = \frac{Q_4}{C_4} = \frac{180\ \mu\text{C}}{4.2\ \mu\text{F}} = 43\ \text{V}$$

Summary: $Q_1 = 550\ \mu C$, $V_1 = 65$ V

$Q_2 = 370\ \mu C$, $V_2 = 87$ V

$Q_3 = 180\ \mu C$, $V_3 = 43$ V

$Q_4 = 180\ \mu C$, $V_4 = 43$ V

$Q_5 = 550\ \mu C$, $V_5 = 65$ V

EVALUATE: $V_3 + V_4 = V_2$ and $V_1 + V_2 + V_5 = 220$ V (apart from some small rounding error)

$Q_1 = Q_2 + Q_3$ and $Q_5 = Q_2 + Q_4$

24.60. **IDENTIFY:** Apply the rules for combining capacitors in series and in parallel.

SET UP: With the switch open, each pair of $3.00\ \mu F$ and $6.00\ \mu F$ capacitors are in series with each other and each pair is in parallel with the other pair. When the switch is closed, each pair of $3.00\ \mu F$ and $6.00\ \mu F$ capacitors are in parallel with each other and the two pairs are in series.

EXECUTE: **(a)** With the switch open $C_{eq} = \left(\left(\frac{1}{3\ \mu F} + \frac{1}{6\ \mu F}\right)^{-1} + \left(\frac{1}{3\ \mu F} + \frac{1}{6\ \mu F}\right)^{-1}\right) = 4.00\ \mu F$.

$Q_{total} = C_{eq}V = (4.00\ \mu F)(210\ V) = 8.40 \times 10^{-4}$ C. By symmetry, each capacitor carries 4.20×10^{-4} C. The voltages are then calculated via $V = Q/C$. This gives $V_{ad} = Q/C_3 = 140$ V and $V_{ac} = Q/C_6 = 70$ V.

$V_{cd} = V_{ad} - V_{ac} = 70$ V.

(b) When the switch is closed, the points *c* and *d* must be at the same potential, so the equivalent capacitance is $C_{eq} = \left(\frac{1}{(3.00 + 6.00)\ \mu F} + \frac{1}{(3.00 + 6.00)\ \mu F}\right)^{-1} = 4.5\ \mu F$.

$Q_{total} = C_{eq}V = (4.50\ \mu F)(210\ V) = 9.5 \times 10^{-4}$ C, and each capacitor has the same potential difference of 105 V (again, by symmetry).

(c) The only way for the sum of the positive charge on one plate of C_2 and the negative charge on one plate of C_1 to change is for charge to flow through the switch. That is, the quantity of charge that flows through the switch is equal to the change in $Q_2 - Q_1$. With the switch open, $Q_1 = Q_2$ and $Q_2 - Q_1 = 0$. After the switch is closed, $Q_2 - Q_1 = 315\ \mu C$, so $315\ \mu C$ of charge flowed through the switch.

EVALUATE: When the switch is closed the charge must redistribute to make points *c* and *d* be at the same potential.

24.65. **(a) IDENTIFY** and **SET UP:** Q is constant. $C = KC_0$; use Eq. (24.1) to relate the dielectric constant K to the ratio of the voltages without and with the dielectric.

EXECUTE: With the dielectric: $V = Q/C = Q/(KC_0)$

without the dielectric: $V_0 = Q/C_0$

$V_0/V = K$, so $K = (45.0\ V)/(11.5\ V) = 3.91$

EVALUATE: Our analysis agrees with Eq. (24.13).

(b) IDENTIFY: The capacitor can be treated as equivalent to two capacitors C_1 and C_2 in parallel, one with area $2A/3$ and air between the plates and one with area $A/3$ and dielectric between the plates.

SET UP: The equivalent network is shown in Figure 24.65.

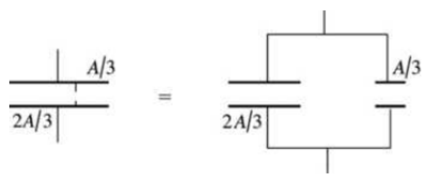


Figure 24.65

EXECUTE: Let $C_0 = \epsilon_0 A/d$ be the capacitance with only air between the plates. $C_1 = KC_0/3$, $C_2 = 2C_0/3$;

$C_{eq} = C_1 + C_2 = (C_0/3)(K + 2)$

$C_{eq} = C_1 + C_2 = (C_0/3)(K+2)$

$$V = \frac{Q}{C_{eq}} = \frac{Q}{C_0}\left(\frac{3}{K+2}\right) = V_0\left(\frac{3}{K+2}\right) = (45.0\text{ V})\left(\frac{3}{5.91}\right) = 22.8\text{ V}$$

EVALUATE: The voltage is reduced by the dielectric. The voltage reduction is less when the dielectric doesn't completely fill the volume between the plates.

25.8. **IDENTIFY:** $I = Q/t$. Positive charge flowing in one direction is equivalent to negative charge flowing in the opposite direction, so the two currents due to Cl^- and Na^+ are in the same direction and add.

SET UP: Na^+ and Cl^- each have magnitude of charge $|q| = +e$.

EXECUTE: **(a)** $Q_{total} = (n_{Cl} + n_{Na})e = (3.92\times10^{16} + 2.68\times10^{16})(1.60\times10^{-19}\text{ C}) = 0.0106\text{ C}$. Then $I = \frac{Q_{total}}{t} = \frac{0.0106\text{ C}}{1.00\text{ s}} = 0.0106\text{ A} = 10.6\text{ mA}$.

(b) Current flows, by convention, in the direction of positive charge. Thus, current flows with Na^+ toward the negative electrode.

EVALUATE: The Cl^- ions have negative charge and move in the direction opposite to the conventional current direction.

25.12. **IDENTIFY:** $E = \rho J$, where $J = I/A$. The drift velocity is given by $I = n|q|v_d A$.

SET UP: For copper, $\rho = 1.72\times10^{-8}\ \Omega\cdot\text{m}$. $n = 8.5\times10^{28}/\text{m}^3$.

EXECUTE: **(a)** $J = \frac{I}{A} = \frac{3.6\text{ A}}{(2.3\times10^{-3}\text{ m})^2} = 6.81\times10^5\text{ A/m}^2$.

(b) $E = \rho J = (1.72\times10^{-8}\ \Omega\cdot\text{m})(6.81\times10^5\text{ A/m}^2) = 0.012\text{ V/m}$.

(c) The time to travel the wire's length l is

$$t = \frac{l}{v_d} = \frac{ln|q|A}{I} = \frac{(4.0\text{ m})(8.5\times10^{28}/\text{m}^3)(1.6\times10^{-19}\text{ C})(2.3\times10^{-3}\text{ m})^2}{3.6\text{ A}} = 8.0\times10^4\text{ s}.$$

$t = 1333\text{ min} \approx 22\text{ hrs!}$

EVALUATE: The currents propagate very quickly along the wire but the individual electrons travel very slowly.

25.18. **IDENTIFY:** $R = \frac{\rho L}{A} = \frac{\rho L}{\pi d^2/4}$.

SET UP: For aluminum, $\rho_{al} = 2.63\times10^{-8}\ \Omega\cdot\text{m}$. For copper, $\rho_c = 1.72\times10^{-8}\ \Omega\cdot\text{m}$.

EXECUTE: $\frac{\rho}{d^2} = \frac{R\pi}{4L} =$ constant, so $\frac{\rho_{al}}{d_{al}^2} = \frac{\rho_c}{d_c^2}$.

$$d_c = d_{al}\sqrt{\frac{\rho_c}{\rho_{al}}} = (3.26\text{ mm})\sqrt{\frac{1.72\times10^{-8}\ \Omega\cdot\text{m}}{2.63\times10^{-8}\ \Omega\cdot\text{m}}} = 2.64\text{ mm}.$$

EVALUATE: Copper has a smaller resistivity, so the copper wire has a smaller diameter in order to have the same resistance as the aluminum wire.

25.53. **IDENTIFY:** $P = I^2R = \frac{V^2}{R} = VI$. $V = IR$.

SET UP: The heater consumes 540 W when $V = 120\text{ V}$. Energy $= Pt$.

EXECUTE: **(a)** $P = \frac{V^2}{R}$ so $R = \frac{V^2}{P} = \frac{(120\text{ V})^2}{540\text{ W}} = 26.7\ \Omega$

(b) $P = VI$ so $I = \frac{P}{V} = \frac{540\text{ W}}{120\text{ V}} = 4.50\text{ A}$

(c) Assuming that R remains $26.7\ \Omega$, $P = \frac{V^2}{R} = \frac{(110\text{ V})^2}{26.7\ \Omega} = 453\text{ W}$. P is smaller by a factor of $(110/120)^2$.

EVALUATE: **(d)** With the lower line voltage the current will decrease and the operating temperature will decrease. R will be less than $26.7\ \Omega$ and the power consumed will be greater than the value calculated in part (c).