Solutions to PC-C Midterm #2 (2021)

1-(a) $C_{12} = C_1 + C_2 = 15\ \mu F$

$C_{45} = C_4 + C_5 = 30\ \mu F$

$$\frac{1}{C_{network}} = \frac{1}{C_{12345}} = \frac{1}{C_{12}} + \frac{1}{C_3} + \frac{1}{C_{45}}$$

$$= \frac{1}{15} + \frac{1}{15} + \frac{1}{30} = \frac{1}{6}$$

$\therefore C_{12345} = 6\ \mu F$

1-(b) The charge on the network capacitor C_{12345}, Q_{net}, is the same as on C_{12}, C_3, and C_{45}.

$$Q_{net} = Q_{45} = V_{45} * C_{1234} = 60\ \mu C$$

$\therefore$ The potential drop across C_4 & C_5 is

$$V_{45} = \frac{Q_{45}}{C_{45}} = \frac{60\ \mu C}{30\ \mu F} = 2V$$

1-(c) The potential drop across C_1 is the same as the potential drop across C_{12}:

$$V_{12} = \frac{Q_{12}}{C_{12}} = \frac{Q_{tot}}{C_{12}} = \frac{60\mu C}{15\mu F} = 4V$$

$$U_{stored}(C_1) = \frac{C_1}{2} V_{12}^2$$

$$= \frac{1}{2} \times 10\mu F * (4V)^2$$

$$= 80\,\mu J$$

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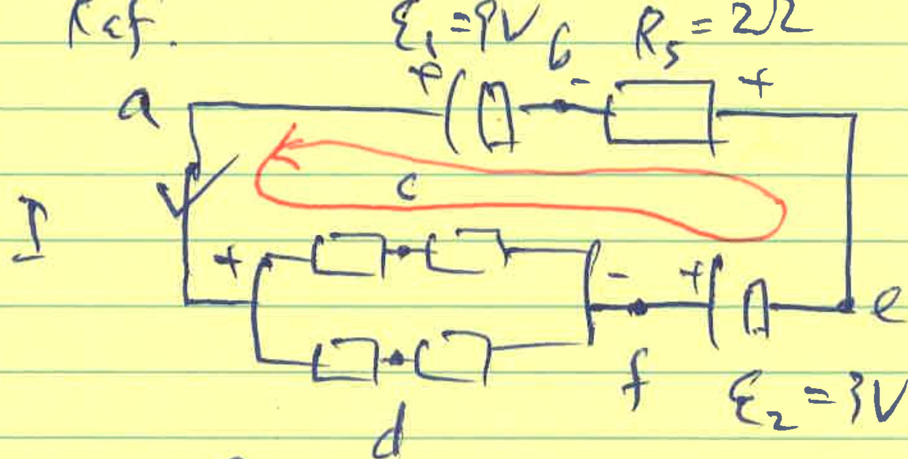
1-(d) The charge stored on C_2 is the potential drop across $V_2 = V_{12}$ times C_2:

$$Q_2 = V_2 \cdot C_2 = 4V * 5\mu F = 20\,\mu C$$

2-(a) The two parallel legs have serial resistances of 6Ω $(3\Omega + 3\Omega)$ and 12Ω $(5\Omega + 7\Omega)$. Thus

$$R_{af} = \frac{6\Omega \cdot 12\Omega}{6\Omega + 12\Omega} = 4\Omega$$

2-(b) We need to know the current through R_{af}.



Let the current be I, counterclockwise. From a counter-clockwise loop:

$$\oint_{afeba} \vec{E}\cdot d\vec{l} = 0 = I\cdot R_{af} + (+3V) + I\cdot R_5 + (-9V)$$

$$\therefore I = \frac{9V - 3V}{R_{af} + R_5} = \frac{6V}{6\Omega} = 1A.$$

$$\therefore V_{af} = V_a - V_f = I\cdot R_{af} = 4V$$

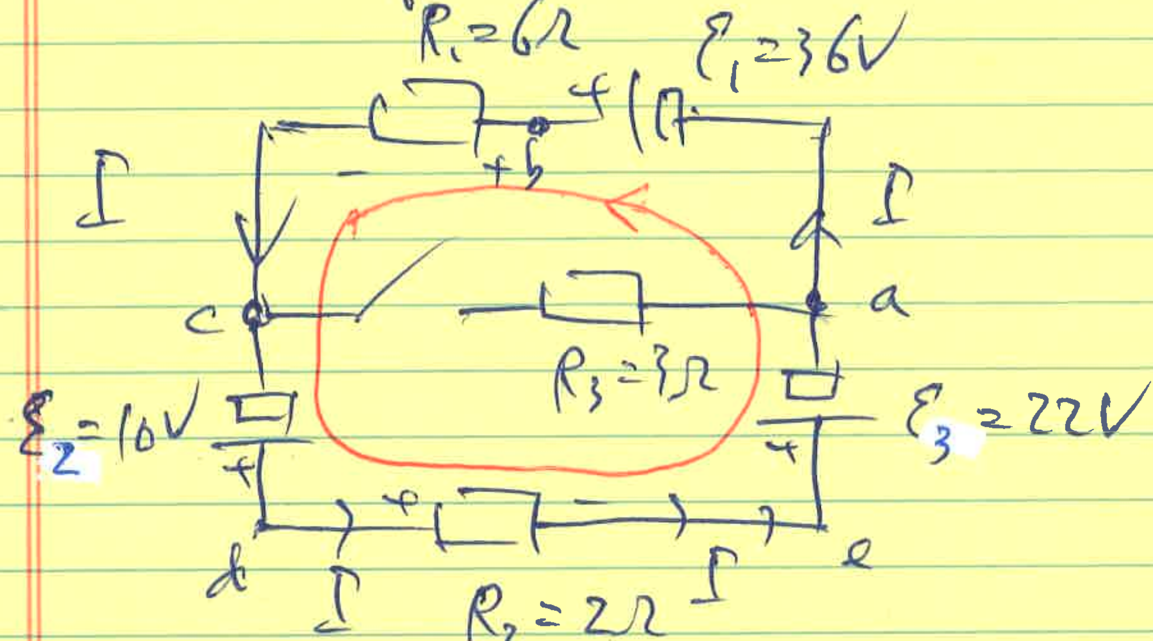
2-(c) The current through 7Ω resistor is the potential drop V_{ef} divided by the resistance of the $5\Omega + 7\Omega$ leg:

$$I(7\Omega) = \frac{V_{ef}}{5\Omega + 7\Omega} = \frac{4V}{12\Omega} = \frac{1}{3}A$$

$$P(7\Omega) = I^2(7\Omega) * 7\Omega = \left(\frac{1}{3}A\right)^2 \cdot 7\Omega$$

$$= \frac{7}{9} \text{ Watt.}$$

3-(a) The only current is the one along the outer loop:



Let the current be I as assigned.

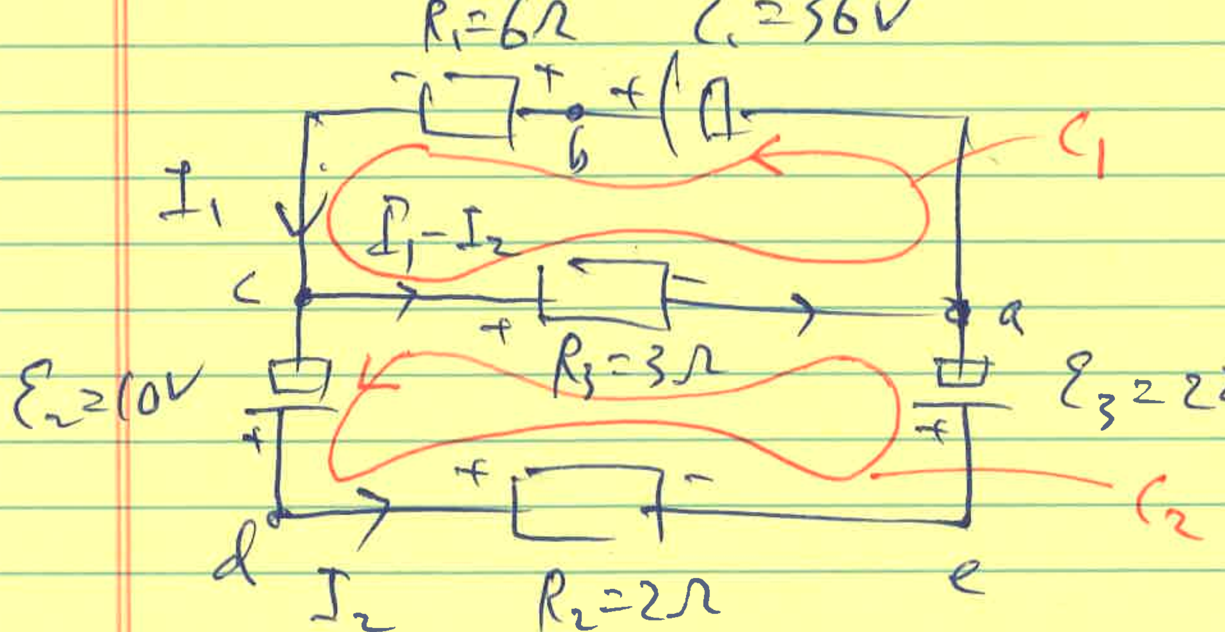
$$\oint_{abcdea} \vec{E}\cdot d\vec{l} = 0 = -36 + 6I - 10 + 2I + 22$$

$$\therefore I = \frac{36+10-22}{6+2} = 3A$$

There is no current through $R_3 = 3\Omega$.

The currents through R_1 and R_2 are the same, $I = 3A$ \#

3-(b) When S is closed, we have two loops and need Kirchoff rules to solve for the currents:



Along loop # 1 :

$$\oint_{abca} \vec{E}\cdot d\vec{l} = -36 + 6I_1 + 3(I_1 - I_2) = 0$$

$$\therefore 9I_1 - 3I_2 = 36$$

$$\Rightarrow \boxed{3I_1 - I_2 = 12} \quad \cdots (1)$$

Along loop # 2 :

$$\oint_{acdea} \vec{E}\cdot d\vec{l} = -3(I_1 - I_2) - 10 + 2I_2 + 22 = 0$$

$$\Rightarrow \boxed{3I_1 - 5I_2 = 12} \quad \cdots (2)$$

① - ② :

$$4I_2 = 0 \quad \therefore I_2 = 0$$

From ① and $I_2 = 0$,

$$I_1 = 4A$$

Thus the currents through R_1 and R_3 are 3A, while there is no current flowing through R_2 ✱

3-(c) $V_{bd} = V_b - V_d = (V_b - V_c) + (V_c - V_d)$

$$= 6I_1 + (-10V) = 14V$$

Or

$$V_{bd} = (V_b - V_a) + (V_a - V_c) + (V_c - V_d)$$

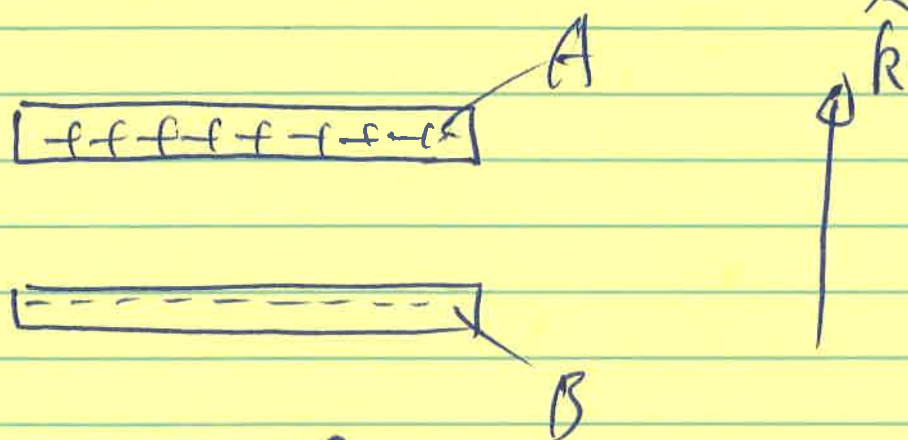
$$= 36V - 12V - 10V = 14V$$

Or

$$V_{bd} = (V_b - V_a) + (V_a - V_e) + (V_e - V_d)$$

$$= 36V + 0 * (2\Omega) - 22V = 14V$$

4-(a) The electric field produced by charges on Plate B at Plate A is



$$\vec{E}_B(\text{at } A) = \frac{Q_B}{2\epsilon_0 S}\cdot\hat{k} = \frac{Q}{2\epsilon_0 S}(-\hat{k})$$

So the force exerted on charge Q on Plate A is

$$\vec{F}_{\text{on}A} = Q\cdot\vec{E}_B(\text{at } A) = \frac{Q^2}{2\epsilon_0 S}(-\hat{k})$$

4-(b) The work done by the external force when plate A is moved up by $\vec{d} = d\,\hat{k}$ is

$$W_{ext} = \vec{d}\cdot\vec{F}_{ext} = \vec{d}\cdot(-\vec{F}_{\text{on}A})$$
$$= \frac{1}{2}\frac{Q^2}{\epsilon_0 S}\cdot d$$

4-(c) $$W_{ext} = \frac{1}{2}\frac{d}{\epsilon_0 S} Q^2 = \frac{Q^2}{2(\epsilon_0 S/d)} = \frac{Q^2}{2C}$$