CAPACITANCE AND DIELECTRICS

24

24.3. **IDENTIFY** and **SET UP:** It is a parallel-plate air capacitor, so we can apply the equations of Sections 24.1.

EXECUTE: **(a)** $C = \frac{Q}{V_{ab}}$ so $V_{ab} = \frac{Q}{C} = \frac{0.148\times10^{-6}\ \text{C}}{245\times10^{-12}\ \text{F}} = 604\ \text{V}$

(b) $C = \frac{\epsilon_0 A}{d}$ so $A = \frac{Cd}{\epsilon_0} = \frac{(245\times10^{-12}\ \text{F})(0.328\times10^{-3}\ \text{m})}{8.854\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2} = 9.08\times10^{-3}\ \text{m}^2 = 90.8\ \text{cm}^2$

(c) $V_{ab} = Ed$ so $E = \frac{V_{ab}}{d} = \frac{604\ \text{V}}{0.328\times10^{-3}\ \text{m}} = 1.84\times10^6\ \text{V/m}$

(d) $E = \frac{\sigma}{\epsilon_0}$ so $\sigma = E\epsilon_0 = (1.84\times10^6\ \text{V/m})(8.854\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2) = 1.63\times10^{-5}\ \text{C/m}^2$

EVALUATE: We could also calculate σ directly as *Q*/*A*. $\sigma = \frac{Q}{A} = \frac{0.148\times10^{-6}\ \text{C}}{9.08\times10^{-3}\ \text{m}^2} = 1.63\times10^{-5}\ \text{C/m}^2$, which checks.

24.5. **IDENTIFY:** $C = \frac{Q}{V_{ab}}$. $C = \frac{\epsilon_0 A}{d}$.

SET UP: When the capacitor is connected to the battery, $V_{ab} = 12.0\ \text{V}$.

EXECUTE: **(a)** $Q = CV_{ab} = (10.0\times10^{-6}\ \text{F})(12.0\ \text{V}) = 1.20\times10^{-4}\ \text{C} = 120\ \mu\text{C}$

(b) When *d* is doubled *C* is halved, so *Q* is halved. $Q = 60\ \mu\text{C}$.

(c) If *r* is doubled, *A* increases by a factor of 4. *C* increases by a factor of 4 and *Q* increases by a factor of 4. $Q = 480\ \mu\text{C}$.

EVALUATE: When the plates are moved apart, less charge on the plates is required to produce the same potential difference. With the separation of the plates constant, the electric field must remain constant to produce the same potential difference. The electric field depends on the surface charge density, σ. To produce the same σ, more charge is required when the area increases.

24.11. **IDENTIFY** and **SET UP:** Use the expression for *C*/*L* derived in Example 24.4. Then use Eq.(24.1) to calculate *Q*.

EXECUTE: **(a)** From Example 24.4, $\frac{C}{L} = \frac{2\pi\epsilon_0}{\ln(r_b/r_a)}$

$$\frac{C}{L} = \frac{2\pi(8.854\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)}{\ln(3.5\ \text{mm}/1.5\ \text{mm})} = 6.57\times10^{-11}\ \text{F/m} = 66\ \text{pF/m}$$

(b) $C = (6.57\times10^{-11}\ \text{F/m})(2.8\ \text{m}) = 1.84\times10^{-10}\ \text{F}$.

$$Q = CV = (1.84\times10^{-10}\ \text{F})(350\times10^{-3}\ \text{V}) = 6.4\times10^{-11}\ \text{C} = 64\ \text{pC}$$

The conductor at higher potential has the positive charge, so there is +64 pC on the inner conductor and -64 pC on the outer conductor.

EVALUATE: *C* depends only on the dimensions of the capacitor. *Q* and *V* are proportional.

24.13. **IDENTIFY:** We can use the definition of capacitance to find the capacitance of the capacitor, and then relate the capacitance to geometry to find the inner radius.

(a) SET UP: By the definition of capacitance, *C* = *Q*/*V*.

EXECUTE: $C = \frac{Q}{V} = \frac{3.30\times10^{-9}\ \text{C}}{2.20\times10^2\ \text{V}} = 1.50\times10^{-11}\ \text{F} = 15.0\ \text{pF}$

(b) SET UP: The capacitance of a spherical capacitor is $C = 4\pi\epsilon_0 \frac{r_a r_b}{r_b - r_a}$.

EXECUTE: Solve for r_a and evaluate using C = 15.0 pF and r_b = 4.00 cm, giving r_a = 3.09 cm.

(c) SET UP: We can treat the inner sphere as a point-charge located at its center and use Coulomb's law, $E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$.

EXECUTE: $E = \frac{(9.00\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(3.30\times10^{-9}\ \text{C})}{(0.0309\ \text{m})^2} = 3.12\times10^4\ \text{N/C}$

EVALUATE: Outside the capacitor, the electric field is zero because the charges on the spheres are equal in magnitude but opposite in sign.

24.25. **IDENTIFY** and **SET UP:** The energy density is given by Eq.(24.11): $u = \frac{1}{2}\epsilon_0 E^2$. Use $V = Ed$ to solve for E.

EXECUTE: Calculate $E: E = \frac{V}{d} = \frac{400\ \text{V}}{5.00\times10^{-3}\ \text{m}} = 8.00\times10^4\ \text{V/m}$.

Then $u = \frac{1}{2}\epsilon_0 E^2 = \frac{1}{2}(8.854\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(8.00\times10^4\ \text{V/m})^2 = 0.0283\ \text{J/m}^3$

EVALUATE: E is smaller than the value in Example 24.8 by about a factor of 6 so u is smaller by about a factor of $6^2 = 36$.

24.36. **IDENTIFY:** Apply Eq.(24.11).

SET UP: Example 24.3 shows that $E = \frac{Q}{4\pi\epsilon_0 r^2}$ between the conducting shells and that $\frac{Q}{4\pi\epsilon_0} = \left(\frac{r_a r_b}{r_b - r_a}\right)V_{ab}$.

EXECUTE: $E = \left(\frac{r_a r_b}{r_b - r_a}\right)\frac{V_{ab}}{r^2} = \left(\frac{[0.125\ \text{m}][0.148\ \text{m}]}{0.148\ \text{m} - 0.125\ \text{m}}\right)\frac{120\ \text{V}}{r^2} = \frac{96.5\ \text{V}\cdot\text{m}}{r^2}$

(a) For $r = 0.126$ m, $E = 6.08\times10^3$ V/m. $u = \frac{1}{2}\epsilon_0 E^2 = 1.64\times10^{-4}\ \text{J/m}^3$.

(b) For $r = 0.147$ m, $E = 4.47\times10^3$ V/m. $u = \frac{1}{2}\epsilon_0 E^2 = 8.85\times10^{-5}\ \text{J/m}^3$.

EVALUATE: **(c)** No, the results of parts (a) and (b) show that the energy density is not uniform in the region between the plates. E decreases as r increases, so u decreases also.

24.53. **IDENTIFY:** $P = E/t$, where E is the total light energy output. The energy stored in the capacitor is $U = \frac{1}{2}CV^2$.

SET UP: $E = 0.95U$

EXECUTE: **(a)** The power output is 600 W, and 95% of the original energy is converted, so $E = Pt = (2.70\times10^5\ \text{W})(1.48\times10^{-3}\ \text{s}) = 400\ \text{J}$. $E_0 = \frac{400\ \text{J}}{0.95} = 421\ \text{J}$.

(b) $U = \frac{1}{2}CV^2$ so $C = \frac{2U}{V^2} = \frac{2(421\ \text{J})}{(125\ \text{V})^2} = 0.054\ \text{F}$.

EVALUATE: For a given V, the stored energy increases linearly with C.

24.70. **IDENTIFY:** The electric field energy density is $u = \frac{1}{2}\epsilon_0 E^2$. $U = \frac{Q^2}{2C}$.

SET UP: For this charge distribution, $E = 0$ for $r < r_a$, $E = \frac{\lambda}{2\pi\epsilon_0 r}$ for $r_a < r < r_b$ and $E = 0$ for $r > r_b$.

Example 24.4 shows that $\frac{U}{L} = \frac{2\pi\epsilon_0}{\ln(r_b/r_a)}$ for a cylindrical capacitor.

EXECUTE: **(a)** $u = \frac{1}{2}\epsilon_0 E^2 = \frac{1}{2}\epsilon_0\left(\frac{\lambda}{2\pi\epsilon_0 r}\right)^2 = \frac{\lambda^2}{8\pi^2\epsilon_0 r^2}$

(b) $U = \int u\,dV = 2\pi L\int u r\,dr = \frac{L\lambda^2}{4\pi\epsilon_0}\int_{r_a}^{r_b}\frac{dr}{r}$ and $\frac{U}{L} = \frac{\lambda^2}{4\pi\epsilon_0}\ln(r_b/r_a)$.

(c) Using Equation (24.9), $U = \frac{Q^2}{2C} = \frac{Q^2}{4\pi\epsilon_0 L}\ln(r_b/r_a) = \frac{\lambda^2 L}{4\pi\epsilon_0}\ln(r_b/r_a)$. This agrees with the result of part (b).

EVALUATE: We could have used the results of part (b) and $U = \frac{Q^2}{2C}$ to calculate U/L and would obtain the same result as in Example 24.4.