24

CAPACITANCE AND DIELECTRICS

24.18. IDENTIFY: For capacitors in parallel the voltages are the same and the charges add. For capacitors in series, the charges are the same and the voltages add. C = Q/V.

SET UP: C_1 and C_2 are in parallel and C_3 is in series with the parallel combination of C_1 and C_2 .

EXECUTE: (a) C_1 and C_2 are in parallel and so have the same potential across them:

 $V_1 = V_2 = \frac{Q_2}{C_2} = \frac{40.0 \times 10^{-6} \text{ C}}{3.00 \times 10^{-6} \text{ F}} = 13.33 \text{ V}.$ Therefore, $Q_1 = V_1 C_1 = (13.33 \text{ V})(3.00 \times 10^{-6} \text{ F}) = 80.0 \times 10^{-6} \text{ C}.$ Since C_3 is

in series with the parallel combination of C_1 and C_2 , its charge must be equal to their combined charge: $C_3 = 40.0 \times 10^{-6} \text{ C} + 80.0 \times 10^{-6} \text{ C} = 120.0 \times 10^{-6} \text{ C}$.

(**b**) The total capacitance is found from $\frac{1}{C_{\text{tot}}} = \frac{1}{C_{12}} + \frac{1}{C_3} = \frac{1}{9.00 \times 10^{-6} \text{ F}} + \frac{1}{5.00 \times 10^{-6} \text{ F}}$ and $C_{\text{tot}} = 3.21 \ \mu\text{F}$.

$$V_{ab} = \frac{Q_{tot}}{C_{tot}} = \frac{120.0 \times 10^{-6} \text{ C}}{3.21 \times 10^{-6} \text{ F}} = 37.4 \text{ V} .$$

EVALUATE: $V_3 = \frac{Q_3}{C_2} = \frac{120.0 \times 10^{-6} \text{ C}}{5.00 \times 10^{-6} \text{ F}} = 24.0 \text{ V} . V_{ab} = V_1 + V_3$

24.28. IDENTIFY: After the two capacitors are connected they must have equal potential difference, and their combined charge must add up to the original charge.

SET UP:
$$C = Q/V$$
. The stored energy is $U = \frac{Q^2}{2C} = \frac{1}{2}CV^2$
EXECUTE: (a) $Q = CV_0$.
(b) $V = \frac{Q_1}{C_1} = \frac{Q_2}{C_2}$ and also $Q_1 + Q_2 = Q = CV_0$. $C_1 = C$ and $C_2 = \frac{C}{2}$ so $\frac{Q_1}{C} = \frac{Q_2}{(C/2)}$ and $Q_2 = \frac{Q_1}{2}$. $Q = \frac{3}{2}Q_1$.
 $Q_1 = \frac{2}{3}Q$ and $V = \frac{Q_1}{C} = \frac{2}{3}\frac{Q}{C} = \frac{2}{3}V_0$.
(c) $U = \frac{1}{2}\left(\frac{Q_1^2}{C_1} + \frac{Q_2^2}{C_2}\right) = \frac{1}{2}\left[\frac{(\frac{2}{3}Q)^2}{C} + \frac{2(\frac{1}{3}Q)^2}{C}\right] = \frac{1}{3}\frac{Q^2}{C} = \frac{1}{3}CV_0^2$
(d) The original U was $U = \frac{1}{2}CV_0^2$, so $\Delta U = -\frac{1}{6}CV_0^2$.

(e) Thermal energy of capacitor, wires, etc., and electromagnetic radiation. **EVALUATE:** The original charge of the charged capacitor must distribute between the two capacitors to make the potential the same across each capacitor. The voltage V for each after they are connected is less than the original voltage V_0 of the charged capacitor.

24.37. IDENTIFY: Use the rules for series and for parallel capacitors to express the voltage for each capacitor in terms of the applied voltage. Express *U*, *Q*, and *E* in terms of the capacitor voltage.

SET UP: Le the applied voltage be V. Let each capacitor have capacitance C. $U = \frac{1}{2}CV^2$ for a single capacitor with voltage V.

EXECUTE: (a) series

Voltage across each capacitor is V/2. The total energy stored is $U_s = 2(\frac{1}{2}C[V/2]^2) = \frac{1}{4}CV^2$

parallel

Voltage across each capacitor is V. The total energy stored is $U_p = 2(\frac{1}{2}CV^2) = CV^2$

 $U_{\rm p} = 4U_{\rm s}$

(b) Q = CV for a single capacitor with voltage V. $Q_s = 2(C[V/2]) = CV$; $Q_p = 2(CV) = 2CV$; $Q_p = 2Q_s$

(c) E = V/d for a capacitor with voltage V. $E_s = V/2d$; $E_p = V/d$; $E_p = 2E_s$

EVALUATE: The parallel combination stores more energy and more charge since the voltage for each capacitor is larger for parallel. More energy stored and larger voltage for parallel means larger electric field in the parallel case.
24.39. IDENTIFY and SET UP: Q is constant so we can apply Eq.(24.14). The charge density on each surface of the dielectric is given by Eq.(24.16).

EXECUTE: $E = \frac{E_0}{K}$ so $K = \frac{E_0}{E} = \frac{3.20 \times 10^5 \text{ V/m}}{2.50 \times 10^5 \text{ V/m}} = 1.28$ (a) $\sigma_i = \sigma(1-1/K)$ $\sigma = P_0 E_0 = (8.854 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(3.20 \times 10^5 \text{ N/C}) = 2.833 \times 10^{-6} \text{ C/m}^2$ $\sigma_i = (2.833 \times 10^{-6} \text{ C/m}^2)(1-1/1.28) = 6.20 \times 10^{-7} \text{ C/m}^2$ (b) As calculated above, K = 1.28.

EVALUATE: The surface charges on the dielectric produce an electric field that partially cancels the electric field produced by the charges on the capacitor plates.

24.44. IDENTIFY: C = Q/V. $C = KC_0$. V = Ed.

SET UP: Table 24.1 gives K = 3.1 for mylar.

EXECUTE: (a)
$$\Delta Q = Q - Q_0 = (K - 1)Q_0 = (K - 1)C_0V_0 = (2.1)(2.5 \times 10^{-7} \text{ F})(12 \text{ V}) = 6.3 \times 10^{-6} \text{ C}$$

(b) $\sigma_i = \sigma(1-1/K)$ so $Q_i = Q(1-1/K) = (9.3 \times 10^{-6} \text{ C})(1-1/3.1) = 6.3 \times 10^{-6} \text{ C}$.

(c) The addition of the mylar doesn't affect the electric field since the induced charge cancels the additional charge drawn to the plates.

EVALUATE: E = V/d and V is constant so E doesn't change when the dielectric is inserted.

24.56. IDENTIFY: Initially the capacitors are connected in parallel to the source and we can calculate the charges Q_1 and

 Q_2 on each. After they are reconnected to each other the total charge is $Q = Q_2 - Q_1$. $U = \frac{1}{2}CV^2 = \frac{Q^2}{2C}$.

SET UP: After they are reconnected, the charges add and the voltages are the same, so $C_{eq} = C_1 + C_2$, as for capacitors in parallel.

EXECUTE: Originally $Q_1 = C_1 V_1 = (9.0 \ \mu\text{F}) (28 \text{ V}) = 2.52 \times 10^{-4} \text{ C}$ and $Q_2 = C_2 V_2 = (4.0 \ \mu\text{F})(28 \text{ V}) = 1.12 \times 10^{-4} \text{ C}$.

 $C_{\rm eq} = C_1 + C_2 = 13.0 \ \mu\text{F}$. The original energy stored is $U = \frac{1}{2}C_{\rm eq}V^2 = \frac{1}{2}(13.0 \times 10^{-6} \text{ F})(28 \text{ V})^2 = 5.10 \times 10^{-3} \text{ J}$.

Disconnect and flip the capacitors, so now the total charge is $Q = Q_2 - Q_1 = 1.4 \times 10^{-4}$ C and the equivalent capacitance

is still the same, $C_{eq} = 13.0 \ \mu\text{F}$. The new energy stored is $U = \frac{Q^2}{2C_{eq}} = \frac{(1.4 \times 10^{-4} \text{ C})^2}{2(13.0 \times 10^{-6} \text{ F})} = 7.54 \times 10^{-4} \text{ J}$. The change in

stored energy is $\Delta U = 7.45 \times 10^{-4} \text{ J} - 5.10 \times 10^{-3} \text{ J} = -4.35 \times 10^{-3} \text{ J}$.

EVALUATE: When they are reconnected, charge flows and thermal energy is generated and energy is radiated as electromagnetic waves.

24.59. (a) **IDENTIFY:** Replace series and parallel combinations of capacitors by their equivalents. **SET UP:** The network is sketched in Figure 24.59a.

$$\bigvee_{V=220 V}^{C_{1}} \bigvee_{C_{5}}^{C_{3}} \bigvee_{C_{4}}^{C_{1}} = C_{5} = 8.4 \ \mu F$$

$$C_{2} = C_{3} = C_{4} = 4.2 \ \mu F$$
Figure 24 59a

EXECUTE: Simplify the circuit by replacing the capacitor combinations by their equivalents: C_3 and C_4 are in series and can be replaced by C_{34} (Figure 24.59b):



Figure 24.59b

 $C_{34} = \frac{C_3 C_4}{C_3 + C_4} = \frac{(4.2 \ \mu\text{F})(4.2 \ \mu\text{F})}{4.2 \ \mu\text{F} + 4.2 \ \mu\text{F}} = 2.1 \ \mu\text{F}$

 C_2 and C_{34} are in parallel and can be replaced by their equivalent (Figure 24.59c):



Figure 24.59c

 C_1 , C_5 and C_{234} are in series and can be replaced by C_{eq} (Figure 24.59d):



Figure 24.59d

EVALUATE: For capacitors in series the equivalent capacitor is smaller than any of those in series. For capacitors in parallel the equivalent capacitance is larger than any of those in parallel.

(b) IDENTIFY and SET UP: In each equivalent network apply the rules for Q and V for capacitors in series and parallel; start with the simplest network and work back to the original circuit. **EXECUTE:** The equivalent circuit is drawn in Figure 24.59e.

$$Q_{eq} = C_{eq}V$$

$$Q_{eq} = (2.5 \ \mu\text{F})(220 \ \text{V}) = 550 \ \mu\text{C}$$

Figure 24.59e

$$Q_1 = Q_5 = Q_{234} = 550 \ \mu C$$
 (capacitors in series have same charge)

$$V_{1} = \frac{Q_{1}}{C_{1}} = \frac{550 \ \mu C}{8.4 \ \mu F} = 65 \ V$$
$$V_{5} = \frac{Q_{5}}{C_{5}} = \frac{550 \ \mu C}{8.4 \ \mu F} = 65 \ V$$
$$V_{234} = \frac{Q_{234}}{C_{234}} = \frac{550 \ \mu C}{6.3 \ \mu F} = 87 \ V$$

Now draw the network as in Figure 24.59f.

$$V = 220 \text{ V} \qquad \begin{array}{c} C_1 \\ C_2 \\ C_5 \\ C$$

Figure 24.59f

 $Q_2 = C_2 V_2 = (4.2 \ \mu F)(87 \ V) = 370 \ \mu C$ $Q_{34} = C_{34}V_{34} = (2.1 \ \mu F)(87 \ V) = 180 \ \mu C$ Finally, consider the original circuit (Figure 24.59g).



 $V_2 = V_{34} = V_{234} = 87$ V capacitors in parallel have the same potential

Figure 24.59g

$$V_{3} = \frac{Q_{3}}{C_{3}} = \frac{180 \ \mu\text{C}}{4.2 \ \mu\text{F}} = 43 \text{ V}$$

$$V_{4} = \frac{Q_{4}}{C_{4}} = \frac{180 \ \mu\text{C}}{4.2 \ \mu\text{F}} = 43 \text{ V}$$
Summary: $Q_{1} = 550 \ \mu\text{C}, V_{1} = 65 \text{ V}$

$$Q_{2} = 370 \ \mu\text{C}, V_{2} = 87 \text{ V}$$

$$Q_{3} = 180 \ \mu\text{C}, V_{3} = 43 \text{ V}$$

$$Q_{4} = 180 \ \mu\text{C}, V_{4} = 43 \text{ V}$$

$$Q_{5} = 550 \ \mu\text{C}, V_{5} = 65 \text{ V}$$
EVALUATE: $V_{3} + V_{4} = V_{2} \text{ and } V_{1} + V_{2} + V_{5} = 220 \text{ V}$ (apart from some small rounding error)
 $Q_{1} = Q_{2} + Q_{3} \text{ and } Q_{5} = Q_{2} + Q_{4}$

24.60. **IDENTIFY:** Apply the rules for combining capacitors in series and in parallel.

SET UP: With the switch open each pair of 3.00 μ F and 6.00 μ F capacitors are in series with each other and each pair is in parallel with the other pair. When the switch is closed each pair of 3.00 μ F and 6.00 μ F capacitors are in parallel with each other and the two pairs are in series.

EXECUTE: (a) With the switch open
$$C_{eq} = \left(\left(\frac{1}{3 \,\mu F} + \frac{1}{6 \,\mu F} \right)^{-1} + \left(\frac{1}{3 \,\mu F} + \frac{1}{6 \,\mu F} \right)^{-1} \right) = 4.00 \,\mu F$$

 $Q_{\text{total}} = C_{\text{eq}}V = (4.00 \ \mu\text{F}) (210 \text{ V}) = 8.40 \times 10^{-4} \text{ C}$. By symmetry, each capacitor carries $4.20 \times 10^{-4} \text{ C}$. The voltages are then calculated via V = Q/C. This gives $V_{ad} = Q/C_3 = 140 \text{ V}$ and $V_{ac} = Q/C_6 = 70 \text{ V}$. $V_{cd} = V_{ad} - V_{ac} = 70 \text{ V}$.

(b) When the switch is closed, the points c and d must be at the same potential, so the equivalent capacitance is

$$C_{\rm eq} = \left(\frac{1}{(3.00 + 6.00)\,\mu\rm{F}} + \frac{1}{(3.00 + 6.00)\,\mu\rm{F}}\right)^{-1} = 4.5\,\mu\rm{F} \cdot Q_{\rm total} = C_{\rm eq}V = (4.50\,\mu\rm{F})(210\,\,\rm{V}) = 9.5 \times 10^{-4}\,\rm{C} \ , \ \text{and each}$$

capacitor has the same potential difference of 105 V (again, by symmetry).

(c) The only way for the sum of the positive charge on one plate of C_2 and the negative charge on one plate of C_1 to change is for charge to flow through the switch. That is, the quantity of charge that flows through the switch is equal to the change in $Q_2 - Q_1$. With the switch open, $Q_1 = Q_2$ and $Q_2 - Q_1 = 0$. After the switch is closed, $Q_2 - Q_1 = 315 \ \mu$ C, so 315 μ C of charge flowed through the switch.

EVALUATE: When the switch is closed the charge must redistribute to make points c and d be at the same potential.

24.61. (a) **IDENTIFY:** Replace the three capacitors in series by their equivalent. The charge on the equivalent capacitor equals the charge on each of the original capacitors.

SET UP: The three capacitors can be replaced by their equivalent as shown in Figure 24.61a.



EXECUTE: $C_3 = C_1/2$ so $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{4}{8.4 \ \mu F}$ and $C_{eq} = 8.4 \ \mu F/4 = 2.1 \ \mu F$

 $Q = C_{eq}V = (2.1 \ \mu F)(36 \ V) = 76 \ \mu C$

The three capacitors are in series so they each have the same charge: $Q_1 = Q_2 = Q_3 = 76 \ \mu\text{C}$ **EVALUATE:** The equivalent capacitance for capacitors in series is smaller than each of the original capacitors. (b) **IDENTIFY** and **SET UP:** Use $U = \frac{1}{2}QV$. We know each Q and we know that $V_1 + V_2 + V_3 = 36 \text{ V}$.

EXECUTE:
$$U = \frac{1}{2}Q_1V_1 + \frac{1}{2}Q_2V_2 + \frac{1}{2}Q_3V_3$$

But $Q_1 = Q_2 = Q_3 = Q$ so $U = \frac{1}{2}Q(V_1 + V_2 + V_3)$

But also $V_1 + V_2 + V_3 = V = 36$ V, so $U = \frac{1}{2}QV = \frac{1}{2}(76 \ \mu\text{C})(36 \text{ V}) = 1.4 \times 10^{-3}$ J.

EVALUATE: We could also use $U = Q^2/2C$ and calculate U for each capacitor.

(c) **IDENTIFY:** The charges on the plates redistribute to make the potentials across each capacitor the same.

SET UP: The capacitors before and after they are connected are sketched in Figure 24.61b.

Figure 24.61b

EXECUTE: The total positive charge that is available to be distributed on the upper plates of the three capacitors is $Q_0 = Q_{01} + Q_{02} + Q_{03} = 3(76 \ \mu\text{C}) = 228 \ \mu\text{C}$. Thus $Q_1 + Q_2 + Q_3 = 228 \ \mu\text{C}$. After the circuit is completed the charge distributes to make $V_1 = V_2 = V_3$. V = Q/C and $V_1 = V_2$ so $Q_1/C_1 = Q_2/C_2$ and then $C_1 = C_2$ says $Q_1 = Q_2$. $V_1 = V_3$ says $Q_1/C_1 = Q_3/C_3$ and $Q_1 = Q_3(C_1/C_3) = Q_3(8.4 \ \mu\text{F}/4.2 \ \mu\text{F}) = 2Q_3$

Using $Q_2 = Q_1$ and $Q_1 = 2Q_3$ in the above equation gives $2Q_3 + 2Q_3 + Q_3 = 228 \mu C$.

 $5Q_3 = 228 \ \mu\text{C}$ and $Q_3 = 45.6 \ \mu\text{C}$, $Q_1 = Q_2 = 91.2 \ \mu\text{C}$

Then
$$V_1 = \frac{Q_1}{C_1} = \frac{91.2 \ \mu C}{8.4 \ \mu F} = 11 \text{ V}, V_2 = \frac{Q_2}{C_2} = \frac{91.2 \ \mu C}{8.4 \ \mu F} = 11 \text{ V}, \text{ and } V_3 = \frac{Q_3}{C_3} = \frac{45.6 \ \mu C}{4.2 \ \mu F} = 11 \text{ V}.$$

The voltage across each capacitor in the parallel combination is 11 V. (d) $U = \frac{1}{2}Q_1V_1 + \frac{1}{2}Q_2V_2 + \frac{1}{2}Q_3V_3$.

But
$$V_1 = V_2 = V_3$$
 so $U = \frac{1}{2}V_1(Q_1 + Q_2 + Q_3) = \frac{1}{2}(11 \text{ V})(228 \ \mu\text{C}) = 1.3 \times 10^{-3} \text{ J}.$

EVALUATE: This is less than the original energy of 1.4×10^{-3} J. The stored energy has decreased, as in Example 24.7.

24.65. (a) **IDENTIFY** and **SET UP**: *Q* is constant. $C = KC_0$; use Eq.(24.1) to relate the dielectric constant *K* to the ratio of the voltages without and with the dielectric.

EXECUTE: With the dielectric: $V = Q/C = Q/(KC_0)$

without the dielectric: $V_0 = Q/C_0$

 $V_0/V = K$, so K = (45.0 V)/(11.5 V) = 3.91

EVALUATE: Our analysis agrees with Eq.(24.13).

(b) IDENTIFY: The capacitor can be treated as equivalent to two capacitors C_1 and C_2 in parallel, one with area 2A/3 and air between the plates and one with area A/3 and dielectric between the plates.

SET UP: The equivalent network is shown in Figure 24.65.



EXECUTE: Let $C_0 = \mathsf{P}_0 A/d$ be the capacitance with only air between the plates. $C_1 = KC_0/3$, $C_2 = 2C_0/3$; $C_{eq} = C_1 + C_2 = (C_0/3)(K+2)$

$$V = \frac{Q}{C_{\text{eq}}} = \frac{Q}{C_0} \left(\frac{3}{K+2}\right) = V_0 \left(\frac{3}{K+2}\right) = (45.0 \text{ V}) \left(\frac{3}{5.91}\right) = 22.8 \text{ V}$$

EVALUATE: The voltage is reduced by the dielectric. The voltage reduction is less when the dielectric doesn't completely fill the volume between the plates.

24.71. IDENTIFY: C = Q/V, so we need to calculate the effect of the dielectrics on the potential difference between the plates.

SET UP: Let the potential of the positive plate be V_a , the potential of the negative plate be V_c , and the potential midway between the plates where the dielectrics meet be V_b , as shown in Figure 24.71.



EXECUTE: The electric field in the absence of any dielectric is $E_0 = \frac{Q}{P_0 A}$. In the first dielectric the electric field is

reduced to $E_1 = \frac{E_0}{K_1} = \frac{Q}{K_1 P_0 A}$ and $V_{ab} = E_1 \left(\frac{d}{2}\right) = \frac{Qd}{K_1 2 P_0 A}$. In the second dielectric the electric field is reduced to $E_2 = \frac{E_0}{K_2} = \frac{Q}{K_2 P_0 A}$ and $V_{bc} = E_2 \left(\frac{d}{2}\right) = \frac{Qd}{K_2 2 P_0 A}$. Thus $V_{ac} = V_{ab} + V_{bc} = \frac{Qd}{K_1 2 P_0 A} + \frac{Qd}{K_2 2 P_0 A} = \frac{Qd}{2 P_0 A} \left(\frac{1}{K_1} + \frac{1}{K_2}\right)$. $V_{ac} = \frac{Qd}{2 P_0 A} \left(\frac{K_1 + K_2}{K_1 K_2}\right)$. This gives $C = \frac{Q}{V_{ac}} = Q \left(\frac{2 P_0 A}{Q d}\right) \left(\frac{K_1 K_2}{K_1 + K_2}\right) = \frac{2 P_0 A}{d} \left(\frac{K_1 K_2}{K_1 + K_2}\right)$.

EVALUATE: An equivalent way to calculate C is to consider the capacitor to be two in series, one with dielectric constant K_1 and the other with dielectric constant K_2 and both with plate separation d/2. (Can imagine inserting a thin conducting plate between the dielectric slabs.)

$$C_{1} = K_{1} \frac{P_{0}A}{d/2} = 2K_{1} \frac{P_{0}A}{d}$$
$$C_{2} = K_{2} \frac{P_{0}A}{d/2} = 2K_{2} \frac{P_{0}A}{d}$$

Since they are in series the total capacitance C is given by $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$ so $C = \frac{C_1 C_2}{C_1 + C_2} = \frac{2P_0 A}{d} \left(\frac{K_1 K_2}{K_1 + K_2}\right)$.