Physics 108 Midterm Examination (Spring, 1998)

1. First lens: $\frac{v_1}{n_1} \quad \frac{v_2}{n_2}$

   $V_1 = \infty$, $V_2 = +0.5\, cm$

   $u_1 = 1.0$, $u_2 = 1.5$

   \[ \frac{1}{f} = \frac{n_2 - n_1}{u_1} \left( \frac{1}{V_1} + \frac{1}{V_2} \right) \]

   \[ = \frac{1.5 - 1}{1} \left( \frac{1}{\infty} + \frac{1}{-0.5} \right) \]

   \[ = -0.8\, (cm^{-1}) \]

   \[ f = -1\, cm \]

   Second lens: $\frac{v_1}{n_1} \quad \frac{v_2}{n_2}$

   $V_1 = -0.5\, cm$, $V_2 = \infty$

   $u_1 = 1.0$, $u_2 = 1.5$

   \[ \frac{1}{f} = \frac{n_2 - n_1}{u_1} \left( \frac{1}{V_1} - \frac{1}{V_2} \right) \]

   \[ = -0.8\, (cm^{-1}) \]

   \[ f = -1\, cm \]

   2. Combination of thin lenses:

   As shown in the following figure, two positive thin lenses are separated by a distance of $d = 30\, cm$. The first one has a focal length of $f_1 = +10\, cm$. The second one has a focal length of $f_2 = +20\, cm$. A small object with a height $y_0$ is placed on the left-side of the first lens at a distance of $20\, cm$.

   (1) (20 points) Use thin lens equation to determine the final image position of the object measured from the center of the second lens.

   (2) (10 points) Use ray diagrams to construct the image of the object after the second lens.

   (3) (10 points) Find the system ABCD matrix for this pair of lenses.

   (4) (10 points) Find the principal points $F_1$, $F_2$, $H_1$, and $H_2$ and mark in the figure.
3. Michelson interferometer

A collimated beam is incident onto a Michelson interferometer as shown in the following figure. The beam has two frequency components: one is blue with a wavelength $\lambda_1 = 4500\text{Å}$ and the other one is green with a wavelength $\lambda_2 = 5000\text{Å}$. The interferometer is in the air so that the indices of refraction for both frequency components are equal to 1. Initially, the interferometer is adjusted such that the difference of the two optical paths is zero, $2(x_2 - x_1) = 0$. As the optical path difference $2\Delta x = 2(x_2 - x_1)$ increases, the intensity at the detector corresponding to each of the two frequency components varies periodically.

(1) (10 points) Write down the intensity at the detector for each of the two frequency components versus $\Delta x = x_2 - x_1$.

(2) (10 points) Find the separation $\Delta x$ between two successive minima in the intensity for both frequency components.

(3) (15 points) Since the separations between two successive minima for the two components are different, what is the smallest value of $\Delta x \equiv x_2 - x_1$ at which a minimum at $\lambda_2 = 5000\text{Å}$ meets with a minimum at $\lambda_1 = 4500\text{Å}$ again?

For $\lambda_1$, Minima occur at

$$\left(\frac{\pi}{\lambda_1} (x_2 - x_1) = 2\pi m\right)$$

$$\Delta x |_{\lambda_1} = \frac{\lambda_1}{2} = 2250\text{Å}$$

$\lambda_1$ same for $\lambda_2$,

$$\Delta x |_{\lambda_2} = \frac{\lambda_2}{2} = 2500\text{Å}$$

(3. (m+1)\lambda_1 = m\lambda_2 \implies m = 9, \quad \Delta x = \frac{m}{2} \lambda_2 = 22,500\text{Å}$$

From

$$x_1' = \frac{D_1}{\alpha_1} = 2\left(30 + 7\left(\frac{\lambda_1}{2}\right)\right)$$

$$x_2' = \frac{D_2}{\alpha_2} = -20\text{ cm}$$

$$x_1' = \frac{D_1}{\alpha_1} = 2\left(30 + 7\left(\frac{\lambda_1}{2}\right)\right)$$

$$x_2' = \frac{D_2}{\alpha_2} = -20\text{ cm}$$