ELECTRIC CHARGE AND ELECTRIC FIELD

21

21.2. IDENTIFY: The charge that flows is the rate of charge flow times the duration of the time interval.

SET UP: The charge of one electron has magnitude $e = 1.60 \times 10^{-19}$ C.

EXECUTE: The rate of charge flow is $20{,}000$ C/s and $t = 100\ \mu\text{s} = 1.00 \times 10^{-4}$ s.

$Q = (20{,}000\ \text{C/s})(1.00 \times 10^{-4}\ \text{s}) = 2.00\ \text{C}$. The number of electrons is $n_\text{e} = \dfrac{Q}{1.60 \times 10^{-19}\ \text{C}} = 1.25 \times 10^{19}$.

EVALUATE: This is a very large amount of charge and a large number of electrons.

21.4. IDENTIFY: Use the mass *m* of the ring and the atomic mass *M* of gold to calculate the number of gold atoms. Each atom has 79 protons and an equal number of electrons.

SET UP: $N_\text{A} = 6.02 \times 10^{23}$ atoms/mol. A proton has charge $+e$.

EXECUTE: The mass of gold is 17.7 g and the atomic weight of gold is 197 g/mol. So the number of atoms is $N_\text{A} n = (6.02 \times 10^{23}\ \text{atoms/mol})\left(\dfrac{17.7\ \text{g}}{197\ \text{g/mol}}\right) = 5.41 \times 10^{22}$ atoms. The number of protons is

$n_\text{p} = (79\ \text{protons/atom})(5.41 \times 10^{22}\ \text{atoms}) = 4.27 \times 10^{24}$ protons. $Q = (n_\text{p})(1.60 \times 10^{-19}\ \text{C/proton}) = 6.83 \times 10^{5}$ C.

(b) The number of electrons is $n_\text{e} = n_\text{p} = 4.27 \times 10^{24}$.

EVALUATE: The total amount of positive charge in the ring is very large, but there is an equal amount of negative charge.

21.7. IDENTIFY: Apply Coulomb's law.

SET UP: Consider the force on one of the spheres.

(a) EXECUTE: $q_1 = q_2 = q$

$$F = \frac{1}{4\pi\epsilon_0}\frac{|q_1 q_2|}{r^2} = \frac{q^2}{4\pi\epsilon_0 r^2} \text{ so } q = r\sqrt{\frac{F}{(1/4\pi\epsilon_0)}} = 0.150\ \text{m}\sqrt{\frac{0.220\ \text{N}}{8.988 \times 10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2}} = 7.42 \times 10^{-7}\ \text{C (on each)}$$

(b) $q_2 = 4q_1$

$$F = \frac{1}{4\pi\epsilon_0}\frac{|q_1 q_2|}{r^2} = \frac{4q_1^2}{4\pi\epsilon_0 r^2} \text{ so } q_1 = r\sqrt{\frac{F}{4(1/4\pi\epsilon_0)}} = \tfrac{1}{2}r\sqrt{\frac{F}{(1/4\pi\epsilon_0)}} = \tfrac{1}{2}(7.42 \times 10^{-7}\ \text{C}) = 3.71 \times 10^{-7}\ \text{C}.$$

And then $q_2 = 4q_1 = 1.48 \times 10^{-6}$ C.

EVALUATE: The force on one sphere is the same magnitude as the force on the other sphere, whether the sphere have equal charges or not.

21.14. IDENTIFY: Apply Coulomb's law and find the vector sum of the two forces on *Q*.

SET UP: The force that q_1 exerts on *Q* is repulsive, as in Example 21.4, but now the force that q_2 exerts is attractive.

EXECUTE: The *x*-components cancel. We only need the *y*-components, and each charge contributes equally.

$F_{1y} = F_{2y} = -\dfrac{1}{4\pi\epsilon_0}\dfrac{(2.0 \times 10^{-6}\ \text{C})\,(4.0 \times 10^{-6}\ \text{C})}{(0.500\ \text{m})^2}\sin\alpha = -0.173\ \text{N (since } \sin\alpha = 0.600)$. Therefore, the total force is $2F = 0.35$ N, in the $-y$-direction.

EVALUATE: If q_1 is $-2.0\ \mu\text{C}$ and q_2 is $+2.0\ \mu\text{C}$, then the net force is in the $+y$-direction.

21.20. **IDENTIFY:** Apply $F = k\dfrac{|qq'|}{r^2}$ to each pair of charges. The net force is the vector sum of the forces due to q_1 and q_2.

SET UP: Like charges repel and unlike charges attract. The charges and their forces on q_3 are shown in Figure 21.20.

EXECUTE: $F_1 = k\dfrac{|q_1q_3|}{r_1^2} = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\dfrac{(4.00\times10^{-9}\ \text{C})(0.600\times10^{-9}\ \text{C})}{(0.200\ \text{m})^2} = 5.394\times10^{-7}\ \text{N}$.

$F_2 = k\dfrac{|q_2q_3|}{r_2^2} = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\dfrac{(5.00\times10^{-9}\ \text{C})(0.600\times10^{-9}\ \text{C})}{(0.300\ \text{m})^2} = 2.997\times10^{-7}\ \text{N}$.

$F_x = F_{1x} + F_{2x} = +F_1 - F_2 = 2.40\times10^{-7}\ \text{N}$. The net force has magnitude $2.40\times10^{-7}\ \text{N}$ and is in the $+x$ direction.

EVALUATE: Each force is attractive, but the forces are in opposite directions because of the placement of the charges. Since the forces are in opposite directions, the net force is obtained by subtracting their magnitudes.

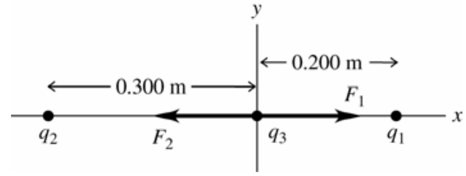

Figure 21.20

21.21. **IDENTIFY:** Apply Coulomb's law to calculate each force on $-Q$.

SET UP: Let $\vec{F}_1$ be the force exerted by the charge at $y = a$ and let $\vec{F}_2$ be the force exerted by the charge at $y = -a$.

EXECUTE: **(a)** The two forces on $-Q$ are shown in Figure 21.21a. $\sin\theta = \dfrac{a}{(a^2+x^2)^{1/2}}$ and $r = (a^2+x^2)^{1/2}$ is the distance between q and $-Q$ and between $-q$ and $-Q$.

(b) $F_x = F_{1x} + F_{2x} = 0$. $F_y = F_{1y} + F_{2y} = 2\dfrac{1}{4\pi\epsilon_0}\dfrac{qQ}{(a^2+x^2)}\sin\theta = \dfrac{1}{4\pi\epsilon_0}\dfrac{2qQa}{(a^2+x^2)^{3/2}}$.

(c) At $x = 0$, $F_y = \dfrac{1}{4\pi\epsilon_0}\dfrac{2qQ}{a^2}$, in the $+y$ direction.

(d) The graph of F_y versus x is given in Figure 21.21b.

EVALUATE: $F_x = 0$ for all values of x and $F_y > 0$ for all x.

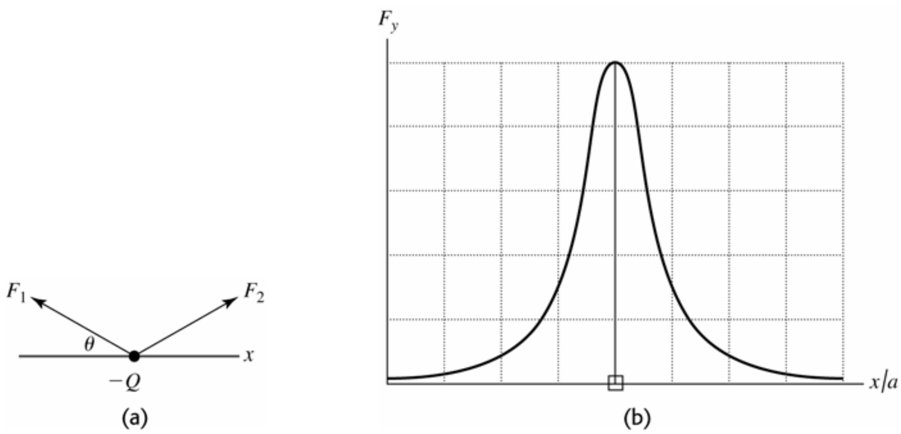

Figure 21.21

21.28. **IDENTIFY:** Use constant acceleration equations to calculate the upward acceleration a and then apply $\vec{F} = q\vec{E}$ to calculate the electric field.

SET UP: Let $+y$ be upward. An electron has charge $q = -e$.

EXECUTE: **(a)** $v_{0y} = 0$ and $a_y = a$, so $y - y_0 = v_{0y}t + \frac{1}{2}a_yt^2$ gives $y - y_0 = \frac{1}{2}at^2$. Then

$a = \dfrac{2(y-y_0)}{t^2} = \dfrac{2(4.50\ \text{m})}{(3.00\times10^{-6}\ \text{s})^2} = 1.00\times10^{12}\ \text{m/s}^2$. $E = \dfrac{F}{q} = \dfrac{ma}{q} = \dfrac{(9.11\times10^{-31}\ \text{kg})(1.00\times10^{12}\ \text{m/s}^2)}{1.60\times10^{-19}\ \text{C}} = 5.69\ \text{N/C}$

The force is up, so the electric field must be *downward* since the electron has negative charge.

(b) The electron's acceleration is $\sim 10^{11} g$, so gravity must be negligibly small compared to the electrical force.

EVALUATE: Since the electric field is uniform, the force it exerts is constant and the electron moves with constant acceleration.

21.30. **IDENTIFY:** Apply $E = \dfrac{1}{4\pi\epsilon_0}\dfrac{|q|}{r^2}$.

SET UP: The iron nucleus has charge $+26e$. A proton has charge $+e$.

EXECUTE: **(a)** $E = \dfrac{1}{4\pi\epsilon_0}\dfrac{(26)(1.60\times10^{-19}\text{ C})}{(6.00\times10^{-10}\text{ m})^2} = 1.04\times10^{11}\text{ N/C.}$

(b) $E_{\text{proton}} = \dfrac{1}{4\pi\epsilon_0}\dfrac{(1.60\times10^{-19}\text{ C})}{(5.29\times10^{-11}\text{ m})^2} = 5.15\times10^{11}\text{ N/C.}$

EVALUATE: These electric fields are very large. In each case the charge is positive and the electric fields are directed away from the nucleus or proton.

21.39. **IDENTIFY:** Find the angle θ that $\hat{\boldsymbol{r}}$ makes with the +x-axis. Then $\hat{\boldsymbol{r}} = (\cos\theta)\hat{\boldsymbol{i}} + (\sin\theta)\hat{\boldsymbol{j}}$.

SET UP: $\tan\theta = y/x$

EXECUTE: **(a)** $\tan^{-1}\left(\dfrac{-1.35}{0}\right) = -\dfrac{\pi}{2}$ rad . $\hat{\boldsymbol{r}} = -\hat{\boldsymbol{j}}$.

(b) $\tan^{-1}\left(\dfrac{12}{12}\right) = \dfrac{\pi}{4}$ rad . $\hat{\boldsymbol{r}} = \dfrac{\sqrt{2}}{2}\hat{\boldsymbol{i}} + \dfrac{\sqrt{2}}{2}\hat{\boldsymbol{j}}$.

(c) $\tan^{-1}\left(\dfrac{2.6}{+1.10}\right) = 1.97\text{ rad} = 112.9°$. $\hat{\boldsymbol{r}} = -0.39\hat{\boldsymbol{i}} + 0.92\hat{\boldsymbol{j}}$ (Second quadrant).

EVALUATE: In each case we can verify that $\hat{\boldsymbol{r}}$ is a unit vector, because $\hat{\boldsymbol{r}}\cdot\hat{\boldsymbol{r}} = 1$.

21.48. **IDENTIFY:** A positive and negative charge, of equal magnitude q, are on the x-axis, a distance a from the origin. Apply Eq.(21.7) to calculate the field due to each charge and then calculate the vector sum of these fields.

SET UP: $\vec{\boldsymbol{E}}$ due to a point charge is directed away from the charge if it is positive and directed toward the charge if it is negative.

EXECUTE: **(a)** Halfway between the charges, both fields are in the $-x$-direction and $E = \dfrac{1}{4\pi\epsilon_0}\dfrac{2q}{a^2}$, in the $-x$-direction .

(b) $E_x = \dfrac{1}{4\pi\epsilon_0}\left(\dfrac{-q}{(a+x)^2} - \dfrac{q}{(a-x)^2}\right)$ for $|x| < a$. $E_x = \dfrac{1}{4\pi\epsilon_0}\left(\dfrac{-q}{(a+x)^2} + \dfrac{q}{(a-x)^2}\right)$ for $x > a$.

$E_x = \dfrac{1}{4\pi\epsilon_0}\left(\dfrac{-q}{(a+x)^2} - \dfrac{q}{(a-x)^2}\right)$ for $x < -a$. E_x is graphed in Figure 21.48.

EVALUATE: At points on the x axis and between the charges, E_x is in the $-x$-direction because the fields from both charges are in this direction. For $x < -a$ and $x > +a$, the fields from the two charges are in opposite directions and the field from the closer charge is larger in magnitude.

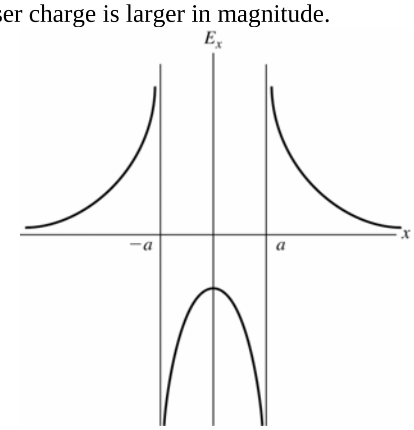

Figure 21.48

21.50. IDENTIFY: Apply Eq.(21.7) to calculate the field due to each charge and then calculate the vector sum of those fields.

SET UP: The fields due to q_1 and to q_2 are sketched in Figure 21.50.

EXECUTE: $\vec{E}_2 = \dfrac{1}{4\pi\epsilon_0}\dfrac{(6.00\times10^{-9}\text{ C})}{(0.6\text{ m})^2}(-\hat{i}) = -150\hat{i}\text{ N/C}$.

$$\vec{E}_1 = \frac{1}{4\pi\epsilon_0}(4.00\times10^{-9}\text{ C})\left(\frac{1}{(1.00\text{ m})^2}(0.600)\hat{i} + \frac{1}{(1.00\text{ m})^2}(0.800)\hat{j}\right) = (21.6\hat{i} + 28.8\hat{j})\text{ N/C}.$$

$\vec{E} = \vec{E}_1 + \vec{E}_2 = (-128.4\text{ N/C})\hat{i} + (28.8\text{ N/C})\hat{j}$. $E = \sqrt{(128.4\text{ N/C})^2 + (28.8\text{ N/C})^2} = 131.6\text{ N/C}$ at $\theta = \tan^{-1}\left(\dfrac{28.8}{128.4}\right) = 12.6°$ above the $-x$ axis and therefore $196.2°$ counterclockwise from the $+x$ axis.

EVALUATE: $\vec{E}_1$ is directed toward q_1 because q_1 is negative and $\vec{E}_2$ is directed away from q_2 because q_2 is positive.

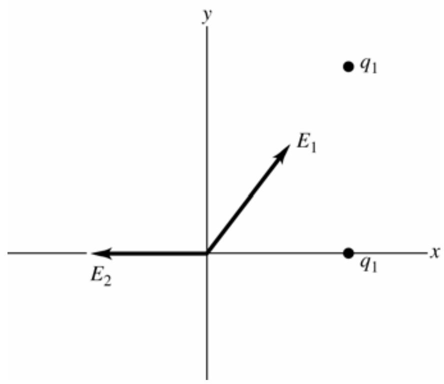

Figure 21.50

21.57. IDENTIFY: By superposition we can add the electric fields from two parallel sheets of charge.

SET UP: The field due to each sheet of charge has magnitude $\sigma / 2\epsilon_0$ and is directed toward a sheet of negative charge and away from a sheet of positive charge.

(a) The two fields are in opposite directions and $E = 0$.

(b) The two fields are in opposite directions and $E = 0$.

(c) The fields of both sheets are downward and $E = 2\dfrac{\sigma}{2\epsilon_0} = \dfrac{\sigma}{\epsilon_0}$, directed downward.

EVALUATE: The field produced by an infinite sheet of charge is uniform, independent of distance from the sheet.

21.74. IDENTIFY: Apply $\sum F_x = 0$ and $\sum F_y = 0$ to one of the spheres.

SET UP: The free-body diagram is sketched in Figure 21.74. F_e is the repulsive Coulomb force between the spheres. For small θ, $\sin\theta \approx \tan\theta$.

EXECUTE: $\sum F_x = T\sin\theta - F_e = 0$ and $\sum F_y = T\cos\theta - mg = 0$. So $\dfrac{mg\sin\theta}{\cos\theta} = F_e = \dfrac{kq^2}{d^2}$. But $\tan\theta \approx \sin\theta = \dfrac{d}{2L}$, so $d^3 = \dfrac{2kq^2L}{mg}$ and $d = \left(\dfrac{q^2L}{2\pi\epsilon_0 mg}\right)^{1/3}$.

EVALUATE: d increases when q increases.

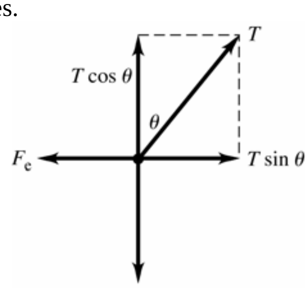

Figure 21.74

21.86. IDENTIFY: Apply constant acceleration equations to a drop to find the acceleration. Then use $F = ma$ to find the force and $F = |q|E$ to find $|q|$.

SET UP: Let $D = 2.0$ cm be the horizontal distance the drop travels and $d = 0.30$ mm be its vertical displacement. Let $+x$ be horizontal and in the direction from the nozzle toward the paper and let $+y$ be vertical, in the direction of the deflection of the drop. $a_x = 0$ and $a_y = a$.

EXECUTE: First, the mass of the drop: $m = \rho V = (1000\ \text{kg/m}^3)\left(\dfrac{4\pi(15.0\times10^{-6}\ \text{m})^3}{3}\right) = 1.41\times10^{-11}\ \text{kg}$. Next, the time of flight: $t = D/v = (0.020\ \text{m})/(20\ \text{m/s}) = 0.00100\ \text{s}$. $d = \dfrac{1}{2}at^2$. $a = \dfrac{2d}{t^2} = \dfrac{2(3.00\times10^{-4}\ \text{m})}{(0.001\,\text{s})^2} = 600\ \text{m/s}^2$.

Then $a = F/m = qE/m$ gives $q = ma/E = \dfrac{(1.41\times10^{-11}\ \text{kg})(600\ \text{m/s}^2)}{8.00\times10^4\ \text{N/C}} = 1.06\times10^{-13}\ \text{C}$.

EVALUATE: Since q is positive the vertical deflection is in the direction of the electric field.

21.104. IDENTIFY: Apply Eq.(21.11) for the electric field of a disk. The hole can be described by adding a disk of charge density $-\sigma$ and radius R_1 to a solid disk of charge density $+\sigma$ and radius R_2.

SET UP: The area of the annulus is $\pi(R_2^2 - R_1^2)\sigma$. The electric field of a disk, Eq.(21.11) is

$$E = \frac{\sigma}{2\epsilon_0}\left[1 - 1\Big/\sqrt{(R/x)^2+1}\right].$$

EXECUTE: (a) $Q = A\sigma = \pi(R_2^2 - R_1^2)\sigma$

(b) $\vec{E}(x) = \dfrac{\sigma}{2\epsilon_0}\left(\left[1-1/\sqrt{(R_2/x)^2+1}\right] - \left[1-1/\sqrt{(R_1/x)^2+1}\right]\right)\dfrac{|x|}{x}\hat{\boldsymbol{i}}$. $\vec{E}(x) = \dfrac{-\sigma}{2\epsilon_0}\left(1/\sqrt{(R_1/x)^2+1} - 1/\sqrt{(R_2/x)^2+1}\right)\dfrac{|x|}{x}\hat{\boldsymbol{i}}$.

The electric field is in the $+x$ direction at points above the disk and in the $-x$ direction at points below the disk, and the factor $\dfrac{|x|}{x}\hat{\boldsymbol{i}}$ specifies these directions.

(c) Note that $1\Big/\sqrt{(R_1/x)^2+1} = \dfrac{|x|}{R_1}(1+(x/R_1)^2)^{-1/2} \approx \dfrac{|x|}{R_1}$. This gives $\vec{E}(x) = \dfrac{\sigma}{2\epsilon_0}\left(\dfrac{x}{R_1} - \dfrac{x}{R_2}\right)\dfrac{|x|^2}{x}\hat{\boldsymbol{i}} = \dfrac{\sigma}{2\epsilon_0}\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)x\hat{\boldsymbol{i}}$.

Sufficiently close means that $(x/R_1)^2 << 1$.

(d) $F_x = qE_x = -\dfrac{q\sigma}{2\epsilon_0}\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)x$. The force is in the form of Hooke's law: $F_x = -kx$, with $k = \dfrac{q\sigma}{2\epsilon_0}\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)$.

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{q\sigma}{2\epsilon_0 m}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}.$$

EVALUATE: The frequency is independent of the initial position of the particle, so long as this position is sufficiently close to the center of the annulus for $(x/R_1)^2$ to be small.