Supplemental Problem Solution

Suppose the lens occupies the shaded region below. Object position \( x_o \) and height \( y_o \) are measured with respect to the left side of the lens. Image position \( x_i \) and height \( y_i \) are measured with respect to the right side.

A ray leaving the object at height \( y_o \) and angle \( \alpha_o \) arrives at the image at height \( y_i \) and angle \( \alpha_i \) after (1) translating by distance \( s_o = -x_o \), (2) passing through the lens, and (3) translating by distance \( s_i = x_i \). Using transfer matrices this is

\[
\begin{bmatrix}
y_i \\
\alpha_i
\end{bmatrix} = \begin{bmatrix}
1 & s_i \\
0 & 1
\end{bmatrix}
\begin{bmatrix}
A & B \\
C & D
\end{bmatrix}
\begin{bmatrix}
y_o \\
\alpha_o
\end{bmatrix}
= \begin{bmatrix}
A + s_i C & (A + s_i C) s_o + B + x_i D \\
C & s_o C + D
\end{bmatrix}
\begin{bmatrix}
y_o \\
\alpha_o
\end{bmatrix}
\]

which gives

\[
y_i = (A + s_i C) y_o + ((A + x_i C) s_o + B + s_i D) \alpha_o
\]

But since the ray begins on the object and ends on the image, any ray starting at height \( y_o \) must end at height \( y_i \), independent of \( \alpha_o \). Therefore

\[
(A + s_i C) s_o + B + s_i D = 0,
\]

or

\[
s_i = \frac{s_o A + B}{s_o C + D}.
\]

The determinant of the lens matrix must be unity (\( AD - BC = 1 \)), since object and image are in the same medium, therefore the four unknowns can be reduced to three.

\[
s_i = \frac{s_o A + B}{s_o C + \frac{1 + BC}{A}}.
\]

Performing a nonlinear fit to the data gives\(^{\dagger}\) \( A = -0.89 \pm 0.07 \), \( B = -1.4 \pm 0.4 \), and \( C = 0.206 \pm 0.005 \). The lens matrix is therefore

\[
\begin{bmatrix}
A & B \\
C & D
\end{bmatrix} = \begin{bmatrix}
-0.89 & -1.4 \\
0.206 & -0.8
\end{bmatrix}
\]

\(^{\dagger}\) The nonlinear fit algorithm used will converge to different solutions depending on the initial guess for the parameters. The values used to find the above fit were \( A = -1 \), \( B = -1 \), and \( C = 0.2 \).
Chapter 7 Solutions

7.1 \( E_0^2 = 36 + 64 + 2(6)(8) \cos \pi/2 = 100, \ E_0 = 10; \tan \alpha = 8/6, \alpha = 53.1^\circ = 0.93 \text{ rad}. \ E = 10 \sin(120\pi t + 0.93). \)

7.2 \( E_1 = E_{01} \cos(\omega t); \quad E_2 = E_{01} \cos(\omega t + \alpha_2). \)

\[
E = E_1 + E_2 = E_{01} \cos \omega t + E_{01} \cos(\omega t + \alpha_2) \\
= E_{01}(2 \cos \frac{1}{2}(\omega t + \omega t + \alpha_2) \cos \frac{1}{2}(\omega t - \omega t - \alpha_2)) \\
= 2E_{01} \cos(\omega t + \alpha_2/2) \cos(-\alpha_2/2).
\]

Recall \( \cos(-\theta) = \cos \theta \), so,

\[
E = (2E_{01} \cos(\alpha_2/2))(\cos(\omega t + \alpha_2/2)) = E_0 \cos(\omega t + \alpha).
\]

To show that this follows from (7.9) and (7.10), recall that \( \cos \theta = \sin(\theta + \pi/2) \) so that

\[
\alpha_1 \to \alpha_1 + \pi/2 = \pi/2, \quad \alpha_2 \to \alpha_2 + \pi/2.
\]

7.3 In phase: \( \alpha_1 = \alpha_2 \cos(\alpha_2 - \alpha_1) = \cos(0) = 1. \)

\[
(7.9) \quad E_0^2 = E_{01}^2 + E_{02}^2 + 2E_{01}E_{02} \cos(\alpha_2 - \alpha_1) \\
= E_{01}^2 + E_{02}^2 + 2E_{01}E_{02} = (E_{01} + E_{02})^2.
\]

Out of phase, \( \alpha_2 - \alpha_1 = \pi, \cos(\alpha_2 - \alpha_1) = \cos \pi = -1. \)

\[
(7.9) \quad E_0^2 = E_{01}^2 + E_{02}^2 - 2E_{01}E_{02} = (E_{01} - E_{02})^2.
\]

7.4 \( OPL = \sum_i n_i x_i = \sum(c/v_i)x_i = \sum_i c t_i \), where \( t_i \) is the time spent in medium \( i \). But \( ct_i \) is also the distance the light would travel, in vacuum.

7.5 \( 1 \text{ m}/500 \text{ nm} = 0.2 \times 10^7 = 2,000,000 \text{ waves}. \) In the glass

\[
0.05/\lambda_0/n = 0.05(1.5)/500 \text{ nm} = 1.5 \times 10^5;
\]
9.4 A bulb at $S$ would produce fringes. We can imagine it as made up of a very large number of incoherent point sources. Each of these would generate an independent pattern, all of which would then overlap. Bulbs at $S_1$ and $S_2$ would be incoherent and could not generate detectable fringes.

9.5 $y_m = s m \lambda/a \approx 14.5 \times 10^{-2}$ m and $\lambda = 0.0145$ m; $\nu = v/\lambda = 23.7$ kHz. This is Young's Experiment with the sources out-of-phase.

9.6 This is comparable to the "two-slit" configuration, (Figure 9.8), so we can use (9.29) $a \sin \theta_m = m \lambda$ ($\theta_m$ may not be "small"). Let $m = 1$,

$$\sin \theta = y/(s^2 + y^2)^{1/2},$$

$$ay = \lambda(s^2 + y^2)^{1/2}; \quad (a^2 - \lambda^2)y^2 = \lambda^2 s^2;$$

$$y = \lambda s/(a^2 - y^2)^{1/2}. \quad c = v \lambda,$$

so $\lambda = c/v = (3 \times 10^8 \text{ m/s})/(1.0 \times 10^6 \text{ Hz}) = 300 \text{ m}.$

$$y = (300 \text{ m})(2000 \text{ m})/((600 \text{ m})^2 - (300 \text{ m})^2)^{1/2} = 1.15 \times 10^3 \text{ m}$$

9.7 (a) $r_1 - r_2 = \pm \lambda/2$, hence $a \sin \theta_1 = \pm \lambda/2$ and

$$\theta_1 \approx \pm \lambda/2a = \pm(1/2)(632.8 \times 10^{-9} \text{ m})/(0.220 \times 10^{-3} \text{ m})$$

$$= \pm 1.58 \times 10^{-3} \text{ rad},$$

or since

$$y_1 = s \theta_1 = (1.00 \text{ m})(\pm 1.58 \times 10^{-3} \text{ rad}) = \pm 1.58 \text{ mm}.$$  

(b) $y_5 = s 5 \lambda/a = (1.00 \text{ m})5(632.8 \times 10^{-9} \text{ m})/(0.200 \times 10^{-3} \text{ m}) = 1.582 \times 10^{-2} \text{ m}.$ (c) Since the fringes vary as cosine-squared and the answer to (a) is half a fringe width, the answer to (b) is 10 times larger.

9.8 $\theta_m$ is "small," so we can use (9.28) $a \theta_m = m \lambda/a$, $\theta_m$ is radian,

$$a = m \lambda/\theta_m = [4(6.943 \times 10^{-7} \text{ m})]/[1^\circ(2\pi \text{ rad}/360^\circ)] = 1.59 \times 10^{-4} \text{ m}.$$  

9.9 $\Delta y \approx (s/a)\lambda$, so,

$$s = a \Delta y/\lambda = [(1.0 \times 10^{-4} \text{ m})(10 \times 10^{-3} \text{ m})]/(4.8799 \times 10^{-7} \text{ m}) = 2.05 \text{ m}.$$
9.10 \( (9.28) \theta_m = m\lambda/a. \) Want \( \theta_{1,\text{red}} = \theta_{2,\text{violet}}; \) \( (1)\lambda_{\text{red}}/a = (2)\lambda_{\text{violet}}/a; \) \( \lambda_{\text{violet}} = 390 \text{ nm}. \)

9.11 Follow section (9.3.1), except that (9.26) becomes \( r_1 - r_2 = (2m' - 1)(\lambda/2) \) for destructive interference, where \( m' = \pm 1, \pm 2, \ldots, \) so that \( (2m' - 1) \) is an odd integer. This leads to an expression equivalent to (9.28), 
\[ \theta_{m'} = (2m - 1)\lambda/2a. \]

9.12 Follow section (9.3.1), except that (9.26) becomes \( r_1 - r_2 + \Lambda = m\lambda, \) where \( \Lambda = \text{Optical path differences in beam}. \) Following \( r_1, \Lambda = nd \) (for \( \theta_m \) "small"),

\[ (r_1 - r_2) = m\lambda - \Lambda; \quad a\theta_m = m\lambda - nd; \quad \theta_m = (m\lambda - nd)/a. \]

9.13 As in section (9.3.1), we have constructive interference when \( \text{OPD} = m\lambda. \) There is an added OPD due to the angle, \( \theta, \) of the plane wave equal to \( a\sin\theta, \) so (9.26) becomes \( r_1 - r_2 + a\sin\theta = m\lambda. \) (9.24) \( \theta_m \approx y/s \) and (9.25) \( r_1 - r_2 \approx ay/s \) are unchanged, for small \( \theta_m, \) so \( r_1 - r_2 = m\lambda - a\sin\theta = a(y/s) = a\theta_m; \) \( \theta_m = (m\lambda/a) - \sin\theta. \)

9.14 \( (9.27) \ y_m = (s/a)m\lambda; \quad y_{1,\text{red}} = [(2.0 \text{ m})/(2.0 \times 10^{-4} \text{ m})](1)(4 \times 10^{-7} \text{ m}) = 4.0 \times 10^{-3} \text{ m}. \)
\[ y_{1,\text{violet}} = [(2.0 \text{ m})/(2.0 \times 10^{-4} \text{ m})](2)(6 \times 10^{-7} \text{ m}) = 12.0 \times 10^{-3} \text{ m}. \]
Distance = \( 8.0 \times 10^{-3} \text{ m}. \)

9.15 \( r_2^2 = a^2 + r_1^2 - 2ar_1\cos(90^\circ - \theta). \) The contribution to \( \cos\delta/2 \) from the third term in the Maclaurin expansion will be negligible if
\[ (k/2)(a^2\cos^2\theta/2r_1) \ll \pi/2; \quad \text{therefore} \quad r_1 \ll a^2/\lambda. \]

9.16 \( E = mv^2/2; \) \( v = 0.42 \times 10^6 \text{ m/s}; \) \( \lambda = h/mv = 1.73 \times 10^{-9} \text{ m}; \)
\[ \Delta y = s\lambda/a = 3.46 \text{ mm}. \]
9.17 $\Delta \nu / \Delta \lambda = \nu / \lambda$; \hspace{1em} $\delta \nu = \nu \Delta \lambda / \lambda = 1 / \Delta t_c$;
\hspace{1em} $c = \nu \lambda$, \hspace{1em} so \hspace{1em} $\nu = c / \lambda$.
\hspace{1em} $\Delta = (c / \lambda) \Delta \lambda / \lambda = c \Delta \lambda / \lambda^2$;
\hspace{1em} $\Delta t_c = \lambda^2 / c \Delta \lambda$; \hspace{1em} $\Delta \ell_c = c \Delta t_c = c (\lambda^2 / \Delta \lambda)$
\hspace{1em} $= \lambda^2 / \Delta \lambda = (500 \text{ nm})^2 / (2.5 \times 10^{-3} \text{ nm})$
\hspace{1em} $= 1 \times 10^8 \text{ nm} = 0.1 \text{ m} \simeq \lambda$.

9.18 $\vec{E} = E_0 e^{i \omega t} + E_0 e^{i \omega t + \delta} + E_0 e^{i (\omega t + 5\delta/2)}$. $I = \langle \vec{E}^2 \rangle_T = \langle \vec{E} \cdot \vec{E} \rangle_T$, so, as in
section 9.1, $I = (3/2) E_0^2 + 2 E_0^2 \{ \frac{1}{2} (\cos \delta + \cos(3\delta/2) + \cos(5\delta/2)) \}$ (three
terms of $\vec{E}_i \cdot \vec{E}_i$, 3 cross terms of $\vec{E}_i \cdot \vec{E}_j$). For each beam,

$I_i = \langle \vec{E}_i^2 \rangle_T = \frac{1}{2} E_0^2$,

at $\theta = 0$, so that for all three together $I(\theta = 0) = \frac{3}{2} E_0^2$. Note that
$(r_2 - r_1) = a \sin \theta$ so that

$\delta_2 = k(r_2 - r_1) = k(a \sin \theta)$; \hspace{1em} $(r_3 - r_1) = (5a/2) \sin \theta$

so that $\delta_3 = k(r_3 - r_1) = k(\frac{5}{2} a \sin \theta)$ where $\delta = ka \sin \theta$. So,

$I(\theta) = I(0)/3 + (2I(0)/9)(\cos \delta + \cos(3\delta/2) + \cos(5\delta/2))$

when $\theta = 0$, the second term is zero.

9.19 A ray form $S$ hits the biprism at an angle $\theta_i$ (w.r.t normal), is refracted at
angle $\theta_i$, and hits the second face at angle $(\theta_i + \alpha)$.
(4.4) $(1) \sin \theta_i = (n) \sin \theta_i$. $(n) \sin(\theta_i + \alpha) = (1) \sin(\theta / 2 + \alpha)$, where angle $\theta$
is defined in Figure 9.13. As $\theta_i \to 0$, $\theta_i \to 0$; $\alpha, \theta$ are both “small.”
$n \sin \alpha = \sin(\theta / 2 + \alpha)$, so $n \alpha \simeq (\theta / 2) + \alpha$, $\theta = 2(n - 1)\alpha$. From the figure
tan($\theta/2$) = ($a/2)/d$, so

$\theta/2 \simeq (a/2)/d$, \hspace{1em} $\theta = a/d$. \hspace{1em} $a/d = 2(n - 1)\alpha$, \hspace{1em} $a = 2d(n - 1)\alpha$. 