2-16. The plane side of the lens has \( R_1 = \infty \). The radius of curvature \( R_2 \) of the convex side is then found from the lensmaker’s equation:

\[
\frac{1}{f} = \frac{n_2 - n_1}{n_1} \left( \frac{1}{R_1} - \frac{1}{R_2} \right) \Rightarrow \frac{1}{f} = \frac{1.52 - 1}{1} \left( \frac{1}{\infty} - \frac{1}{R_2} \right) \Rightarrow R_2 = -13\text{cm}
\]

2-17. In general the lensmaker’s equation gives

\[
\frac{1}{f} = \frac{n_2 - n_1}{n_1} \left( \frac{1}{R_1} - \frac{1}{R_2} \right)
\]

For the positive meniscus lens shown to the right, \( R_1 = 5\text{ cm} \) and \( R_2 = 10\text{ cm} \).

Then, \( \frac{1}{f} = \frac{1.50 - 1}{1} \left( \frac{1}{5} - \frac{1}{10} \right) \Rightarrow f = +20\text{ cm} \)

For the negative meniscus lens shown to the right, \( R_1 = 10\text{ cm} \) and \( R_2 = 5\text{ cm} \).

For this case, \( \frac{1}{f} = \frac{1.50 - 1}{1} \left( \frac{1}{10} - \frac{1}{5} \right) \Rightarrow f = -20\text{ cm} \)

2-20. See Figure 2-36 in the text. Consider the three media as a sequence of three thin lenses. Each has a focal length given by the lensmaker’s equation, and the equivalent focal length is given Eq. (2-33) as,

\[
\frac{1}{f_{eq}} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3}
\]

Then, \( \frac{1}{f_1} = (1.5 - 1) \left( \frac{1}{\infty} - \frac{1}{-15} \right) \Rightarrow f_1 = 30\text{ cm} \),

\[
\frac{1}{f_2} = (1.65 - 1) \left( \frac{1}{-15} - \frac{1}{-15} \right) \text{ or } f_2 = -150\text{ cm} \text{, and } \frac{1}{f_3} \text{ same as for } f_1 : f_3 = 30\text{ cm} \text{. Then,}
\]

\[
\frac{1}{f_{eq}} = \frac{1}{30} + \frac{-13}{150} + \frac{1}{30} \text{ and so } f_{eq} = 50\text{ cm}.
\]

2-22. Refer to Figure 2-37 in the text.

(b) Lens heading towards mirror:

\[
\frac{1}{3f/2} = \frac{1}{s'} = \frac{1}{s} = \frac{s'}{s}
\]

Therefore, \( s' = -3f/5 \). \( m_1 = -s'/s = -3f/5 \times -3f/5 = 2/5 \)

Mirror:

\[
s = 3f + 3f/5 = 18f/5 \Rightarrow \frac{1}{s} = \frac{5}{18f}, \text{ and } s' = 18f/13, m_2 = -s'/s = -(18f/13)/(18f/5) = -5/13
\]

Lens after reflection:

\[
s = 3f - 18f/13 = 21f/13 \Rightarrow \frac{1}{s} = \frac{13}{21f}, \text{ or } s' = 21f/34, m_3 = -s'/s = -(21f/34)/(21f/13) = 13/34
\]

\[
m_f = \left( \frac{2}{5} \right) \left( \frac{13}{34} \right) = -17. \text{ The image is inverted, (21/34) f behind (right of) lens, inverted, and 1/17 original size.}
\]

2-27. See Figure 2-38 in the text. The applicable relations are:

Lenses equations: \( \frac{1}{s_1} + \frac{1}{s_2} = \frac{1}{f} \) and \( \frac{1}{s_2} + \frac{1}{s_2'} = \frac{1}{f} \)

Geometrical: \( L = s_1 + s_2 = s_2' + s_2 \). \( D = s_2 - s_1 = s_2' - s_2 \)

Thus,

\[
f = \frac{s_1 s_2'}{s_1 + s_2'} = \frac{s_1 s_2}{s_2 + s_2'} = \frac{s_2 s_2'}{s_2 + s_2'}
\]

(1)

Because the lens equation can be satisfied the second time by simply interchanging object and image distances,

\[
s_2 = s_1' \text{ and } s_2' = s_1
\]

(2)

Adding and subtracting the equations \( L = s_2 + s_2' \) and \( D = -s_1 + s_2 \), we get,

\( L - D = 2s \) and \( L + D = 2s_2 \). Their product is by Eq. (1), \( L^2 - D^2 = 4s_1 s_2' \), or by Eq. (2), \( L^2 - D^2 = 4fL \).

Thus, \( f = \frac{L^2 - D^2}{4L} \).
2-23. The arrangement of the object and lenses is shown below.

\[ \begin{array}{ccc}
\uparrow & 20 \text{ cm} & \downarrow \\
\downarrow & 30 \text{ cm} & \uparrow \\
1 & 2 & 3 \\
\end{array} \]

(a) \( f_1 = +10 \text{ cm}, f_2 = +15 \text{ cm}, f_3 = +20 \text{ cm} \)

1st lens: \( \frac{1}{20} + \frac{1}{s} = 1 \]
\[ s' = 20 \] \[ m_1 = -\frac{20}{20} = -1 \]

2nd lens: \( \frac{1}{10} + \frac{1}{s'} = \frac{1}{15} \]
\[ s' = -30 \] \[ m_2 = -\frac{(-30)/10}{15} = +3 \]

3rd lens: \( \frac{1}{50} + \frac{1}{s'} = \frac{1}{20} \]
\[ s' = 100/3 \] \[ m_3 = -\frac{100/3(50)}{3} = -\frac{2}{3} \]
\[ m_T = m_1 m_2 m_3 = +2 \]

(b) \( f_1 = +10 \text{ cm}, f_2 = -15 \text{ cm}, f_3 = +20 \text{ cm} \)

1st lens: \( \frac{1}{20} + \frac{1}{s} = \frac{1}{10} \]
\[ s' = 20 \] \[ m_1 = -\frac{20}{20} = -1 \]

2nd lens: \( \frac{1}{10} + \frac{1}{s'} = -\frac{1}{15} \]
\[ s' = -6 \] \[ m_2 = -\frac{(-6)/10}{15} = +0.6 \]

3rd lens: \( \frac{1}{20} + \frac{1}{s'} = \frac{1}{20} \]
\[ s' = 520/6 \] \[ m_3 = -\frac{520/(6 \times 26)}{3} = -\frac{10}{3} \]
\[ m_T = m_1 m_2 m_3 = +2 \]

(c) \( f_1 = -10 \text{ cm}, f_2 = +15 \text{ cm}, f_3 = -20 \text{ cm} \)

1st lens: \( \frac{1}{20} + \frac{1}{s} = -\frac{1}{10} \]
\[ s' = 20/3 \] \[ m_1 = -\frac{(-20)/(20)}{3} = \frac{1}{3} \]

2nd lens: \( \frac{3}{110} + \frac{1}{s'} = \frac{1}{15} \]
\[ s' = 330/13 \] \[ m_2 = -\frac{(330)(13)}{13} = -\frac{9}{13} \]

3rd lens: \( \frac{-13}{70} + \frac{1}{s'} = \frac{1}{-20} \]
\[ s' = 140/19 \] \[ m_3 = -\frac{(140)(19)}{19} = -\frac{26}{19} \]
\[ m_T = m_1 m_2 m_3 = -\frac{6}{19} \]

2-31. The distance between the object and the image is \( D = s + s' = s + \frac{f s}{s - f} \). This is minimized when,

\[ \frac{dD}{ds} = 1 + \frac{(s - f) f - f s}{(s - f)^2} = 0 \Rightarrow s(s - 2f) = 0 \Rightarrow s = f, 2f. \]

The minimum distance \( D \) occurs when \( s = 2f \) and has the value \( D = 2f + \frac{f(2f)}{2f - f} = 4f. \) That is, in this configuration \( s = s' = 2f. \)
3-4. (a) The vertical and horizontal scales in the figure differ by a factor of 4.

(b) Image Plane:

$$s_f' = \frac{s_f}{s} = \frac{100}{20} = 5 \Rightarrow s_1' = \frac{5}{40} = 0.125 \text{ cm (right of Lens 1)} $$

$$s_f' = \frac{s_f}{s} = \frac{100}{20} = 5 \Rightarrow s_2' = \frac{5}{40} = 0.125 \text{ cm (right of Lens 2)} $$

(c) Candidates for AS: Lens 1, Aperture, or Lens 2.

Lenses subtend at an angle of $\theta_{l1} = \frac{2}{40} = 0.05 \text{ rad}$. Aperture image in Lens 1: $s' = \frac{20}{40/3} = 30 \text{ cm}, \text{ or at object plane, then } \theta_{Aperture} = 0^\circ$

Lenses 2 image through Lens 1: $s = \frac{30}{40/3} = 22.5 \text{ cm left of Lens 1 or 16 cm right of object}$. 

$$m = -s'/s = -2.25 = -0.8 \text{ so size } = 0.8 \times 2 = 1.6 \text{ cm} \Rightarrow \theta_{l2} = \frac{1.6}{16} = 0.1 > 0.05$$

Thus Lens 1 behaves as the AS. It is also the $E_A P$, being the first in line.

(d) Exit Pupil $E_A P$: Image of AS (i.e., Lens 1) in Lens 2:

$$s' = \frac{s_f' 2}{s - f_2} = \frac{30}{20/3} = 8.57 \text{ cm, right of Lens 2} \Rightarrow m = -s'/s = -\frac{8.57}{30} = 0.2857 \Rightarrow D_{exit} = \frac{2}{7} \times 2 = \frac{4}{7} \text{ cm}$$

(e) Field Stop: Either Aperture or Lens 2, whichever subtends the smaller angle at center of $E_A P$-Lens 1:

$$\theta_{Aperture} = \frac{0.5}{20} = 0.025^\circ; \theta_{Lens 2} = \frac{2}{30} = 0.067^\circ$$

Thus the Aperture Stop behaves as the field stop (FS). *Continued on next page*
3-4. (c) continued.


\[ s' = \frac{(10 \times 20)}{30 - 20} = 20 \text{ cm}, \text{ right of Lens 2, in image plane,} \]

\[ m = -\frac{s'}{s} = 20/10 = -2 \text{ so } D_{EwP} = 2 \times 0.5 = 1.0 \text{ cm} \]

(f) Angular field of view: Drawn not to scale,

\[ \alpha = \frac{1}{40} = 0.0125 \text{ rad} \]
\[ \alpha = 1.43^\circ \]

3-16. Using Eq. (3-35),

\[ \frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{L}{f_1 f_2} = \frac{1}{20} + \frac{1}{-8} = \frac{15}{20} \Rightarrow f = 53.3 \text{ cm} \]

If the film plane is in the image position for an object at infinity, its position can be found as follows. The image formed by the first lens of the object at infinity falls at the focal point of the first lens which is 5 cm past the second lens. This intermediate image serves as the object for the second lens. That is \( s_2 = -5 \text{ cm} \).

The final image position relative to the second lens can be found using the thin lens formula,

\[ \frac{1}{s_2} + \frac{1}{s'_2} - \frac{1}{f_2} = \frac{1}{-5} + \frac{1}{s'_2} = \frac{1}{13.33} \Rightarrow s'_2 = 13.33 \text{ cm} \]

That is, the film should be placed 13.33 cm past the negative lens of the combination. The image size \( h' \) of a distant object subtending an angle of \( 2^\circ \) at the camera is found using the effective focal length of the lens.

\[ \tan 2^\circ = \frac{|h'|}{f} = \frac{h'}{53.33} \Rightarrow 1.86 \text{ cm} \]

Alternately one may find the image height as,

\[ h' = m_1 m_2 b = -\frac{s_1}{s_2} \left( -\frac{s_2}{s_2} \right) = \frac{20 \times 13.33}{s_1} = \frac{h}{s_1} = \frac{(53.33 \text{ cm})}{s_1} \Rightarrow h = (53.33 \text{ cm}) \tan 2^\circ = 1.86 \text{ cm} \]

3-22. The focal lengths of the two lenses are, \( f_1 = f_2 = 3 \text{ cm} \) and they are separated by \( L = 2.8 \text{ cm} \).

(a) The equivalent focal length is,

\[ \frac{1}{f_{eq}} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{L}{f_1 f_2} = \frac{1}{3} + \frac{1}{3} = \frac{2.8}{9} \Rightarrow f_{eq} = 2.8125 \text{ cm} \]

(b) The magnification is,

\[ M = \frac{25}{f_{eq}} + 1 = \frac{25}{2.8125} + 1 = 9.9 \approx 10 \times \]

3-23. (a) The system is sketched below.

\[ M = m_{obj} \times M_{eq} = \frac{s'}{s} \times 10 \text{ where } s' = f + 16 = 16.5 \text{ cm} \]

\[ s = \frac{s'}{f} = \frac{(16.5) (0.5)}{16.5 - 0.5} = \frac{8.25}{16} \]

\[ |m| = \frac{s'}{s} = \frac{(16.5) (16)}{8.25} = 32, M = (32)(16) = 320 \times \]

(b) \[ s = \frac{8.25}{16} = 0.516 \text{ cm} \]