CURRENT, RESISTANCE, AND ELECTROMOTIVE FORCE

25

25.2. **IDENTIFY:** $I = Q/t$. Use $I = n|q|v_{\text{d}}A$ to calculate the drift velocity v_{d}.

SET UP: $n = 5.8\times10^{28}\ \text{m}^{-3}$. $|q| = 1.60\times10^{-19}\ \text{C}$.

EXECUTE: **(a)** $I = \dfrac{Q}{t} = \dfrac{420\ \text{C}}{80(60\ \text{s})} = 8.75\times10^{-2}\ \text{A}.$

(b) $I = n|q|v_{\text{d}}A.$ This gives $v_{\text{d}} = \dfrac{I}{nqA} = \dfrac{8.75\times10^{-2}\ \text{A}}{(5.8\times10^{28})(1.60\times10^{-19}\ \text{C})(\pi(1.3\times10^{-3}\ \text{m})^2)} = 1.78\times10^{-6}\ \text{m/s}.$

EVALUATE: v_{d} is smaller than in Example 25.1, because I is smaller in this problem.

25.8. **IDENTIFY:** $I = Q/t$. Positive charge flowing in one direction is equivalent to negative charge flowing in the opposite direction, so the two currents due to Cl^- and Na^+ are in the same direction and add.

SET UP: Na^+ and Cl^- each have magnitude of charge $|q| = +e$

EXECUTE: **(a)** $Q_{\text{total}} = (n_{\text{Cl}} + n_{\text{Na}})e = (3.92\times10^{16} + 2.68\times10^{16})(1.60\times10^{-19}\ \text{C}) = 0.0106\ \text{C}.$ Then

$I = \dfrac{Q_{\text{total}}}{t} = \dfrac{0.0106\ \text{C}}{1.00\ \text{s}} = 0.0106\ \text{A} = 10.6\ \text{mA}.$

(b) Current flows, by convention, in the direction of positive charge. Thus, current flows with Na^+ toward the negative electrode.

EVALUATE: The Cl^- ions have negative charge and move in the direction opposite to the conventional current direction.

25.12. **IDENTIFY:** $E = \rho J$, where $J = I/A$. The drift velocity is given by $I = n|q|v_{\text{d}}A.$

SET UP: For copper, $\rho = 1.72\times10^{-8}\ \Omega\cdot\text{m}$. $n = 8.5\times10^{28}/\text{m}^3$.

EXECUTE: **(a)** $J = \dfrac{I}{A} = \dfrac{3.6\ \text{A}}{(2.3\times10^{-3}\ \text{m})^2} = 6.81\times10^5\ \text{A/m}^2.$

(b) $E = \rho J = (1.72\times10^{-8}\ \Omega\cdot\text{m})(6.81\times10^5\ \text{A/m}^2) = 0.012\ \text{V/m}.$

(c) The time to travel the wire's length l is

$t = \dfrac{l}{v_{\text{d}}} = \dfrac{ln|q|A}{I} = \dfrac{(4.0\ \text{m})(8.5\times10^{28}/\text{m}^3)(1.6\times10^{-19}\ \text{C})(2.3\times10^{-3}\ \text{m})^2}{3.6\ \text{A}} = 8.0\times10^4\ \text{s}.$

$t = 1333\ \text{min} \approx 22\ \text{hrs!}$

EVALUATE: The currents propagate very quickly along the wire but the individual electrons travel very slowly.

25.16. **IDENTIFY:** Apply $R = \dfrac{\rho L}{A}$ and solve for L.

SET UP: $A = \pi D^2/4$, where $D = 0.462\ \text{mm}$.

EXECUTE: $L = \dfrac{RA}{\rho} = \dfrac{(1.00\ \Omega)(\pi/4)(0.462\times10^{-3}\ \text{m})^2}{1.72\times10^{-8}\ \Omega\cdot\text{m}} = 9.75\ \text{m}.$

EVALUATE: The resistance is proportional to the length of the wire.

25.36. **IDENTIFY:** The sum of the potential changes around the circuit loop is zero. Potential decreases by IR when going through a resistor in the direction of the current and increases by $\mathcal{E}$ when passing through an emf in the direction from the $-$ to + terminal.

SET UP: The current is counterclockwise, because the 16 V battery determines the direction of current flow.

EXECUTE: $+16.0\text{ V}-8.0\text{ V}-I(1.6\ \Omega+5.0\ \Omega+1.4\ \Omega+9.0\ \Omega)=0$

$$I=\frac{16.0\text{ V}-8.0\text{ V}}{1.6\ \Omega+5.0\ \Omega+1.4\ \Omega+9.0\ \Omega}=0.47\text{ A}$$

(b) $V_b+16.0\text{ V}-I(1.6\ \Omega)=V_a$, so $V_a-V_b=V_{ab}=16.0\text{ V}-(1.6\ \Omega)(0.47\text{ A})=15.2\text{ V}$.

(c) $V_c+8.0\text{ V}+I(1.4\ \Omega+5.0\ \Omega)=V_a$ so $V_{ac}=(5.0\ \Omega)(0.47\text{ A})+(1.4\ \Omega)(0.47\text{ A})+8.0\text{ V}=11.0\text{ V}$.

(d) The graph is sketched in Figure 25.36.

EVALUATE: $V_{cb}=(0.47\text{ A})(9.0\ \Omega)=4.2\text{ V}$. The potential at point *b* is 15.2 V below the potential at point *a* and the potential at point *c* is 11.0 V below the potential at point *a*, so the potential of point *c* is $15.2\text{ V}-11.0\text{ V}=4.2\text{ V}$ above the potential of point *b*.

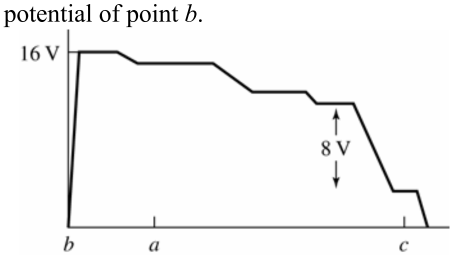

Figure 25.36

25.47. **IDENTIFY** and **SET UP:** By definition $p=\dfrac{P}{LA}$. Use $P=VI$, $E=VL$ and $I=JA$ to rewrite this expression in terms of the specified variables.

EXECUTE: **(a)** *E* is related to *V* and *J* is related to *I*, so use $P=VI$. This gives $p=\dfrac{VI}{LA}$

$\dfrac{V}{L}=E$ and $\dfrac{I}{A}=J$ so $p=EJ$

(b) *J* is related to *I* and ρ is related to *R*, so use $P=IR^2$. This gives $p=\dfrac{I^2R}{LA}$.

$I=JA$ and $R=\dfrac{\rho L}{A}$ so $p=\dfrac{J^2A^2\rho L}{LA^2}\rho J^2$

(c) *E* is related to *V* and ρ is related to *R*, so use $P=V^2/R$. This gives $p=\dfrac{V^2}{RLA}$.

$V=EL$ and $R=\dfrac{\rho L}{A}$ so $p=\dfrac{E^2L^2}{LA}\left(\dfrac{A}{\rho L}\right)=\dfrac{E^2}{\rho}$.

EVALUATE: For a given material (ρ constant), *p* is proportional to J^2 or to E^2.

25.55. **IDENTIFY:** $P=I^2R=\dfrac{V^2}{R}=VI$. $V=IR$.

SET UP: The heater consumes 540 W when $V=120\text{ V}$. Energy $=Pt$.

EXECUTE: **(a)** $P=\dfrac{V^2}{R}$ so $R=\dfrac{V^2}{P}=\dfrac{(120\text{ V})^2}{540\text{ W}}=26.7\ \Omega$

(b) $P=VI$ so $I=\dfrac{P}{V}=\dfrac{540\text{ W}}{120\text{ V}}=4.50\text{ A}$

(c) Assuming that *R* remains $26.7\ \Omega$, $P=\dfrac{V^2}{R}=\dfrac{(110\text{ V})^2}{26.7\ \Omega}=453\text{ W}$. *P* is smaller by a factor of $(110/120)^2$.

EVALUATE: **(d)** With the lower line voltage the current will decrease and the operating temperature will decrease. *R* will be less than $26.7\ \Omega$ and the power consumed will be greater than the value calculated in part (c).

25.60. **IDENTIFY:** Conservation of charge requires that the current is the same in both sections. The voltage drops across each section add, so $R=R_{\text{Cu}}+R_{\text{Ag}}$. The total resistance is the sum of the resistances of each section.

$E=\rho J=\dfrac{\rho I}{A}$, so $E=\dfrac{IR}{L}$, where *R* is the resistance of a section and *L* is its length.

SET UP: For copper, $\rho_{\text{Cu}}=1.72\times10^{-8}\ \Omega\cdot\text{m}$. For silver, $\rho_{\text{Ag}}=1.47\times10^{-8}\ \Omega\cdot\text{m}$.

EXECUTE: **(a)** $I = \dfrac{V}{R} = \dfrac{V}{R_{\rm Cu} + R_{\rm Ag}}$. $R_{\rm Cu} = \dfrac{\rho_{\rm Cu} L_{\rm Cu}}{A_{\rm Cu}} = \dfrac{(1.72\times10^{-8}\ \Omega\cdot\text{m})(0.8\ \text{m})}{(\pi/4)(6.0\times10^{-4}\text{m})^2} = 0.049\ \Omega$ and

$R_{\rm Ag} = \dfrac{\rho_{\rm Ag} L_{\rm Ag}}{A_{\rm Ag}} = \dfrac{(1.47\times10^{-8}\ \Omega\cdot\text{m})(1.2\ \text{m})}{(\pi/4)(6.0\times10^{-4}\ \text{m})^2} = 0.062\ \Omega$. This gives $I = \dfrac{5.0\ \text{V}}{0.049\ \Omega + 0.062\ \Omega} = 45\ \text{A}$.

The current in the copper wire is 45 A.

(b) The current in the silver wire is 45 A, the same as that in the copper wire or else charge would build up at their interface.

(c) $E_{\rm Cu} = J\rho_{\rm Cu} = \dfrac{IR_{\rm Cu}}{L_{\rm Cu}} = \dfrac{(45\ \text{A})(0.049\ \Omega)}{0.8\ \text{m}} = 2.76\ \text{V/m}$.

(d) $E_{\rm Ag} = J\rho_{\rm Ag} = \dfrac{IR_{\rm Ag}}{L_{\rm Ag}} = \dfrac{(45\ \text{A})(0.062\ \Omega)}{1.2\ \text{m}} = 2.33\ \text{V/m}$.

(e) $V_{\rm Ag} = IR_{\rm Ag} = (45\ \text{A})(0.062\ \Omega) = 2.79\ \text{V}$.

EVALUATE: For the copper section, $V_{\rm Cu} = IR_{\rm Cu} = 2.21\ \text{V}$. Note that $V_{\rm Cu} + V_{\rm Ag} = 5.0\ \text{V}$, the voltage applied across the ends of the composite wire.

25.79. **(a) IDENTIFY:** Set the sum of the potential rises and drops around the circuit equal to zero and solve for the resulting equation for the current I. Apply Eq. (25.17) to each circuit element to find the power associated with it.

SET UP: The circuit is sketched in Figure 25.79.

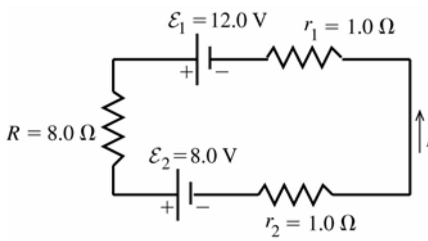

Figure 25.79

EXECUTE:

$$\mathcal{E}_1 - \mathcal{E}_2 - I(r_1 + r_2 + R) = 0$$

$$I = \frac{\mathcal{E}_1 - \mathcal{E}_2}{r_1 + r_2 + R}$$

$$I = \frac{12.0\ \text{V} - 8.0\ \text{V}}{1.0\ \Omega + 1.0\ \Omega + 8.0\ \Omega}$$

$$I = 0.40\ \text{A}$$

(b) $P = I^2R + I^2r_1 + I^2r_2 = I^2(R + r_1 + r_2) = (0.40\ \text{A})^2(8.0\ \Omega + 1.0\ \Omega + 1.0\ \Omega)$

$P = 1.6\ \text{W}$

(c) Chemical energy is converted to electrical energy in a battery when the current goes through the battery from the negative to the positive terminal, so the electrical energy of the charges increases as the current passes through. This happens in the 12.0 V battery, and the rate of production of electrical energy is

$P = \mathcal{E}_1 I = (12.0\ \text{V})(0.40\ \text{A}) = 4.8\ \text{W}$.

(d) Electrical energy is converted to chemical energy in a battery when the current goes through the battery from the positive to the negative terminal, so the electrical energy of the charges decreases as the current passes through. This happens in the 8.0 V battery, and the rate of consumption of electrical energy is

$P = \mathcal{E}_2 I = (8.0\ \text{V})(0.40\ \text{V}) = 3.2\ \text{W}$.

(e) EVALUATE: Total rate of production of electrical energy = 4.8 W. Total rate of consumption of electrical energy = 1.6 W + 3.2 W = 4.8 W, which equals the rate of production, as it must.

DIRECT-CURRENT CIRCUITS

26

26.12. **IDENTIFY:** Replace the series combinations of resistors by their equivalents. In the resulting parallel network the battery voltage is the voltage across each resistor.

SET UP: The circuit is sketched in Figure 26.12a.

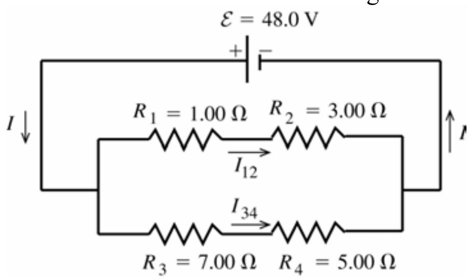

Figure 26.12a

EXECUTE: R_1 and R_2 in series have an equivalent resistance of $R_{12} = R_1 + R_2 = 4.00\ \Omega$

R_3 and R_4 in series have an equivalent resistance of $R_{34} = R_3 + R_4 = 12.0\ \Omega$

The circuit is equivalent to the circuit sketched in Figure 26.12b.

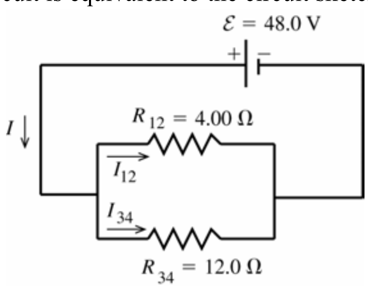

Figure 26.12b

R_{12} and R_{34} in parallel are equivalent to R_{eq} given by $\dfrac{1}{R_{eq}} = \dfrac{1}{R_{12}} + \dfrac{1}{R_{34}} = \dfrac{R_{12}+R_{34}}{R_{12}R_{34}}$

$$R_{eq} = \frac{R_{12}R_{34}}{R_{12}+R_{34}}$$

$$R_{eq} = \frac{(4.00\ \Omega)(12.0\ \Omega)}{4.00\ \Omega + 12.0\ \Omega} = 3.00\ \Omega$$

The voltage across each branch of the parallel combination is $\mathcal{E}$, so $\mathcal{E} - I_{12}R_{12} = 0$.

$$I_{12} = \frac{\mathcal{E}}{R_{12}} = \frac{48.0\ \text{V}}{4.00\ \Omega} = 12.0\ \text{A}$$

$$\mathcal{E} - I_{34}R_{34} = 0 \ \text{ so } \ I_{34} = \frac{\mathcal{E}}{R_{34}} = \frac{48.0\ \text{V}}{12.0\ \Omega} = 4.0\ \text{A}$$

The current is 12.0 A through the 1.00 Ω and 3.00 Ω resistors, and it is 4.0 A through the 7.00 Ω and 5.00 Ω resistors.

EVALUATE: The current through the battery is $I = I_{12} + I_{34} = 12.0\ \text{A} + 4.0\ \text{A} = 16.0\ \text{A}$, and this is equal to $\mathcal{E}/R_{eq} - 48.0\ \text{V}/3.00\ \Omega = 16.0\ \text{A}$.

26.14. **IDENTIFY:** Apply Ohm's law to each resistor.

SET UP: For resistors in parallel the voltages are the same and the currents add. For resistors in series the currents are the same and the voltages add.

EXECUTE: From Ohm's law, the voltage drop across the 6.00 Ω resistor is $V = IR = (4.00\ \text{A})(6.00\ \Omega) = 24.0\ \text{V}$. The voltage drop across the 8.00 Ω resistor is the same, since these two resistors are wired in parallel. The current through the 8.00 Ω resistor is then $I = V/R = 24.0\ \text{V}/8.00\ \Omega = 3.00\ \text{A}$. The current through the 25.0 Ω resistor is the sum of these two currents: 7.00 A. The voltage drop across the 25.0 Ω resistor is $V = IR = (7.00\ \text{A})(25.0\ \Omega) = 175\ \text{V}$, and total voltage drop across the top branch of the circuit is 175 V + 24.0 V = 199 V, which is also the voltage drop across the 20.0 Ω resistor. The current through the 20.0 Ω resistor is then $I = V/R = 199\ \text{V}/20\ \Omega = 9.95\ \text{A}$.

EVALUATE: The total current through the battery is $7.00\ \text{A} + 9.95\ \text{A} = 16.95\ \text{A}$. Note that we did not need to calculate the emf of the battery.

26.19. **IDENTIFY** and **SET UP:** Replace series and parallel combinations of resistors by their equivalents until the circuit is reduced to a single loop. Use the loop equation to find the current through the $20.0\ \Omega$ resistor. Set $P = I^2R$ for the $20.0\ \Omega$ resistor equal to the rate Q/t at which heat goes into the water and set $Q = mc\Delta T$.

EXECUTE: Replace the network by the equivalent resistor, as shown in Figure 26.19.

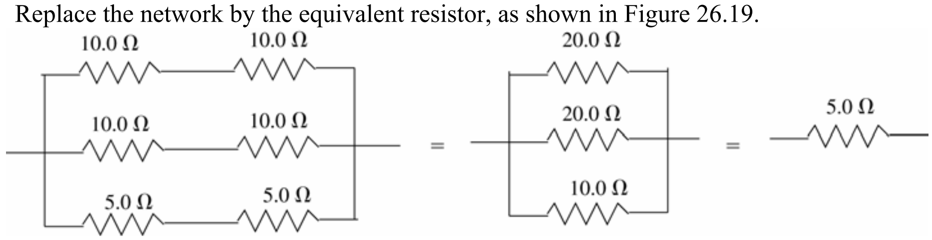

Figure 26.19

$$30.0\ \text{V} - I(20.0\ \Omega + 5.0\ \Omega + 5.0\ \Omega) = 0;\ I = 1.00\ \text{A}$$

For the $20.0\text{-}\Omega$ resistor thermal energy is generated at the rate $P = I^2R = 20.0\text{ W}$. $Q = Pt$ and $Q = mc\Delta T$ gives

$$t = \frac{mc\Delta T}{P} = \frac{(0.100\text{ kg})(4190\text{ J/kg}\cdot\text{ K})(48.0\text{ C}^\circ)}{20.0\text{ W}} = 1.01\times 10^3\text{ s}$$

EVALUATE: The battery is supplying heat at the rate $P = \mathcal{E}I = 30.0\text{ W}$. In the series circuit, more energy is dissipated in the larger resistor $(20.0\ \Omega)$ than in the smaller ones $(5.00\ \Omega)$.

26.22. **IDENTIFY:** Apply the loop rule and junction rule.
SET UP: The circuit diagram is given in Figure 26.22. The junction rule has been used to find the magnitude and direction of the current in the middle branch of the circuit. There are no remaining unknown currents.
EXECUTE: The loop rule applied to loop (1) gives:

$+20.0\text{ V} - (1.00\text{ A})(1.00\ \Omega) + (1.00\text{ A})(4.00\ \Omega) + (1.00\text{ A})(1.00\ \Omega) - \mathcal{E}_1 - (1.00\text{ A})(6.00\ \Omega) = 0$

$\mathcal{E}_1 = 20.0\text{ V} - 1.00\text{ V} + 4.00\text{ V} + 1.00\text{ V} - 6.00\text{ V} = 18.0\text{ V}$. The loop rule applied to loop (2) gives:

$+20.0\text{ V} - (1.00\text{ A})(1.00\ \Omega) - (2.00\text{ A})(1.00\ \Omega) - \mathcal{E}_2 - (2.00\text{ A})(2.00\ \Omega) - (1.00\text{ A})(6.00\ \Omega) = 0$

$\mathcal{E}_2 = 20.0\text{ V} - 1.00\text{ V} - 2.00\text{ V} - 4.00\text{ V} - 6.00\text{ V} = 7.0\text{ V}$. Going from b to a along the lower branch,

$V_b + (2.00\text{ A})(2.00\ \Omega) + 7.0\text{ V} + (2.00\text{ A})(1.00\ \Omega) = V_a$. $V_b - V_a = -13.0\text{ V}$; point b is at 13.0 V lower potential than point a.

EVALUATE: We can also calculate $V_b - V_a$ by going from b to a along the upper branch of the circuit.

$V_b - (1.00\text{ A})(6.00\ \Omega) + 20.0\text{ V} - (1.00\text{ A})(1.00\ \Omega) = V_a$ and $V_b - V_a = -13.0\text{ V}$. This agrees with $V_b - V_a$ calculated along a different path between b and a.

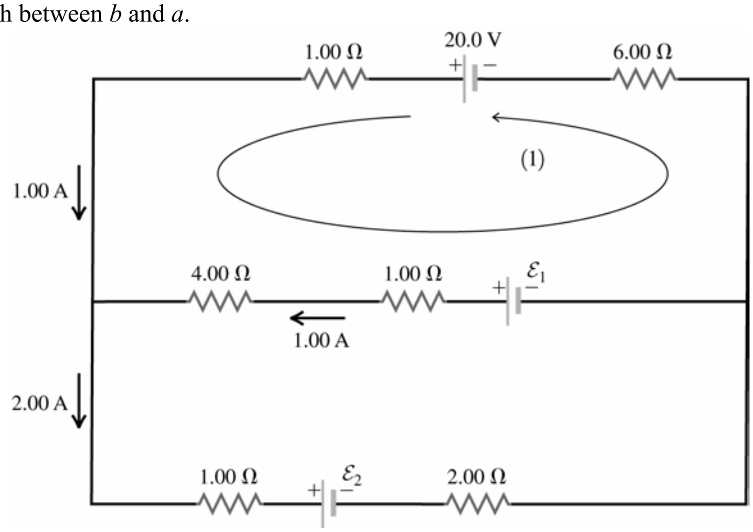

Figure 26.22

26.23. **IDENTIFY:** Apply the junction rule at points *a, b, c* and *d* to calculate the unknown currents. Then apply the loop rule to three loops to calculate $\mathcal{E}_1$, $\mathcal{E}_2$ and R.
(a) SET UP: The circuit is sketched in Figure 26.23.

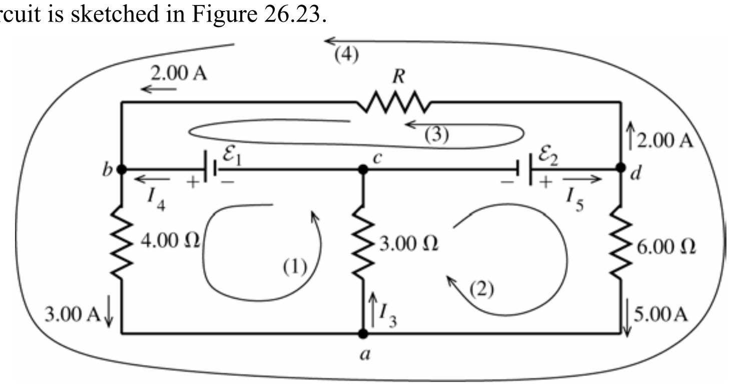

Figure 26.23

EXECUTE: Apply the junction rule to point a: $3.00\text{ A} + 5.00\text{ A} - I_3 = 0$

$I_3 = 8.00\text{ A}$

Apply the junction rule to point *b*: $2.00\text{ A}+I_4-3.00\text{ A}=0$

$I_4=1.00\text{ A}$

Apply the junction rule to point *c*: $I_3-I_4-I_5=0$

$I_5=I_3-I_4=8.00\text{ A}-1.00\text{ A}=7.00\text{ A}$

EVALUATE: As a check, apply the junction rule to point *d*: $I_5-2.00\text{ A}-5.00\text{ A}=0$

$I_5=7.00\text{ A}$

(b) EXECUTE: Apply the loop rule to loop (1): $\mathcal{E}_1-(3.00\text{ A})(4.00\ \Omega)-I_3(3.00\ \Omega)=0$

$\mathcal{E}_1=12.0\text{ V}+(8.00\text{ A})(3.00\ \Omega)=36.0\text{ V}$

Apply the loop rule to loop (2): $\mathcal{E}_2-(5.00\text{ A})(6.00\ \Omega)-I_3(3.00\ \Omega)=0$

$\mathcal{E}_2=30.0\text{ V}+(8.00\text{ A})(3.00\ \Omega)=54.0\text{ V}$

(c) Apply the loop rule to loop (3): $-(2.00\text{ A})R-\mathcal{E}_1+\mathcal{E}_2=0$

$$R=\frac{\mathcal{E}_2-\mathcal{E}_1}{2.00\text{ A}}=\frac{54.0\text{ V}-36.0\text{ V}}{2.00\text{ A}}=9.00\ \Omega$$

EVALUATE: Apply the loop rule to loop (4) as a check of our calculations:

$-(2.00\text{ A})R-(3.00\text{ A})(4.00\ \Omega)+(5.00\text{ A})(6.00\ \Omega)=0$

$-(2.00\text{ A})(9.00\ \Omega)-12.0\text{ V}+30.0\text{ V}=0$

$-18.0\text{ V}+18.0\text{ V}=0$

26.59. **IDENTIFY:** The ohmmeter reads the equivalent resistance between points *a* and *b*. Replace series and parallel combinations by their equivalent.

SET UP: For resistors in parallel, $\frac{1}{R_{eq}}=\frac{1}{R_1}+\frac{1}{R_2}$. For resistors in series, $R_{eq}=R_1+R_2$

EXECUTE: Circuit (a): The $75.0\ \Omega$ and $40.0\ \Omega$ resistors are in parallel and have equivalent resistance $26.09\ \Omega$. The $25.0\ \Omega$ and $50.0\ \Omega$ resistors are in parallel and have an equivalent resistance of $16.67\ \Omega$. The equivalent network is given in Figure 26.59a. $\frac{1}{R_{eq}}=\frac{1}{100.0\ \Omega}+\frac{1}{23.05\ \Omega}$, so $R_{eq}=18.7\ \Omega$.

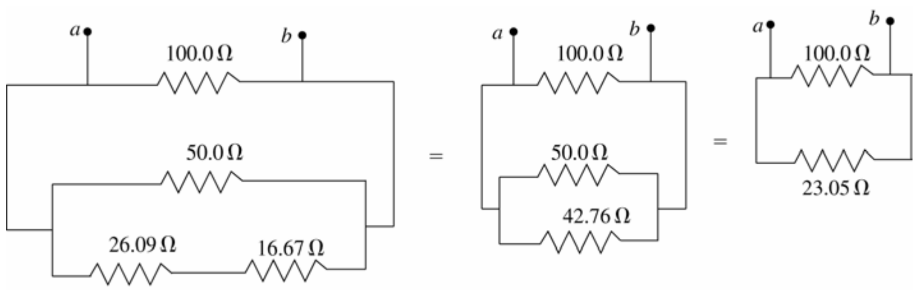

Figure 26.59a

Circuit (b): The $30.0\ \Omega$ and $45.0\ \Omega$ resistors are in parallel and have equivalent resistance $18.0\ \Omega$. The equivalent network is given in Figure 26.59b. $\frac{1}{R_{eq}}=\frac{1}{10.0\ \Omega}+\frac{1}{30.3\ \Omega}$, so $R_{eq}=7.5\ \Omega$.

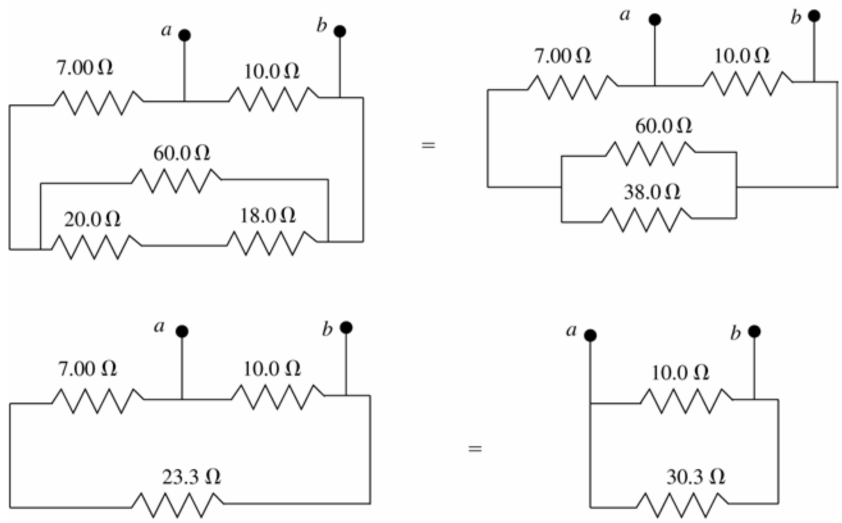

Figure 26.59b

EVALUATE: In circuit (a) the resistance along one path between *a* and *b* is $100.0\ \Omega$, but that is not the equivalent resistance between these points. A similar comment can be made about circuit (b).

26.61. **IDENTIFY:** Apply the junction rule to express the currents through the $5.00\ \Omega$ and $8.00\ \Omega$ resistors in terms of I_1, I_2 and I_3. Apply the loop rule to three loops to get three equations in the three unknown currents.

SET UP: The circuit is sketched in Figure 26.61.

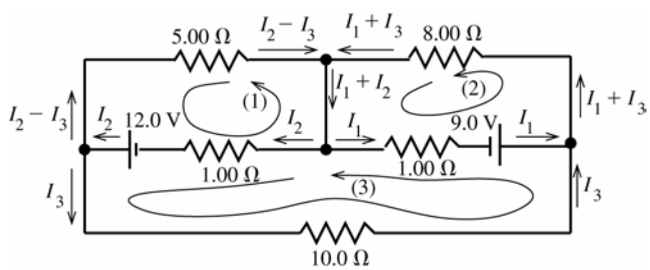

Figure 26.61

The current in each branch has been written in terms of I_1, I_2 and I_3 such that the junction rule is satisfied at each junction point.

EXECUTE: Apply the loop rule to loop (1).

$-12.0\text{ V} + I_2(1.00\ \Omega) + (I_2 - I_3)(5.00\ \Omega) = 0$

$I_2(6.00\ \Omega) - I_3(5.00\ \Omega) = 12.0\text{ V}$ eq.(1)

Apply the loop rule to loop (2).

$-I_1(1.00\ \Omega) + 9.00\text{ V} - (I_1 + I_3)(8.00\ \Omega) = 0$

$I_1(9.00\ \Omega) + I_3(8.00\ \Omega) = 9.00\text{ V}$ eq.(2)

Apply the loop rule to loop (3).

$-I_3(10.0\ \Omega) - 9.00\text{ V} + I_1(1.00\ \Omega) - I_2(1.00\ \Omega) + 12.0\text{ V} = 0$

$-I_1(1.00\ \Omega) + I_2(1.00\ \Omega) + I_3(10.0\ \Omega) = 3.00\text{ V}$ eq.(3)

Eq.(1) gives $I_2 = 2.00\text{ A} + \frac{5}{6}I_3$; eq.(2) gives $I_1 = 1.00\text{ A} - \frac{8}{9}I_3$

Using these results in eq.(3) gives $-(1.00\text{ A} - \frac{8}{9}I_3)(1.00\ \Omega) + (2.00\text{ A} + \frac{5}{6}I_3)(1.00\ \Omega) + I_3(10.0\ \Omega) = 3.00\text{ V}$

$\left(\frac{16+15+180}{18}\right)I_3 = 2.00\text{ A}$; $I_3 = \frac{18}{211}(2.00\text{ A}) = 0.171\text{ A}$

Then $I_2 = 2.00\text{ A} + \frac{5}{6}I_3 = 2.00\text{ A} + \frac{5}{6}(0.171\text{ A}) = 2.14\text{ A}$ and $I_1 = 1.00\text{ A} - \frac{8}{9}I_3 = 1.00\text{ A} - \frac{8}{9}(0.171\text{ A}) = 0.848\text{ A}$.

EVALUATE: We could check that the loop rule is satisfied for a loop that goes through the $5.00\ \Omega$, $8.00\ \Omega$ and $10.0\ \Omega$ resistors. Going around the loop clockwise: $-(I_2 - I_3)(5.00\ \Omega) + (I_1 + I_3)(8.00\ \Omega) + I_3(10.0\ \Omega) =$ $-9.85\text{ V} + 8.15\text{ V} + 1.71\text{ V}$, which does equal zero, apart from rounding.

26.67. **IDENTIFY:** In Figure 26.67, points *a* and *c* are at the same potential and points *d* and *b* are at the same potential, so we can calculate V_{ab} by calculating V_{cd}. We know the current through the resistor that is between points *c* and *d*. We thus can calculate the terminal voltage of the 24.0 V battery without calculating the current through it.

SET UP:

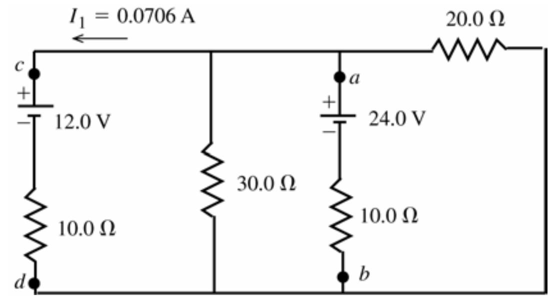

Figure 26.67

EXECUTE: $V_d + I_1(10.0\ \Omega) + 12.0\ \text{V} = V_c$

$V_c - V_d = 12.7\ \text{V}$; $V_a - V_b = V_c - V_d = 12.7\ \text{V}$

EVALUATE: The voltage across each parallel branch must be the same. The current through the 24.0 V battery must be $(24.0\ \text{V} - 12.7\ \text{V})/(10.0\ \Omega) = 1.13\ \text{A}$ in the direction *b* to *a*.

26.73. **(a) IDENTIFY** and **SET UP:** The circuit is sketched in Figure 26.73a.

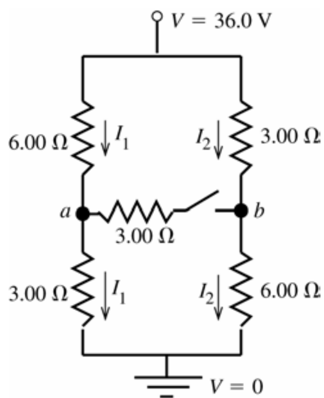

Figure 26.73a

With the switch open there is no current through it and there are only the two currents I_1 and I_2 indicated in the sketch.

The potential drop across each parallel branch is 36.0 V. Use this fact to calculate I_1 and I_2. Then travel from point *a* to point *b* and keep track of the potential rises and drops in order to calculate V_{ab}.

EXECUTE: $-I_1(6.00\ \Omega + 3.00\ \Omega) + 36.0\ \text{V} = 0$

$$I_1 = \frac{36.0\ \text{V}}{6.00\ \Omega + 3.00\ \Omega} = 4.00\ \text{A}$$

$$-I_2(3.00\ \Omega + 6.00\ \Omega) + 36.0\ \text{V} = 0$$

$$I_2 = \frac{36.0\ \text{V}}{3.00\ \Omega + 6.00\ \Omega} = 4.00\ \text{A}$$

To calculate $V_{ab} = V_a - V_b$ start at point *b* and travel to point *a*, adding up all the potential rises and drops along the way. We can do this by going from *b* up through the $3.00\ \Omega$ resistor:

$V_b + I_2(3.00\ \Omega) - I_1(6.00\ \Omega) = V_a$

$V_a - V_b = (4.00\ \text{A})(3.00\ \Omega) - (4.00\ \text{A})(6.00\ \Omega) = 12.0\ \text{V} - 24.0\ \text{V} = -12.0\ \text{V}$

$V_{ab} = -12.0\ \text{V}$ (point *a* is 12.0 V lower in potential than point *b*)

EVALUATE: Alternatively, we can go from point *b* down through the $6.00\ \Omega$ resistor.

$V_b - I_2(6.00\ \Omega) + I_1(3.00\ \Omega) = V_a$

$V_a - V_b = -(4.00\ \text{A})(6.00\ \Omega) + (4.00\ \text{A})(3.00\ \Omega) = -24.0\ \text{V} + 12.0\ \text{V} = -12.0\ \text{V}$, which checks.

(b) IDENTIFY: Now there are multiple current paths, as shown in Figure 26.73b. Use junction rule to write the current in each branch in terms of three unknown currents I_1, I_2, and I_3. Apply the loop rule to three loops to get three equations for the three unknowns. The target variable is I_3, the current through the switch. R_{eq} is calculated from $V = IR_{eq}$, where *I* is the total current that passes through the network.

SET UP:

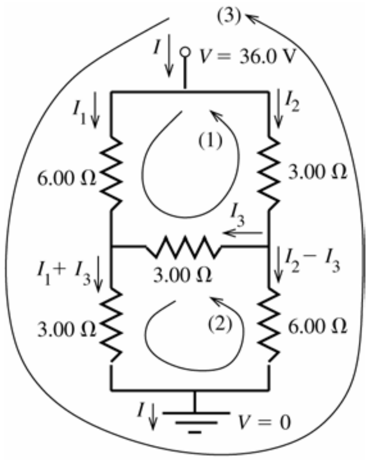

Figure 26.73b

The three unknown currents $I_1, I_2,$ and I_3 are labeled on Figure 26.73b.

EXECUTE: Apply the loop rule to loops (1), (2), and (3).

loop (1): $-I_1(6.00\ \Omega)+I_3(3.00\ \Omega)+I_2(3.00\ \Omega)=0$

$I_2 = 2I_1 - I_3$ eq.(1)

loop (2): $-(I_1+I_3)(3.00\ \Omega)+(I_2-I_3)(6.00\ \Omega)-I_3(3.00\ \Omega)=0$

$6I_2 - 12I_3 - 3I_1 = 0$ so $2I_2 - 4I_3 - I_1 = 0$

Use eq(1) to replace I_2:

$4I_1 - 2I_3 - 4I_3 - I_1 = 0$

$3I_1 = 6I_3$ and $I_1 = 2I_3$ eq.(2)

loop (3) (This loop is completed through the battery [not shown], in the direction from the $-$ to the $+$ terminal.):

$-I_1(6.00\ \Omega)-(I_1+I_3)(3.00\ \Omega)+36.0\ \text{V}=0$

$9I_1 + 3I_3 = 36.0$ A and $3I_1 + I_3 = 12.0$ A eq.(3)

Use eq.(2) in eq.(3) to replace I_1:

$3(2I_3)+I_3 = 12.0\ \text{A}$

$I_3 = 12.0\ \text{A}/7 = 1.71\ \text{A}$

$I_1 = 2I_3 = 3.42\ \text{A}$

$I_2 = 2I_1 - I_3 = 2(3.42\ \text{A}) - 1.71\ \text{A} = 5.13\ \text{A}$

The current through the switch is $I_3 = 1.71$ A.

(c) From the results in part (a) the current through the battery is $I = I_1 + I_2 = 3.42\ \text{A} + 5.13\ \text{A} = 8.55\ \text{A}$. The equivalent circuit is a single resistor that produces the same current through the 36.0 V battery, as shown in Figure 26.73c.

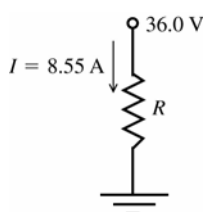

Figure 26.73c

$$-IR + 36.0\ \text{V} = 0$$

$$R = \frac{36.0\ \text{V}}{I} = \frac{36.0\ \text{V}}{8.55\ \text{A}} = 4.21\ \Omega$$

EVALUATE: With the switch open (part a), point *b* is at higher potential than point *a*, so when the switch is closed the current flows in the direction from *b* to *a*. With the switch closed the circuit cannot be simplified using series and parallel combinations but there is still an equivalent resistance that represents the network.