ELECTRIC CHARGE AND ELECTRIC FIELD

21.63. **(a) IDENTIFY** and **SET UP:** Use Eq.(21.14) to relate the dipole moment to the charge magnitude and the separation d of the two charges. The direction is from the negative charge toward the positive charge.
EXECUTE: $p = qd = (4.5\times10^{-9}\ \text{C})(3.1\times10^{-3}\ \text{m}) = 1.4\times10^{-11}\ \text{C}\cdot\text{m}$; The direction of $\vec{p}$ is from q_1 toward q_2.
(b) IDENTIFY and **SET UP:** Use Eq. (21.15) to relate the magnitudes of the torque and field.
EXECUTE: $\tau = pE\sin\phi$, with ϕ as defined in Figure 21.63, so

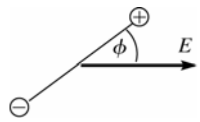


Figure 21.63

$$E = \frac{\tau}{p\sin\phi}$$

$$E = \frac{7.2\times10^{-9}\ \text{N}\cdot\text{m}}{(1.4\times10^{-11}\ \text{C}\cdot\text{m})\sin 36.9^\circ} = 860\ \text{N/C}$$

EVALUATE: Eq.(21.15) gives the torque about an axis through the center of the dipole. But the forces on the two charges form a couple (Problem 11.53) and the torque is the same for any axis parallel to this one. The force on each charge is $|q|E$ and the maximum moment arm for an axis at the center is $d/2$, so the maximum torque is $2(|q|E)(d/2) = 1.2\times10^{-8}\ \text{N}\cdot\text{m}$. The torque for the orientation of the dipole in the problem is less than this maximum.

21.64. **(a) IDENTIFY:** The potential energy is given by Eq.(21.17).
SET UP: $U(\phi) = -\vec{p}\cdot\vec{E} = -pE\cos\phi$, where ϕ is the angle between $\vec{p}$ and $\vec{E}$.
EXECUTE: parallel: $\phi = 0$ and $U(0^\circ) = -pE$

perpendicular: $\phi = 90^\circ$ and $U(90^\circ) = 0$

$$\Delta U = U(90^\circ) - U(0^\circ) = pE = (5.0\times10^{-30}\ \text{C}\cdot\text{m})(1.6\times10^{6}\ \text{N/C}) = 8.0\times10^{-24}\ \text{J}.$$

(b) $\frac{3}{2}kT = \Delta U$ so $T = \dfrac{2\Delta U}{3k} = \dfrac{2(8.0\times10^{-24}\ \text{J})}{3(1.381\times10^{-23}\ \text{J/K})} = 0.39\ \text{K}$

EVALUATE: Only at very low temperatures are the dipoles of the molecules aligned by a field of this strength. A much larger field would be required for alignment at room temperature.

21.90. **IDENTIFY:** Use Eq. (21.7) to calculate the electric field due to a small slice of the line of charge and integrate as in Example 21.11. Use Eq. (21.3) to calculate $\vec{F}$.
SET UP: The electric field due to an infinitesimal segment of the line of charge is sketched in Figure 21.90.

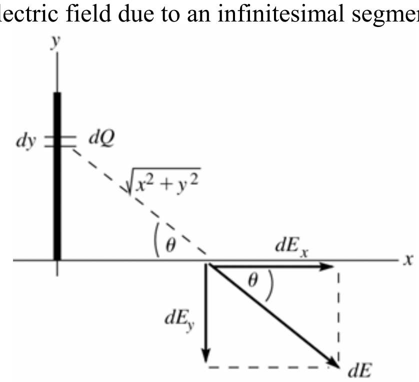


Figure 21.90

$$\sin\theta = \frac{y}{\sqrt{x^2+y^2}}$$

$$\cos\theta = \frac{x}{\sqrt{x^2+y^2}}$$

Slice the charge distribution up into small pieces of length dy. The charge dQ in each slice is $dQ = Q(dy/a)$. The electric field this produces at a distance x along the x-axis is dE. Calculate the components of $d\vec{E}$ and then integrate over the charge distribution to find the components of the total field.

EXECUTE: $dE = \dfrac{1}{4\pi\epsilon_0}\left(\dfrac{dQ}{x^2+y^2}\right) = \dfrac{Q}{4\pi\epsilon_0 a}\left(\dfrac{dy}{x^2+y^2}\right)$

$$dE_x = dE\cos\theta = \frac{Qx}{4\pi\epsilon_0 a}\left(\frac{dy}{(x^2+y^2)^{3/2}}\right)$$

$$dE_y = -dE\sin\theta = -\frac{Q}{4\pi\epsilon_0 a}\left(\frac{ydy}{(x^2+y^2)^{3/2}}\right)$$

$$E_x = \int dE_x = -\frac{Qx}{4\pi\epsilon_0 a}\int_0^a \frac{dy}{(x^2+y^2)^{3/2}} = \frac{Qx}{4\pi\epsilon_0 a}\left[\frac{1}{x^2}\frac{y}{\sqrt{x^2+y^2}}\right]_0^a = \frac{Q}{4\pi\epsilon_0 x}\frac{1}{\sqrt{x^2+a^2}}$$

$$E_y = \int dEy = -\frac{Q}{4\pi\epsilon_0 a}\int_0^a \frac{ydy}{(x^2+y^2)^{3/2}} = -\frac{Q}{4\pi\epsilon_0 a}\left[-\frac{1}{\sqrt{x^2+y^2}}\right]_0^a = -\frac{Q}{4\pi\epsilon_0 a}\left(\frac{1}{x} - \frac{1}{\sqrt{x^2+a^2}}\right)$$

(b) $\vec{F} = q_0\vec{E}$

$$F_x = -qE_x = \frac{-qQ}{4\pi\epsilon_0 x}\frac{1}{\sqrt{x^2+a^2}};\ F_y = -qE_y = \frac{qQ}{4\pi\epsilon_0 a}\left(\frac{1}{x} - \frac{1}{\sqrt{x^2+a^2}}\right)$$

(c) For $x >> a$, $\dfrac{1}{\sqrt{x^2+a^2}} = \dfrac{1}{x}\left(1+\dfrac{a^2}{x^2}\right)^{-1/2} = \dfrac{1}{x}\left(1-\dfrac{a^2}{2x^2}\right) = \dfrac{1}{x} - \dfrac{a^2}{2x^3}$

$$F_x \approx -\frac{qQ}{4\pi\epsilon_0 x^2},\ F_y \approx \frac{qQ}{4\pi\epsilon_0 a}\left(\frac{1}{x} - \frac{1}{x} + \frac{a^2}{2x^3}\right) = \frac{qQa}{8\pi\epsilon_0 x^3}$$

EVALUATE: For $x >> a$, $F_y << F_x$ and $F \approx |F_x| = \dfrac{qQ}{4\pi\epsilon_0 x^2}$ and $\vec{F}$ is in the $-x$-direction. For $x >> a$ the charge distribution Q acts like a point charge.

22

GAUSS'S LAW

22.4. **IDENTIFY:** Use Eq.(22.3) to calculate the flux for each surface. Use Eq.(22.8) to calculate the total enclosed charge.

SET UP: $\vec{E} = (-5.00\ \text{N/C}\cdot\text{m})x\hat{i} + (3.00\ \text{N/C}\cdot\text{m})z\hat{k}$. The area of each face is L^2, where $L = 0.300\ \text{m}$.

EXECUTE: $\hat{n}_{S_1} = -\hat{j} \Rightarrow \Phi_1 = \vec{E}\cdot\hat{n}_{S_1}A = 0$.

$\hat{n}_{S_2} = +\hat{k} \Rightarrow \Phi_2 = \vec{E}\cdot\hat{n}_{S_2}A = (3.00\ \text{N/C}\cdot\text{m})(0.300\ \text{m})^2 z = (0.27\ (\text{N/C})\cdot\text{m})z$.

$\Phi_2 = (0.27\ (\text{N/C})\text{m})(0.300\ \text{m}) = 0.081\ (\text{N/C})\cdot\text{m}^2$.

$\hat{n}_{S_3} = +\hat{j} \Rightarrow \Phi_3 = \vec{E}\cdot\hat{n}_{S_3}A = 0$.

$\hat{n}_{S_4} = -\hat{k} \Rightarrow \Phi_4 = \vec{E}\cdot\hat{n}_{S_4}A = -(0.27\ (\text{N/C})\cdot\text{m})z = 0$ (since $z = 0$).

$\hat{n}_{S_5} = +\hat{i} \Rightarrow \Phi_5 = \vec{E}\cdot\hat{n}_{S_5}A = (-5.00\ \text{N/C}\cdot\text{m})(0.300\ \text{m})^2 x = -(0.45\ (\text{N/C})\cdot\text{m})x$.

$\Phi_5 = -(0.45\ (\text{N/C})\cdot\text{m})(0.300\ \text{m}) = -(0.135\ (\text{N/C})\cdot\text{m}^2)$.

$\hat{n}_{S_6} = -\hat{i} \Rightarrow \Phi_6 = \vec{E}\cdot\hat{n}_{S_6}A = +(0.45\ (\text{N/C})\cdot\text{m})x = 0$ (since $x = 0$).

(b) Total flux: $\Phi = \Phi_2 + \Phi_5 = (0.081 - 0.135)(\text{N/C})\cdot\text{m}^2 = -0.054\ \text{N}\cdot\text{m}^2/\text{C}$. Therefore, $q = \epsilon_0\Phi = -4.78\times10^{-13}$ C.

EVALUATE: Flux is positive when $\vec{E}$ is directed out of the volume and negative when it is directed into the volume.

22.8. **IDENTIFY:** Apply Gauss's law to each surface.

SET UP: Q_{encl} is the algebraic sum of the charges enclosed by each surface. Flux out of the volume is positive and flux into the enclosed volume is negative.

EXECUTE: **(a)** $\Phi_{S_1} = q_1/\epsilon_0 = (4.00\times10^{-9}\ \text{C})/\epsilon_0 = 452\ \text{N}\cdot\text{m}^2/\text{C}.$

(b) $\Phi_{S_2} = q_2/\epsilon_0 = (-7.80\times10^{-9}\ \text{C})/\epsilon_0 = -881\ \text{N}\cdot\text{m}^2/\text{C}.$

(c) $\Phi_{S_3} = (q_1+q_2)/\epsilon_0 = ((4.00-7.80)\times10^{-9}\ \text{C})/\epsilon_0 = -429\ \text{N}\cdot\text{m}^2/\text{C}.$

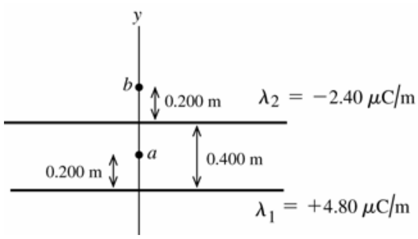
(d) $\Phi_{S_4} = (q_1+q_3)/\epsilon_0 = ((4.00+2.40)\times10^{-9}\ \text{C})/\epsilon_0 = 723\ \text{N}\cdot\text{m}^2/\text{C}.$

(e) $\Phi_{S_5} = (q_1+q_2+q_3)/\epsilon_0 = ((4.00-7.80+2.40)\times10^{-9}\ \text{C})/\epsilon_0 = -158\ \text{N}\cdot\text{m}^2/\text{C}.$

EVALUATE: **(f)** All that matters for Gauss's law is the total amount of charge enclosed by the surface, not its distribution within the surface.

22.21. **IDENTIFY:** Add the vector electric fields due to each line of charge. $E(r)$ for a line of charge is given by Example 22.6 and is directed toward a negative line of chage and away from a positive line.

SET UP: The two lines of charge are shown in Figure 22.21.



$$E = \frac{1}{2\pi\epsilon_0}\frac{\lambda}{r}$$

Figure 22.21

EXECUTE: **(a)** At point *a*, $\vec{E}_1$ and $\vec{E}_2$ are in the $+y$-direction (toward negative charge, away from positive charge).

$E_1 = (1/2\pi\epsilon_0)\left[(4.80\times10^{-6}\ \text{C/m})/(0.200\ \text{m})\right] = 4.314\times10^5\ \text{N/C}$

$E_2 = (1/2\pi\epsilon_0)\left[(2.40\times10^{-6}\ \text{C/m})/(0.200\ \text{m})\right] = 2.157\times10^5\ \text{N/C}$

$E = E_1 + E_2 = 6.47\times10^5\ \text{N/C},$ in the y-direction.

(b) At point *b*, $\vec{E}_1$ is in the $+y$-direction and $\vec{E}_2$ is in the $-y$-direction.

$E_1 = (1/2\pi\epsilon_0)\left[(4.80\times10^{-6}\ \text{C/m})/(0.600\ \text{m})\right] = 1.438\times10^5\ \text{N/C}$

$E_2 = (1/2\pi\epsilon_0)\left[(2.40\times10^{-6}\ \text{C/m})/(0.200\ \text{m})\right] = 2.157\times10^5\ \text{N/C}$

$E = E_2 - E_1 = 7.2\times10^4\ \text{N/C},$ in the $-y$-direction.

EVALUATION: At point *a* the two fields are in the same direction and the magnitudes add. At point *b* the two fields are in opposite directions and the magnitudes subtract.

22.30. **IDENTIFY:** The net electric field is the vector sum of the fields due to each of the four sheets of charge.

SET UP: The electric field of a large sheet of charge is $E = \sigma/2\epsilon_0$. The field is directed away from a positive sheet and toward a negative sheet.

EXECUTE: **(a)** At A: $E_A = \frac{|\sigma_2|}{2\epsilon_0} + \frac{|\sigma_3|}{2\epsilon_0} + \frac{|\sigma_4|}{2\epsilon_0} - \frac{|\sigma_1|}{2\epsilon_0} = \frac{1}{2\epsilon_0}(|\sigma_2| + |\sigma_3| + |\sigma_4| - |\sigma_1|)$.

$E_A = \frac{1}{2\epsilon_0}(5\ \mu\text{C/m}^2 + 2\ \mu\text{C/m}^2 + 4\ \mu\text{C/m}^2 - 6\ \mu\text{C/m}^2) = 2.82\times10^5\ \text{N/C}$ to the left.

(b) $E_B = \frac{|\sigma_1|}{2\epsilon_0} + \frac{|\sigma_3|}{2\epsilon_0} + \frac{|\sigma_4|}{2\epsilon_0} - \frac{|\sigma_2|}{2\epsilon_0} = \frac{1}{2\epsilon_0}(|\sigma_1| + |\sigma_3| + |\sigma_4| - |\sigma_2|)$.

$E_B = \frac{1}{2\epsilon_0}(6\ \mu\text{C/m}^2 + 2\ \mu\text{C/m}^2 + 4\ \mu\text{C/m}^2 - 5\ \mu\text{C/m}^2) = 3.95\times10^5\ \text{N/C}$ to the left.

(c) $E_C = \frac{|\sigma_4|}{2\epsilon_0} + \frac{|\sigma_1|}{2\epsilon_0} - \frac{|\sigma_2|}{2\epsilon_0} - \frac{|\sigma_3|}{2\epsilon_0} = \frac{1}{2\epsilon_0}(|\sigma_2| + |\sigma_3| - |\sigma_4| - |\sigma_1|)$.

$E_C = \frac{1}{2\epsilon_0}(4\ \mu\text{C/m}^2 + 6\ \mu\text{C/m}^2 - 5\ \mu\text{C/m}^2 - 2\ \mu\text{C/m}^2) = 1.69\times 10^5\ \text{N/C}$ to the left

EVALUATE: The field at C is not zero. The pieces of plastic are not conductors.

22.32. **IDENTIFY** and **SET UP:** Eq.(22.3) to calculate the flux. Identify the direction of the normal unit vector $\hat{\boldsymbol{n}}$ for each surface.

EXECUTE: **(a)** $\vec{E} = -B\hat{\boldsymbol{i}} + C\hat{\boldsymbol{j}} - D\hat{\boldsymbol{k}};\ A = L^2$

face S_1: $\hat{\boldsymbol{n}} = -\hat{\boldsymbol{j}}$

$\Phi_E = \vec{E}\cdot\vec{A} = \vec{E}\cdot(A\hat{\boldsymbol{n}}) = (-B\hat{\boldsymbol{i}} + C\hat{\boldsymbol{j}} - D\hat{\boldsymbol{k}})\cdot(-A\hat{\boldsymbol{j}}) = -CL^2.$

face S_2: $\hat{\boldsymbol{n}} = +\hat{\boldsymbol{k}}$

$\Phi_E = \vec{E}\cdot\vec{A} = \vec{E}\cdot(A\hat{\boldsymbol{n}}) = (-B\hat{\boldsymbol{i}} + C\hat{\boldsymbol{j}} - D\hat{\boldsymbol{k}})\cdot(A\hat{\boldsymbol{k}}) = -DL^2.$

face S_3: $\hat{\boldsymbol{n}} = +\hat{\boldsymbol{j}}$

$\Phi_E = \vec{E}\cdot\vec{A} = \vec{E}\cdot(A\hat{\boldsymbol{n}}) = (-B\hat{\boldsymbol{i}} + C\hat{\boldsymbol{j}} - D\hat{\boldsymbol{k}})\cdot(A\hat{\boldsymbol{j}}) = +CL^2.$

face S_4: $\hat{\boldsymbol{n}} = -\hat{\boldsymbol{k}}$

$\Phi_E = \vec{E}\cdot\vec{A} = \vec{E}\cdot(A\hat{\boldsymbol{n}}) = (-B\hat{\boldsymbol{i}} + C\hat{\boldsymbol{j}} - D\hat{\boldsymbol{k}})\cdot(-A\hat{\boldsymbol{k}}) = +DL^2.$

face S_5: $\hat{\boldsymbol{n}} = +\hat{\boldsymbol{i}}$

$\Phi_E = \vec{E}\cdot\vec{A} = \vec{E}\cdot(A\hat{\boldsymbol{n}}) = (-B\hat{\boldsymbol{i}} + C\hat{\boldsymbol{j}} - D\hat{\boldsymbol{k}})\cdot(A\hat{\boldsymbol{i}}) = -BL^2.$

face S_6: $\hat{\boldsymbol{n}} = -\hat{\boldsymbol{i}}$

$\Phi_E = \vec{E}\cdot\vec{A} = \vec{E}\cdot(A\hat{\boldsymbol{n}}) = (-B\hat{\boldsymbol{i}} + C\hat{\boldsymbol{j}} - D\hat{\boldsymbol{k}})\cdot(-A\hat{\boldsymbol{i}}) = +BL^2.$

(b) Add the flux through each of the six faces: $\Phi_E = -CL^2 - DL^2 + CL^2 + DL^2 - BL^2 + BL^2 = 0$

The total electric flux through all sides is zero.

EVALUATE: All electric field lines that enter one face of the cube leave through another face. No electric field lines terminate inside the cube and the net flux is zero.

22.37. **(a) IDENTIFY:** Apply Gauss's law to a Gaussian cylinder of length l and radius r, where $a < r < b$, and calculate E on the surface of the cylinder.

SET UP: The Gaussian surface is sketched in Figure 22.37a.

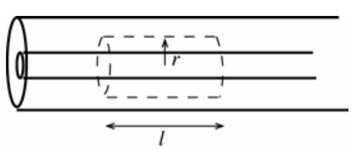


Figure 22.37a

EXECUTE: $\Phi_E = E(2\pi r l)$

$Q_{\text{encl}} = \lambda l$ (the charge on the length l of the inner conductor that is inside the Gaussian surface).

$\Phi_E = \frac{Q_{\text{encl}}}{\epsilon_0}$ gives $E(2\pi r l) = \frac{\lambda l}{\epsilon_0}$

$E = \frac{\lambda}{2\pi\epsilon_0 r}$. The enclosed charge is positive so the direction of $\vec{E}$ is radially outward.

(b) SET UP: Apply Gauss's law to a Gaussian cylinder of length l and radius r, where $r > c$, as shown in Figure 22.37b.

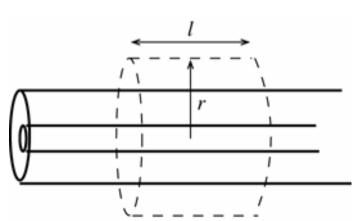


Figure 22.37b

EXECUTE: $\Phi_E = E(2\pi r l)$

$Q_{\text{encl}} = \lambda l$ (the charge on the length l of the inner conductor that is inside the Gaussian surface; the outer conductor carries no net charge).

$\Phi_E = \frac{Q_{\text{encl}}}{\epsilon_0}$ gives $E(2\pi rl) = \frac{\lambda l}{\epsilon_0}$

$E = \frac{\lambda}{2\pi\epsilon_0 r}$. The enclosed charge is positive so the direction of $\vec{E}$ is radially outward.

(c) $E = 0$ within a conductor. Thus $E = 0$ for $r < a$;

$E = \frac{\lambda}{2\pi\epsilon_0 r}$ for $a < r < b$; $E = 0$ for $b < r < c$;

$E = \frac{\lambda}{2\pi\epsilon_0 r}$ for $r > c$. The graph of E versus r is sketched in Figure 22.37c.

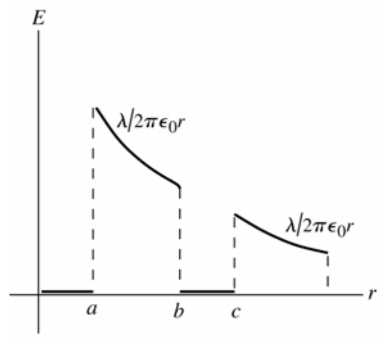


Figure 22.37c

EVALUATE: Inside either conductor $E = 0$. Between the conductors and outside both conductors the electric field is the same as for a line of charge with linear charge density λ lying along the axis of the inner conductor.

(d) IDENTIFY and **SET UP:** inner surface: Apply Gauss's law to a Gaussian cylinder with radius r, where $b < r < c$. We know E on this surface; calculate Q_{encl}.

EXECUTE: This surface lies within the conductor of the outer cylinder, where $E = 0$, so $\Phi_E = 0$. Thus by Gauss's law $Q_{\text{encl}} = 0$. The surface encloses charge λl on the inner conductor, so it must enclose charge $-\lambda l$ on the inner surface of the outer conductor. The charge per unit length on the inner surface of the outer cylinder is $-\lambda$.

outer surface: The outer cylinder carries no net charge. So if there is charge per unit length $-\lambda$ on its inner surface there must be charge per until length $+\lambda$ on the outer surface.

EVALUATE: The electric field lines between the conductors originate on the surface charge on the outer surface of the inner conductor and terminate on the surface charges on the inner surface of the outer conductor. These surface charges are equal in magnitude (per unit length) and opposite in sign. The electric field lines outside the outer conductor originate from the surface charge on the outer surface of the outer conductor.

22.40. **IDENTIFY:** Apply Gauss's law.

SET UP: Use a Gaussian surface that is a cylinder of radius r and length l, and that is coaxial with the cylindrical charge distributions. The volume of the Gaussian cylinder is $\pi r^2 l$ and the area of its curved surface is $2\pi rl$. The charge on a length l of the charge distribution is $q = \lambda l$, where $\lambda = \rho\pi R^2$.

EXECUTE: **(a)** For $r < R$, $Q_{\text{encl}} = \rho\pi r^2 l$ and Gauss's law gives $E(2\pi rl) = \frac{Q_{\text{encl}}}{\epsilon_0} = \frac{\rho\pi r^2 l}{\epsilon_0}$ and $E = \frac{\rho r}{2\epsilon_0}$, radially outward.

(b) For $r > R$, $Q_{\text{encl}} = \lambda l = \rho\pi R^2 l$ and Gauss's law gives $E(2\pi rl) = \frac{q}{\epsilon_0} = \frac{\rho\pi R^2 l}{\epsilon_0}$ and $E = \frac{\rho R^2}{2\epsilon_0 r} = \frac{\lambda}{2\pi\epsilon_0 r}$, radially outward.

(c) At $r = R$, the electric field for BOTH regions is $E = \frac{\rho R}{2\epsilon_0}$, so they are consistent.

(d) The graph of E versus r is sketched in Figure 22.40.

EVALUATE: For $r > R$ the field is the same as for a line of charge along the axis of the cylinder.

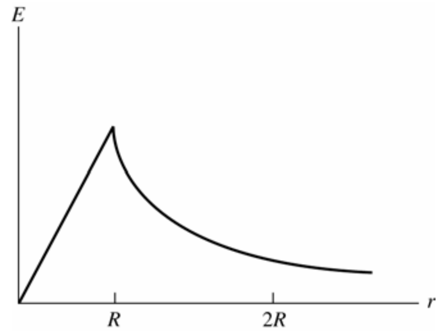


Figure 22.40

22.42. **IDENTIFY:** Apply Gauss's law.

SET UP: Use a Gaussian surface that is a sphere of radius r and that is concentric with the conducting spheres.

EXECUTE: **(a)** For $r < a$, $E = 0$, since these points are within the conducting material.

For $a < r < b$, $E = \dfrac{1}{4\pi\epsilon_0}\dfrac{q}{r^2}$, since there is $+q$ inside a radius r.

For $b < r < c$, $E = 0$, since since these points are within the conducting material

For $r > c$, $E = \dfrac{1}{4\pi\epsilon_0}\dfrac{q}{r^2}$, since again the total charge enclosed is $+q$.

(b) The graph of E versus r is sketched in Figure 22.42a.

(c) Since the Gaussian sphere of radius r, for $b < r < c$, must enclose zero net charge, the charge on inner shell surface is $-q$.

(d) Since the hollow sphere has no net charge and has charge $-q$ on its inner surface, the charge on outer shell surface is $+q$.

(e) The field lines are sketched in Figure 22.42b. Where the field is nonzero, it is radially outward.

EVALUATE: The net charge on the inner solid conducting sphere is on the surface of that sphere. The presence of the hollow sphere does not affect the electric field in the region $r < b$.

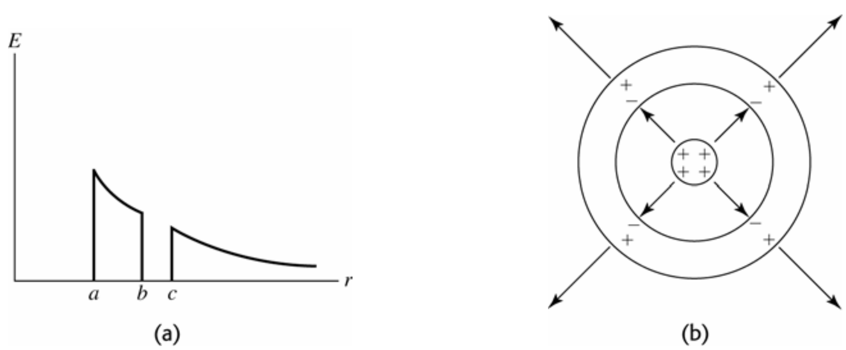


Figure 22.42

22.48. **IDENTIFY:** Apply Gauss's law.

SET UP: Use a Gaussian surface that is a sphere of radius r and that is concentric with the sphere and shell. The volume of the insulating shell is $V = \frac{4}{3}\pi\left([2R]^3 - R^3\right) = \frac{28\pi}{3}R^3$.

EXECUTE: **(a)** Zero net charge requires that $-Q = \dfrac{28\pi\rho R^3}{3}$, so $\rho = -\dfrac{3Q}{28\pi R^3}$.

(b) For $r < R$, $E = 0$ since this region is within the conducting sphere. For $r > 2R$, $E = 0$, since the net charge enclosed by the Gaussian surface with this radius is zero. For $R < r < 2R$, Gauss's law gives $E(4\pi r^2) = \dfrac{Q}{\epsilon_0} + \dfrac{4\pi\rho}{3\epsilon_0}(r^3 - R^3)$ and $E = \dfrac{Q}{4\pi\epsilon_0 r^2} + \dfrac{\rho}{3\epsilon_0 r^2}(r^3 - R^3)$. Substituting ρ from part (a) gives $E = \dfrac{2}{7\pi\epsilon_0}\dfrac{Q}{r^2} - \dfrac{Qr}{28\pi\epsilon_0 R^3}$. The net enclosed charge for each r in this range is positive and the electric field is outward.

(c) The graph is sketched in Figure 22.48. We see a discontinuity in going from the conducting sphere to the insulator due to the thin surface charge of the conducting sphere. But we see a smooth transition from the uniform insulator to the surrounding space.

EVALUATE: The expression for E within the insulator gives $E = 0$ at $r = 2R$.

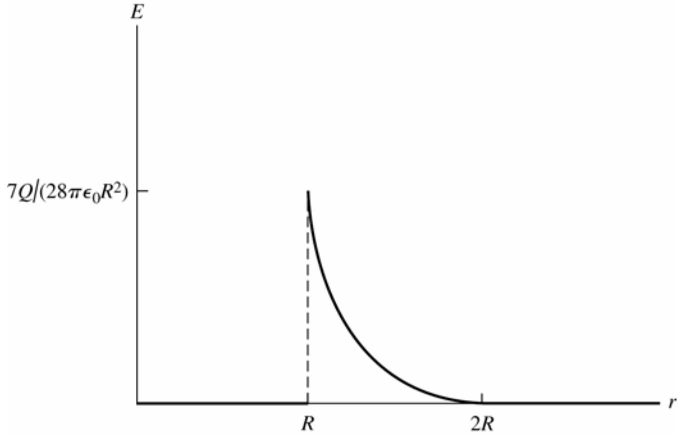


Figure 22.48

22.51. **IDENTIFY:** The net electric field is the vector sum of the fields due to the sheet of charge on each surface of the plate.

SET UP: The electric field due to the sheet of charge on each surface is $E = \sigma / 2\epsilon_0$ and is directed away from the surface.

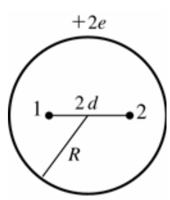
EXECUTE: **(a)** For the conductor the charge sheet on each surface produces fields of magnitude $\sigma / 2\epsilon_0$ and in the same direction, so the total field is twice this, or σ / ϵ_0.

(b) At points inside the plate the fields of the sheets of charge on each surface are equal in magnitude and opposite in direction, so their vector sum is zero. At points outside the plate, on either side, the fields of the two sheets of charge are in the same direction so their magnitudes add, giving $E = \sigma / \epsilon_0$.

EVALUATE: Gauss's law can also be used directly to determine the fields in these regions.

22.53. **IDENTIFY:** There is a force on each electron due to the other electron and a force due to the sphere of charge. Use Coulomb's law for the force between the electrons. Example 22.9 gives E inside a uniform sphere and Eq.(21.3) gives the force.

SET UP: The positions of the electrons are sketched in Figure 22.53a.



If the electrons are in equilibrium the net force on each one is zero.

Figure 22.53a

EXECUTE: Consider the forces on electron 2. There is a repulsive force F_1 due to the other electron, electron 1.

$$F_1 = \frac{1}{4\pi\epsilon_0}\frac{e^2}{(2d)^2}$$

The electric field inside the uniform distribution of positive charge is $E = \dfrac{Qr}{4\pi\epsilon_0 R^3}$ (Example 22.9), where $Q = +2e$. At the position of electron 2, $r = d$. The force F_{cd} exerted by the positive charge distribution is $F_{\text{cd}} = eE = \dfrac{e(2e)d}{4\pi\epsilon_0 R^3}$ and is attractive.

The force diagram for electron 2 is given in Figure 22.53b.

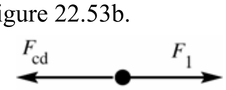


Figure 22.53b

Net force equals zero implies $F_1 = F_{\text{cd}}$ and $\dfrac{1}{4\pi\epsilon_0}\dfrac{e^2}{4d^2} = \dfrac{2e^2 d}{4\pi\epsilon_0 R^3}$

Thus $\left(1/4d^2\right) = 2d/R^3$, so $d^3 = R^3/8$ and $d = R/2$.

EVALUATE: The electric field of the sphere is radially outward; it is zero at the center of the sphere and increases with distance from the center. The force this field exerts on one of the electrons is radially inward and increases as the electron is farther from the center. The force from the other electron is radially outward, is infinite when $d = 0$ and decreases as d increases. It is reasonable therefore for there to be a value of d for which these forces balance.

22.61. **(a) IDENTIFY:** Use $\vec{E}(\vec{r})$ from Example (22.9) (inside the sphere) and relate the position vector of a point inside the sphere measured from the origin to that measured from the center of the sphere.

SET UP: For an insulating sphere of uniform charge density ρ and centered at the origin, the electric field inside the sphere is given by $E = Qr'/4\pi\epsilon_0 R^3$ (Example 22.9), where $\vec{r}'$ is the vector from the center of the sphere to the point where E is calculated.

But $\rho = 3Q/4\pi R^3$ so this may be written as $E = \rho r/3\epsilon_0$. And $\vec{E}$ is radially outward, in the direction of $\vec{r}'$, so $\vec{E} = \rho\vec{r}'/3\epsilon_0$.

For a sphere whose center is located by vector $\vec{b}$, a point inside the sphere and located by $\vec{r}$ is located by the vector $\vec{r}' = \vec{r} - \vec{b}$ relative to the center of the sphere, as shown in Figure 22.61.

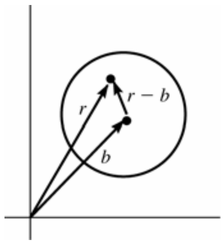


Figure 22.61

EXECUTE: Thus $\vec{E} = \dfrac{\rho\left(\vec{r} - \vec{b}\right)}{3\epsilon_0}$

EVALUATE: When $b = 0$ this reduces to the result of Example 22.9. When $\vec{r} = \vec{b}$, this gives $E = 0$, which is correct since we know that $E = 0$ at the center of the sphere.

(b) IDENTIFY: The charge distribution can be represented as a uniform sphere with charge density ρ and centered at the origin added to a uniform sphere with charge density $-\rho$ and centered at $\vec{r} = \vec{b}$.

SET UP: $\vec{E} = \vec{E}_{\text{uniform}} + \vec{E}_{\text{hole}}$, where $\vec{E}_{\text{uniform}}$ is the field of a uniformly charged sphere with charge density ρ and $\vec{E}_{\text{hole}}$ is the field of a sphere located at the hole and with charge density $-\rho$. (Within the spherical hole the net charge density is $+\rho - \rho = 0$.)

EXECUTE: $\vec{E}_{\text{uniform}} = \dfrac{\rho\vec{r}}{3\epsilon_0}$, where $\vec{r}$ is a vector from the center of the sphere.

$\vec{E}_{\text{hole}} = \dfrac{-\rho\left(\vec{r} - \vec{b}\right)}{3\epsilon_0}$, at points inside the hole.

Then $\vec{E} = \dfrac{\rho\vec{r}}{3\epsilon_0} + \left(\dfrac{-\rho\left(\vec{r} - \vec{b}\right)}{3\epsilon_0}\right) = \dfrac{\rho\vec{b}}{3\epsilon_0}$.

EVALUATE: $\vec{E}$ is independent of $\vec{r}$ so is uniform inside the hole. The direction of $\vec{E}$ inside the hole is in the direction of the vector $\vec{b}$, the direction from the center of the insulating sphere to the center of the hole.