27

MAGNETIC FIELD AND MAGNETIC FORCES

27.1. **IDENTIFY** and **SET UP:** Apply Eq.(27.2) to calculate $\vec{F}$. Use the cross products of unit vectors from Section 1.10.

EXECUTE: $\vec{v} = (+4.19\times10^4\ \text{m/s})\hat{i} + (-3.85\times10^4\ \text{m/s})\hat{j}$

(a) $\vec{B} = (1.40\ \text{T})\hat{i}$

$\vec{F} = q\vec{v}\times\vec{B} = (-1.24\times10^{-8}\ \text{C})(1.40\ \text{T})\left[(4.19\times10^4\ \text{m/s})\hat{i}\times\hat{i} - (3.85\times10^4\ \text{m/s})\hat{j}\times\hat{i}\right]$

$\hat{i}\times\hat{i} = 0,\ \hat{j}\times\hat{i} = -\hat{k}$

$\vec{F} = (-1.24\times10^{-8}\ \text{C})(1.40\ \text{T})(-3.85\times10^4\ \text{m/s})(-\hat{k}) = (-6.68\times10^{-4}\ \text{N})\hat{k}$

EVALUATE: The directions of $\vec{v}$ and $\vec{B}$ are shown in Figure 27.1a.

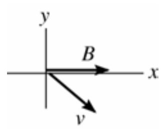

Figure 27.1a

The right-hand rule gives that $\vec{v}\times\vec{B}$ is directed out of the paper (+z-direction). The charge is negative so $\vec{F}$ is opposite to $\vec{v}\times\vec{B}$;

$\vec{F}$ is in the $-z$-direction. This agrees with the direction calculated with unit vectors.

(b) EXECUTE: $\vec{B} = (1.40\ \text{T})\hat{k}$

$\vec{F} = q\vec{v}\times\vec{B} = (-1.24\times10^{-8}\ \text{C})(1.40\ \text{T})\left[(+4.19\times10^4\ \text{m/s})\hat{i}\times\hat{k} - (3.85\times10^4\ \text{m/s})\hat{j}\times\hat{k}\right]$

$\hat{i}\times\hat{k} = -\hat{j},\ \hat{j}\times\hat{k} = \hat{i}$

$\vec{F} = (-7.27\times10^{-4}\ \text{N})(-\hat{j}) + (6.68\times10^{-4}\ \text{N})\hat{i} = \left[(6.68\times10^{-4}\ \text{N})\hat{i} + (7.27\times10^{-4}\ \text{N})\hat{j}\right]$

EVALUATE: The directions of $\vec{v}$ and $\vec{B}$ are shown in Figure 27.1b.

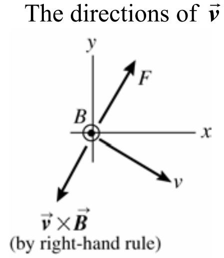

Figure 27.1b

The direction of $\vec{F}$ is opposite to $\vec{v}\times\vec{B}$ since q is negative. The direction of $\vec{F}$ computed from the right-hand rule agrees qualitatively with the direction calculated with unit vectors.

27.3. **IDENTIFY:** The force $\vec{F}$ on the particle is in the direction of the deflection of the particle. Apply the right-hand rule to the directions of $\vec{v}$ and $\vec{B}$. See if your thumb is in the direction of $\vec{F}$, or opposite to that direction. Use $F = |q|vB\sin\phi$ with $\phi = 90°$ to calculate F.

SET UP: The directions of $\vec{v}$, $\vec{B}$ and $\vec{F}$ are shown in Figure 27.3.

EXECUTE: **(a)** When you apply the right-hand rule to $\vec{v}$ and $\vec{B}$, your thumb points east. $\vec{F}$ is in this direction, so the charge is positive.

(b) $F = |q|vB\sin\phi = (8.50\times10^{-6}\ \text{C})(4.75\times10^3\ \text{m/s})(1.25\ \text{T})\sin 90° = 0.0505\ \text{N}$

EVALUATE: If the particle had negative charge and $\vec{v}$ and $\vec{B}$ are unchanged, the particle would be deflected toward the west.

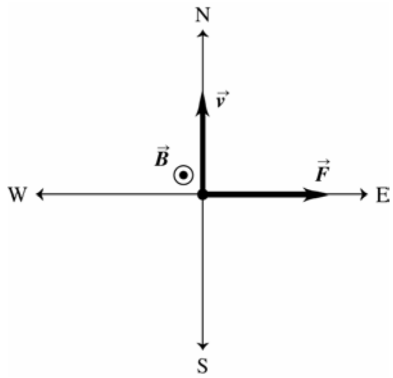

Figure 27.3

27.5. **IDENTIFY:** Apply $F = |q|vB\sin\phi$ and solve for v.

SET UP: An electron has $q = -1.60\times10^{-19}$ C.

EXECUTE: $v = \dfrac{F}{|q|B\sin\phi} = \dfrac{4.60\times10^{-15}\text{ N}}{(1.6\times10^{-19}\text{ C})(3.5\times10^{-3}\text{ T})\sin60^\circ} = 9.49\times10^{6}\text{ m/s}$

EVALUATE: Only the component $B\sin\phi$ of the magnetic field perpendicular to the velocity contributes to the force.

27.7. **IDENTIFY:** Apply $\vec{F} = q\vec{v}\times\vec{B}$.

SET UP: $\vec{v} = v_y\hat{j}$, with $v_y = -3.80\times10^3\text{ m/s}$. $F_x = +7.60\times10^{-3}$ N, $F_y = 0$, and $F_z = -5.20\times10^{-3}$ N.

EXECUTE: **(a)** $F_x = q(v_yB_z - v_zB_y) = qv_yB_z$.

$B_z = F_x/qv_y = (7.60\times10^{-3}\text{ N})/([7.80\times10^{-6}\text{ C})(-3.80\times10^3\text{ m/s})] = -0.256\text{ T}$

$F_y = q(v_zB_x - v_xB_z) = 0$, which is consistent with $\vec{F}$ as given in the problem. There is no force component along the direction of the velocity.

$F_z = q(v_xB_y - v_yB_x) = -qv_yB_x$. $B_x = -F_z/qv_y = -0.175\text{ T}$.

(b) B_y is not determined. No force due to this component of $\vec{B}$ along $\vec{v}$; measurement of the force tells us nothing about B_y.

(c) $\vec{B}\cdot\vec{F} = B_xF_x + B_yF_y + B_zF_z = (-0.175\text{ T})(+7.60\times10^{-3}\text{ N}) + (-0.256\text{ T})(-5.20\times10^{-3}\text{ N})$

$\vec{B}\cdot\vec{F} = 0$. $\vec{B}$ and $\vec{F}$ are perpendicular (angle is 90°).

EVALUATE: The force is perpendicular to both $\vec{v}$ and $\vec{B}$, so $\vec{v}\cdot\vec{F}$ is also zero.

27.15. **(a) IDENTIFY:** Apply Eq.(27.2) to relate the magnetic force $\vec{F}$ to the directions of $\vec{v}$ and $\vec{B}$. The electron has negative charge so $\vec{F}$ is opposite to the direction of $\vec{v}\times\vec{B}$. For motion in an arc of a circle the acceleration is toward the center of the arc so $\vec{F}$ must be in this direction. $a = v^2/R$.

SET UP:

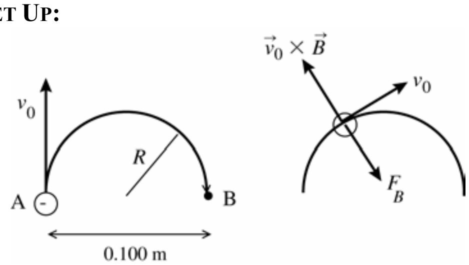

Figure 27.15

As the electron moves in the semicircle, its velocity is tangent to the circular path. The direction of $\vec{v}_0\times\vec{B}$ at a point along the path is shown in Figure 27.15.

EXECUTE: For circular motion the acceleration of the electron $\vec{a}_{\text{rad}}$ is directed in toward the center of the circle. Thus the force $\vec{F}_B$ exerted by the magnetic field, since it is the only force on the electron, must be radially inward. Since q is negative, $\vec{F}_B$ is opposite to the direction given by the right-hand rule for $\vec{v}_0\times\vec{B}$. Thus $\vec{B}$ is directed into the page. Apply Newton's 2nd law to calculate the magnitude of $\vec{B}$: $\sum\vec{F} = m\vec{a}$ gives $\sum F_{\text{rad}} = ma$

$F_B = m(v^2/R)$

$F_B = |q|vB\sin\phi = |q|vB,$ so $|q|vB = m(v^2/R)$

$$B = \frac{mv}{|q|R} = \frac{(9.109\times10^{-31}\text{ kg})(1.41\times10^{6}\text{ m/s})}{(1.602\times10^{-19}\text{ C})(0.050\text{ m})} = 1.60\times10^{-4}\text{ T}$$

(b) IDENTIFY and **SET UP:** The speed of the electron as it moves along the path is constant. ($\vec{F}_B$ changes the direction of $\vec{v}$ but not its magnitude.) The time is given by the distance divided by v_0.

EXECUTE: The distance along the semicircular path is $\pi R,$ so $t = \dfrac{\pi R}{v_0} = \dfrac{\pi(0.050\text{ m})}{1.41\times10^{6}\text{ m/s}} = 1.11\times10^{-7}\text{ s}$

EVALUATE: The magnetic field required increases when v increases or R decreases and also depends on the mass to charge ratio of the particle.

27.22. IDENTIFY: For motion in an arc of a circle, $a = \dfrac{v^2}{R}$ and the net force is radially inward, toward the center of the circle.

SET UP: The direction of the force is shown in Figure 27.22. The mass of a proton is 1.67×10^{-27} kg.

EXECUTE: **(a)** $\vec{F}$ is opposite to the right-hand rule direction, so the charge is negative. $\vec{F} = m\vec{a}$ gives $|q|vB\sin\phi = m\dfrac{v^2}{R}$. $\phi = 90°$ and $v = \dfrac{|q|BR}{m} = \dfrac{3(1.60\times10^{-19}\text{ C})(0.250\text{ T})(0.475\text{ m})}{12(1.67\times10^{-27}\text{ kg})} = 2.84\times10^{6}\text{ m/s}$.

(b) $F_B = |q|vB\sin\phi = 3(1.60\times10^{-19}\text{ C})(2.84\times10^{6}\text{ m/s})(0.250\text{ T})\sin90° = 3.41\times10^{-13}\text{ N}$.

$w = mg = 12(1.67\times10^{-27}\text{ kg})(9.80\text{ m/s}^2) = 1.96\times10^{-25}\text{ N}$. The magnetic force is much larger than the weight of the particle, so it is a very good approximation to neglect gravity.

EVALUATE: **(c)** The magnetic force is always perpendicular to the path and does no work. The particles move with constant speed.

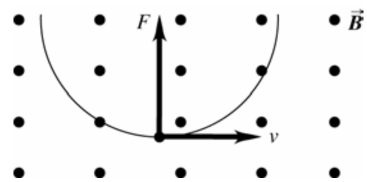

Figure 27.22

27.30. IDENTIFY: For no deflection the magnetic and electric forces must be equal in magnitude and opposite in direction.

SET UP: $v = E/B$ for no deflection.

EXECUTE: To pass undeflected in both cases, $E = vB = (5.85\times10^{3}\text{ m/s})(1.35\text{ T}) = 7898\text{ N/C}$.

(a) If $q = 0.640\times10^{-9}$ C, the electric field direction is given by $-(\hat{\boldsymbol{j}}\times(-\hat{\boldsymbol{k}})) = \hat{\boldsymbol{i}},$ since it must point in the opposite direction to the magnetic force.

(b) If $q = -0.320\times10^{-9}$ C, the electric field direction is given by $((-\hat{\boldsymbol{j}})\times(-\hat{\boldsymbol{k}})) = \hat{\boldsymbol{i}},$ since the electric force must point in the opposite direction as the magnetic force. Since the particle has negative charge, the electric force is opposite to the direction of the electric field and the magnetic force is opposite to the direction it has in part (a).

EVALUATE: The same configuration of electric and magnetic fields works as a velocity selector for both positively and negatively charged particles.

27.35. IDENTIFY: Apply $F = IlB\sin\phi$.

SET UP: Label the three segments in the field as a, b, and c. Let x be the length of segment a. Segment b has length 0.300 m and segment c has length $0.600\text{ cm} - x$. Figure 27.35a shows the direction of the force on each segment. For each segment, $\phi = 90$? The total force on the wire is the vector sum of the forces on each segment.

EXECUTE: $F_a = IlB = (4.50\text{ A})x(0.240\text{ T})$. $F_c = (4.50\text{ A})(0.600\text{ m} - x)(0.240\text{ T})$. Since $\vec{F}_a$ and $\vec{F}_c$ are in the same direction their vector sum has magnitude $F_{ac} = F_a + F_c = (4.50\text{ A})(0.600\text{ m})(0.240\text{ T}) = 0.648\text{ N}$ and is directed toward the bottom of the page in Figure 27.35a. $F_b = (4.50\text{ A})(0.300\text{ m})(0.240\text{ T}) = 0.324\text{ N}$ and is directed to the right. The vector addition diagram for $\vec{F}_{ac}$ and $\vec{F}_b$ is given in Figure 27.35b.

$F = \sqrt{F_{ac}^2 + F_b^2} = \sqrt{(0.648\text{ N})^2 + (0.324\text{ N})^2} = 0.724\text{ N}$. $\tan\theta = \dfrac{F_{ac}}{F_b} = \dfrac{0.648\text{ N}}{0.324\text{ N}}$ and $\theta = 63.4$? The net force has magnitude 0.724 N and its direction is specified by $\theta = 63.4°$ in Figure 27.35b.

EVALUATE: All three current segments are perpendicular to the magnetic field, so $\phi = 90°$ for each in the force equation. The direction of the force on a segment depends on the direction of the current for that segment.

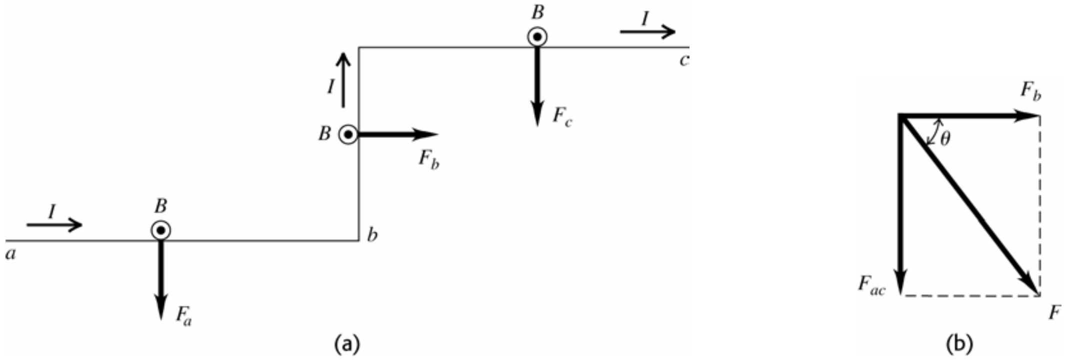

Figure 27.35

27.40. **IDENTIFY:** The magnetic force $\vec{F}_B$ must be upward and equal to mg. The direction of $\vec{F}_B$ is determined by the direction of I in the circuit.

SET UP: $F_B = IlB\sin\phi$, with $\phi = 90°$. $I = \dfrac{V}{R}$, where V is the battery voltage.

EXECUTE: **(a)** The forces are shown in Figure 27.40. The current I in the bar must be to the right to produce $\vec{F}_B$ upward. To produce current in this direction, point a must be the positive terminal of the battery.

(b) $F_B = mg$. $IlB = mg$. $m = \dfrac{IlB}{g} = \dfrac{VlB}{Rg} = \dfrac{(175\text{ V})(0.600\text{ m})(1.50\text{ T})}{(5.00\ \Omega)(9.80\text{ m/s}^2)} = 3.21\text{ kg}$.

EVALUATE: If the battery had opposite polarity, with point a as the negative terminal, then the current would be clockwise and the magnetic force would be downward.

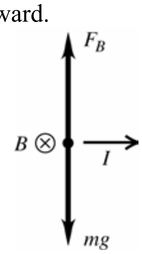

Figure 27.40

27.43. **IDENTIFY:** The period is $T = 2\pi r/v$, the current is Q/t and the magnetic moment is $\mu = IA$

SET UP: The electron has charge $-e$. The area enclosed by the orbit is πr^2.

EXECUTE: **(a)** $T = 2\pi r/v = 1.5 \times 10^{-16}$ s

(b) Charge $-e$ passes a point on the orbit once during each period, so $I = Q/t = e/t = 1.1$ mA .

(c) $\mu = IA = I\pi r^2 = 9.3 \times 10^{-24}\ \text{A} \cdot \text{m}^2$

EVALUATE: Since the electron has negative charge, the direction of the current is opposite to the direction of motion of the electron.

27.44. **IDENTIFY:** $\tau = IAB\sin\phi$, where ϕ is the angle between $\vec{B}$ and the normal to the loop.

SET UP: The coil as viewed along the axis of rotation is shown in Figure 27.44a for its original position and in Figure 27.44b after it has rotated $30.0°$.

EXECUTE: **(a)** The forces on each side of the coil are shown in Figure 27.44a. $\vec{F}_1 + \vec{F}_2 = 0$ and $\vec{F}_3 + \vec{F}_4 = 0$. The net force on the coil is zero. $\phi = 0°$ and $\sin\phi = 0$, so $\tau = 0$. The forces on the coil produce no torque.

(b) The net force is still zero. $\phi = 30.0°$ and the net torque is

$\tau = (1)(1.40\text{ A})(0.220\text{ m})(0.350\text{ m})(1.50\text{ T})\sin 30.0° = 0.0808\text{ N} \cdot \text{m}$. The net torque is clockwise in Figure 27.44b and is directed so as to increase the angle ϕ .

EVALUATE: For any current loop in a uniform magnetic field the net force on the loop is zero. The torque on the loop depends on the orientation of the plane of the loop relative to the magnetic field direction.

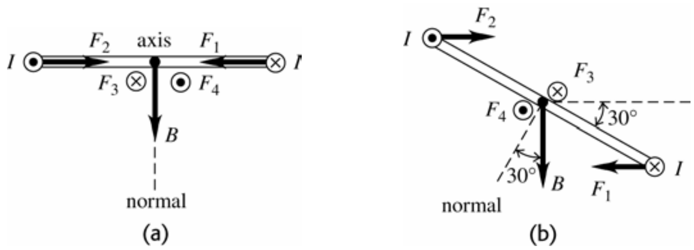

Figure 27.44

27.46. **IDENTIFY:** $\vec{\tau} = \vec{\mu} \times \vec{B}$ and $U = -\mu B\cos\phi$, where $\mu = NIB$. $\tau = \mu B\sin\phi$.

SET UP: ϕ is the angle between $\vec{B}$ and the normal to the plane of the loop.

EXECUTE: **(a)** $\phi = 90°$. $\tau = NIAB\sin(90°) = NIAB$, direction $\hat{k}\times\hat{j} = -\hat{i}$. $U = -\mu B\cos\phi = 0$.

(b) $\phi = 0$. $\tau = NIAB\sin(0) = 0$, no direction. $U = -\mu B\cos\phi = -NIAB$.

(c) $\phi = 90°$. $\tau = NIAB\sin(90°) = NIAB$, direction $-\hat{k}\times\hat{j} = \hat{i}$. $U = -\mu B\cos\phi = 0$.

(d) $\phi = 180°$: $\tau = NIAB\sin(180°) = 0$, no direction, $U = -\mu B\cos(180°) = NIAB$.

EVALUATE: When τ is maximum, $U = 0$. When $|U|$ is maximum, $\tau = 0$.

27.52. **IDENTIFY:** Apply Eq.(27.30).

SET UP: $A = y_1 z_1$. $E = \mathcal{E}/z_1$. $|q| = e$.

EXECUTE: $n = \dfrac{J_x B_y}{|q|E_z} = \dfrac{IB_y}{A|q|E_z} = \dfrac{IB_y z_1}{A|q|\mathcal{E}} = \dfrac{IB_y}{y_1|q|\mathcal{E}}$

$$n = \frac{(78.0\ \text{A})(2.29\ \text{T})}{(2.3\times10^{-4}\ \text{m})(1.6\times10^{-19}\ \text{C})(1.31\times10^{-4}\ \text{V})} = 3.7\times10^{28}\ \text{electrons/m}^3$$

EVALUATE: The value of n for this metal is about one-third the value of n calculated in Example 27.12 for copper.

27.61. **IDENTIFY** and **SET UP:** Use Eq.(27.2) to relate $q, \vec{v}, \vec{B}$ and $\vec{F}$. The force $\vec{F}$ and $\vec{a}$ are related by Newton's 2nd law.

$\vec{B} = -(0.120\ \text{T})\hat{k}$, $\vec{v} = (1.05\times10^6\ \text{m/s})(-3\hat{i} + 4\hat{j} + 12\hat{k})$, $F = 1.25$ N

(a) EXECUTE: $\vec{F} = q\vec{v}\times\vec{B}$

$\vec{F} = q(-0.120\ \text{T})(1.05\times10^6\ \text{m/s})(-3\hat{i}\times\hat{k} + 4\hat{j}\times\hat{k} + 12\hat{k}\times\hat{k})$

$\hat{i}\times\hat{k} = -\hat{j}, \hat{j}\times\hat{k} = \hat{i}, \hat{k}\times\hat{k} = 0$

$\vec{F} = -q(1.26\times10^5\ \text{N/C})(+3\hat{j} + 4\hat{i}) = -q(1.26\times10^5\ \text{N/C})(+4\hat{i} + 3\hat{j})$

The magnitude of the vector $+4\hat{i} + 3\hat{j}$ is $\sqrt{3^2+4^2} = 5$. Thus $F = -q(1.26\times10^5\ \text{N/C})(5)$.

$$q = -\frac{F}{5(1.26\times10^5\ \text{N/C})} = -\frac{1.25\ \text{N}}{5(1.26\times10^5\ \text{N/C})} = -1.98\times10^{-6}\ \text{C}$$

(b) $\sum\vec{F} = m\vec{a}$ so $\vec{a} = \vec{F}/m$

$\vec{F} = -q(1.26\times10^5\ \text{N/C})(+4\hat{i} + 3\hat{j}) = -(-1.98\times10^{-6}\ \text{C})(1.26\times10^5\ \text{N/C})(+4\hat{i} + 3\hat{j}) = +0.250\ \text{N}(+4\hat{i} + 3\hat{j})$

Then $\vec{a} = \vec{F}/m = \left(\dfrac{0.250\ \text{N}}{2.58\times10^{-15}\ \text{kg}}\right)(+4\hat{i} + 3\hat{j}) = (9.69\times10^{13}\ \text{m/s}^2)(+4\hat{i} + 3\hat{j})$

(c) IDENTIFY and **SET UP:** $\vec{F}$ is in the xy-plane, so in the z-direction the particle moves with constant speed 12.6×10^6 m/s. In the xy-plane the force $\vec{F}$ causes the particle to move in a circle, with $\vec{F}$ directed in towards the center of the circle.

EXECUTE: $\sum\vec{F} = m\vec{a}$ gives $F = m(v^2/R)$ and $R = mv^2/F$

$v^2 = v_x^2 + v_y^2 = (-3.15\times10^6\ \text{m/s})^2 + (+4.20\times10^6\ \text{m/s})^2 = 2.756\times10^{13}\ \text{m}^2/\text{s}^2$

$F = \sqrt{F_x^2 + F_y^2} = (0.250\ \text{N})\sqrt{4^2+3^2} = 1.25\ \text{N}$

$$R = \frac{mv^2}{F} = \frac{(2.58\times10^{-15}\ \text{kg})(2.756\times10^{13}\ \text{m}^2/\text{s}^2)}{1.25\ \text{N}} = 0.0569\ \text{m} = 5.69\ \text{cm}$$

(d) IDENTIFY and **SET UP:** By Eq.(27.12) the cyclotron frequency is $f = \omega/2\pi = v/2\pi R$.

EXECUTE: The circular motion is in the xy-plane, so $v = \sqrt{v_x^2 + v_y^2} = 5.25\times10^6$ m/s.

$$f = \frac{v}{2\pi R} = \frac{5.25\times10^{6}\ \text{m/s}}{2\pi(0.0569\ \text{m})} = 1.47\times10^{7}\ \text{Hz, and } \omega = 2\pi f = 9.23\times10^{7}\ \text{rad/s}$$

(e) IDENTIFY and **SET UP** Compare t to the period T of the circular motion in the xy-plane to find the x and y coordinates at this t. In the z-direction the particle moves with constant speed, so $z = z_0 + v_z t$.

EXECUTE: The period of the motion in the xy-plane is given by $T = \frac{1}{f} = \frac{1}{1.47\times10^{7}\ \text{Hz}} = 6.80\times10^{-8}\ \text{s}$

In $t = 2T$ the particle has returned to the same x and y coordinates. The z-component of the motion is motion with a constant velocity of $v_z = +12.6\times10^{6}\ \text{m/s}$. Thus $z = z_0 + v_z t = 0 + (12.6\times10^{6}\ \text{m/s})(2)(6.80\times10^{-8}\ \text{s}) = +1.71\ \text{m}$.

The coordinates at $t = 2T$ are $x = R, y = 0, z = +1.71$ m.

EVALUATE: The circular motion is in the plane perpendicular to $\vec{B}$. The radius of this motion gets smaller when B increases and it gets larger when v increases. There is no magnetic force in the direction of $\vec{B}$ so the particle moves with constant velocity in that direction. The superposition of circular motion in the xy-plane and constant speed motion in the z-direction is a helical path.

27.64. IDENTIFY: Apply $\vec{F} = q\vec{v}\times\vec{B}$

SET UP: $\vec{v} = v\hat{k}$

EXECUTE: **(a)** $\vec{F} = -qvB_y\hat{i} + qvB_x\hat{j}$. But $\vec{F} = 3F_0\hat{i} + 4F_0\hat{j}$, so $3F_0 = -qvB_y$ and $4F_0 = qvB_x$

Therefore, $B_y = -\frac{3F_0}{qv}$, $B_x = \frac{4F_0}{qv}$ and B_z is undetermined.

(b) $B = \frac{6F_0}{qv} = \sqrt{B_x^{2} + B_y^{2} + B_z^{2}} = \frac{F_0}{qv}\sqrt{9 + 16 + \left(\frac{qv}{F_0}\right)^{2} B_z^{2}} = \frac{F_0}{qv}\sqrt{25 + \left(\frac{qv}{F_0}\right)^{2} B_z^{2}}$, so $B_z = \pm\frac{11F_0}{qv}$.

EVALUATE: The force doesn't depend on B_z, since $\vec{v}$ is along the z-direction.

27.67. IDENTIFY: The force exerted by the magnetic field is given by Eq.(27.19). The net force on the wire must be zero.
SET UP: For the wire to remain at rest the force exerted on it by the magnetic field must have a component directed up the incline. To produce a force in this direction, the current in the wire must be directed from right to left in Figure 27.61 in the textbook. Or, viewing the wire from its left-hand end the directions are shown in Figure 27.67a.

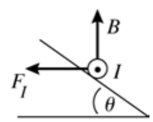

Figure 27.67a

The free-body diagram for the wire is given in Figure 27.67b.

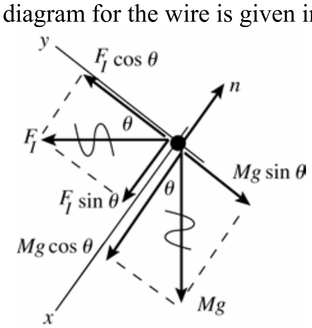

Figure 27.67b

EXECUTE: $\sum F_y = 0$

$F_I\cos\theta - Mg\sin\theta = 0$

$F_I = ILB\sin\phi$

$\phi = 90°$ since $\vec{B}$ is perpendicular to the current direction.

Thus $(ILB)\cos\theta - Mg\sin\theta = 0$ and $I = \frac{Mg\tan\theta}{LB}$

EVALUATE: The magnetic and gravitational forces are in perpendicular directions so their components parallel to the incline involve different trig functions. As the tilt angle θ increases there is a larger component of Mg down the incline and the component of F_I up the incline is smaller; I must increase with θ to compensate. As $\theta\to0$, $I\to0$ and as $\theta\to90°, I\to\infty$.

27.75. IDENTIFY: For the loop to be in equilibrium the net torque on it must be zero. Use Eq.(27.26) to calculate the torque due to the magnetic field and use Eq.(10.3) for the torque due to the gravity force.
SET UP: See Figure 27.75a.

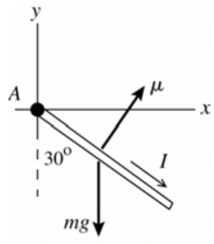

Figure 27.75a

Use $\sum \tau_A = 0$, where point A is at the origin.

EXECUTE: See Figure 27.75b.

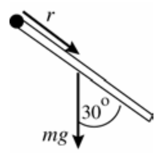

Figure 27.75b

$\tau_{mg} = mgr\sin\phi = mg(0.400 \text{ m})\sin 30.0°$

The torque is clockwise; $\vec{\tau}_{mg}$ is directed into the paper.

For the loop to be in equilibrium the torque due to $\vec{B}$ must be counterclockwise (opposite to $\vec{\tau}_{mg}$) and it must be that $\tau_B = \tau_{mg}$. See Figure 27.75c.

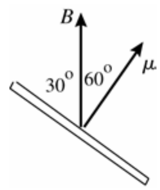

Figure 27.75c

$\vec{\tau}_B = \vec{\mu} \times \vec{B}$. For this torque to be counterclockwise ($\vec{\tau}_B$ directed out of the paper), $\vec{B}$ must be in the $+y$-direction.

$\tau_B = \mu B\sin\phi = IAB\sin 60.0°$

$\tau_B = \tau_{mg}$ gives $IAB\sin 60.0° = mg(0.0400 \text{ m})\sin 30.0°$

$m = (0.15 \text{ g/cm})2(8.00 \text{ cm} + 6.00 \text{ cm}) = 4.2 \text{ g} = 4.2\times10^{-3} \text{ kg}$

$A = (0.800 \text{ m})(0.0600 \text{ m}) = 4.80\times10^{-3} \text{ m}^2$

$$B = \frac{mg(0.0400 \text{ m})(\sin 30.0°)}{IA\sin 60.0°}$$

$$B = \frac{(4.2\times10^{-3} \text{ kg})(9.80 \text{ m/s}^2)(0.0400 \text{ m})\sin 30.0°}{(8.2 \text{ A})(4.80\times10^{-3} \text{ m}^2)\sin 60.0°} = 0.024 \text{ T}$$

EVALUATE: As the loop swings up the torque due to $\vec{B}$ decreases to zero and the torque due to mg increases from zero, so there must be an orientation of the loop where the net torque is zero.

27.78. **IDENTIFY:** Apply $\vec{F} = I\vec{l} \times \vec{B}$ to calculate the force on each side of the loop.

SET UP: The net force is the vector sum of the forces on each side of the loop.

EXECUTE: **(a)** $F_{PQ} = (5.00 \text{ A})(0.600 \text{ m})(3.00 \text{ T})\sin(0°) = 0 \text{ N}$.

$F_{RP} = (5.00 \text{ A})\,(0.800 \text{ m})\,(3.00 \text{ T})\,\sin(90°) = 12.0 \text{ N}$, into the page.

$F_{QR} = (5.00 \text{ A})(1.00 \text{ m})(3.00 \text{ T})(0.800/1.00) = 12.0 \text{ N}$, out of the page.

(b) The net force on the triangular loop of wire is zero.

(c) For calculating torque on a straight wire we can assume that the force on a wire is applied at the wire's center. Also, note that we are finding the torque with respect to the PR-axis (not about a point), and consequently the lever arm will be the distance from the wire's center to the x-axis. $\tau = rF\sin\phi$ gives $\tau_{PQ} = r(0 \text{ N}) = 0$,

$\tau_{RP} = (0 \text{ m})F\sin\phi = 0$ and $\tau_{QR} = (0.300 \text{ m})(12.0 \text{ N})\sin(90°) = 3.60 \text{ N}\cdot\text{m}$. The net torque is $3.60 \text{ N}\cdot\text{m}$.

(d) According to Eq.(27.28), $\tau = NIAB\sin\phi = (1)(5.00 \text{ A})\left(\tfrac{1}{2}\right)(0.600 \text{ m})(0.800 \text{ m})(3.00 \text{ T})\sin(90°) = 3.60 \text{ N}\cdot\text{m}$, which agrees with part (c).

(e) Since F_{QR} is out of the page and since this is the force that produces the net torque, the point Q will be rotated out of the plane of the figure.

EVALUATE: In the expression $\tau = NIAB\sin\phi$, ϕ is the angle between the plane of the loop and the direction of $\vec{\boldsymbol{B}}$. In this problem, $\phi = 90°$.

27.81. IDENTIFY: Apply $d\vec{\boldsymbol{F}} = Id\vec{\boldsymbol{l}} \times \vec{\boldsymbol{B}}$ to each side of the loop.

SET UP: For each side of the loop, $d\vec{\boldsymbol{l}}$ is parallel to that side of the loop and is in the direction of *I*. Since the loop is in the *xy*-plane, $z = 0$ at the loop and $B_y = 0$ at the loop.

EXECUTE: **(a)** The magnetic field lines in the *yz*-plane are sketched in Figure 27.81.

(b) Side 1, that runs from (0,0) to (0,*L*): $\vec{\boldsymbol{F}} = \int_0^L Id\vec{\boldsymbol{l}} \times \vec{\boldsymbol{B}} = I\int_0^L \frac{B_0 y\, dy}{L}\hat{\boldsymbol{i}} = \frac{1}{2}B_0LI\boldsymbol{i}$.

Side 2, that runs from (0,*L*) to (*L*,*L*): $\vec{\boldsymbol{F}} = \int_{0,y=L}^{L} Id\vec{\boldsymbol{l}} \times \vec{\boldsymbol{B}} = I\int_{0,y=L}^{L} \frac{B_0 y\, dx}{L}\hat{\boldsymbol{j}} = -IB_0L\boldsymbol{j}$.

Side 3, that runs from (*L*,*L*) to (*L*,0): $\vec{\boldsymbol{F}} = \int_{L,x=L}^{0} Id\vec{\boldsymbol{l}} \times \vec{\boldsymbol{B}} = I\int_{L,x=L}^{0} \frac{B_0 y\, dy}{L}(-\hat{\boldsymbol{i}}) = -\frac{1}{2}IB_0L\boldsymbol{i}$.

Side 4, that runs from (*L*,0) to (0,0): $\vec{\boldsymbol{F}} = \int_{L,y=0}^{0} Id\vec{\boldsymbol{l}} \times \vec{\boldsymbol{B}} = I\int_{L,y=0}^{0} \frac{B_0 y\, dx}{L}\hat{\boldsymbol{j}} = 0$.

(c) The sum of all forces is $\vec{\boldsymbol{F}}_{\text{total}} = -IB_0L\hat{\boldsymbol{j}}$.

EVALUATE: The net force on sides 1 and 3 is zero. The force on side 4 is zero, since $y = 0$ and $z = 0$ at that side and therefore $B = 0$ there. The net force on the loop equals the force on side 2.

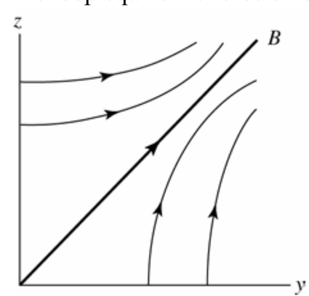

Figure 27.81