28.2. **IDENTIFY:** A moving charge creates a magnetic field as well as an electric field.

SET UP: The magnetic field caused by a moving charge is $B = \frac{\mu_0}{4\pi}\frac{qv\sin\phi}{r^2}$, and its electric field is $E = \frac{1}{4\pi P_0}\frac{e}{r^2}$ since $q = e$.

EXECUTE: Substitute the appropriate numbers into the above equations.

$$B = \frac{\mu_0}{4\pi}\frac{qv\sin\phi}{r^2} = \frac{4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}}{4\pi}\frac{(1.60\times10^{-19}\,\text{C})(2.2\times10^{6}\,\text{m/s})\sin 90^\circ}{(5.3\times10^{-11}\text{m})^2} = 13\ \text{T, out of the page.}$$

$$E = \frac{1}{4\pi P_0}\frac{e}{r^2} = \frac{(9.00\times10^9\,\text{燦}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\,\text{烫})}{(5.3\times10^{-11}\,\text{爛})^2} = 5.1\times10^{11}\,\text{燦/C, toward the electron.}$$

EVALUATE: There are enormous fields within the atom!

28.8. **IDENTIFY:** Both moving charges create magnetic fields, and the net field is the vector sum of the two. The magnetic force on a moving charge is $F_{\text{mag}} = qvB\sin\phi$ and the electrical force obeys Coulomb's law.

SET UP: The magnetic field due to a moving charge is $B = \frac{\mu_0}{4\pi}\frac{qv\sin\phi}{r^2}$.

EXECUTE: **(a)** Both fields are into the page, so their magnitudes add, giving

$$B = B_\text{e} + B_\text{p} = \frac{\mu_0}{4\pi}\left(\frac{ev}{r_\text{e}^2} + \frac{ev}{r_\text{p}^2}\right)\sin 90^\circ$$

$$B = \frac{\mu_0}{4\pi}\left(1.60\times10^{-19}\,\text{烫}\right)\left(845{,}000\ \text{m/s}\right)\left[\frac{1}{(5.00\times10^{-9}\,\text{爛})^2} + \frac{1}{(4.00\times10^{-9}\,\text{爛})^2}\right]$$

$B = 1.39 \times 10^{-3}$ T = 1.39 mT, into the page.

(b) Using $B = \frac{\mu_0}{4\pi}\frac{qv\sin\phi}{r^2}$, where $r = \sqrt{41}$ nm and $\phi = 180^\circ - \arctan(5/4) = 128.7^\circ$, we get

$$B = \frac{4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}}{4\pi}?\frac{(1.6\times10^{-19}\,\text{烫})(845{,}000\ \text{m/s})\sin 128.7^\circ}{(\sqrt{41}\times10^{-9}\,\text{爛})^2} = \ .58\times10^{-4}\ \text{T, into the page.}$$

(c) $F_{\text{mag}} = qvB\sin 90^\circ = (1.60\times10^{-19}\ \text{C})(845{,}000\ \text{m/s})(2.58\times10^{-4}\ \text{T}) = 3.48\times10^{-17}\ \text{N}$, in the $+x$ direction.

$F_{\text{elec}} = (1/4\pi P_0)e^2/r^2 = \frac{(9.00\times10^9\,\text{燦}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\,\text{烫})^2}{(\sqrt{41}\times10^{-9}\,\text{爛})^2} = 5.62\times10^{-12}\ \text{N}$, at 51.3° below the $+x$-axis measured clockwise.

EVALUATE: The electric force is much stronger than the magnetic force.

28.23. **IDENTIFY:** The net magnetic field at the center of the square is the vector sum of the fields due to each wire.

SET UP: For each wire, $B = \frac{\mu_0 I}{2\pi r}$ and the direction of $\vec{B}$ is given by the right-hand rule that is illustrated in Figure 28.6 in the textbook.

EXECUTE: **(a)** and **(b)** $B = 0$ since the magnetic fields due to currents at opposite corners of the square cancel.
(c) The fields due to each wire are sketched in Figure 28.23.

$$B = B_a\cos 45^\circ + B_b\cos 45^\circ + B_c\cos 45^\circ + B_d\cos 45^\circ = 4B_a\cos 45^\circ = 4\left(\frac{\mu_0 I}{2\pi r}\right)\cos 45^\circ .$$

$r = \sqrt{(10\ \text{cm})^2 + (10\ \text{cm})^2} = 10\sqrt{2}\ \text{cm} = 0.10\sqrt{2}\ \text{m}$, so

$$B = 4\frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})\,(100\ \text{A})}{2\pi(0.10\sqrt{2}\ \text{m})}\cos 45^\circ = 4.0\times10^{-4}\ \text{T, to the left.}$$

EVALUATE: In part (c), if all four currents are reversed in direction, the net field at the center of the square would be to the right.

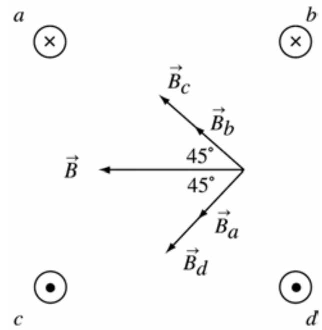

Figure 28.23

28.30. **IDENTIFY:** The magnetic field at the center of a circular loop is $B = \dfrac{\mu_0 I}{2R}$. By symmetry each segment of the loop that has length Δl contributes equally to the field, so the field at the center of a semicircle is $\frac{1}{2}$ that of a full loop.

SET UP: Since the straight sections produce no field at P, the field at P is $B = \dfrac{\mu_0 I}{4R}$.

EXECUTE: $B = \dfrac{\mu_0 I}{4R}$. The direction of $\vec{B}$ is given by the right-hand rule: $\vec{B}$ is directed into the page.

EVALUATE: For a quarter-circle section of wire the magnetic field at its center of curvature is $B = \dfrac{\mu_0 I}{8R}$.

28.31. **IDENTIFY:** Calculate the magnetic field vector produced by each wire and add these fields to get the total field.

SET UP: First consider the field at P produced by the current I_1 in the upper semicircle of wire. See Figure 28.31a.

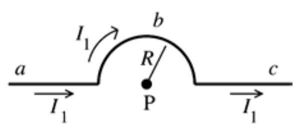

Figure 28.31a

Consider the three parts of this wire
a: long straight section,
b: semicircle
c: long, straight section

Apply the Biot-Savart law $d\vec{B} = \dfrac{\mu_0}{4\pi}\dfrac{I d\vec{l} \times \hat{r}}{r^2} = \dfrac{\mu_0}{4\pi}\dfrac{I d\vec{l} \times \vec{r}}{r^3}$ to each piece.

EXECUTE: part *a* See Figure 28.31b.

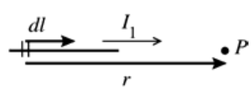

Figure 28.31b

$d\vec{l} \times \vec{r} = 0$,
so $dB = 0$

The same is true for all the infinitesimal segments that make up this piece of the wire, so $B = 0$ for this piece.

part *c* See Figure 28.31c.

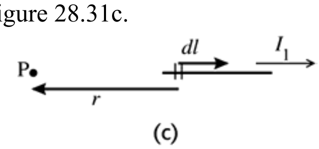

Figure 28.31c

$d\vec{l} \times \vec{r} = 0$,
so $dB = 0$ and $B = 0$ for this piece.

part *b* See Figure 28.31d.

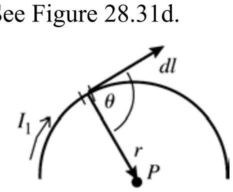

Figure 28.31d

$d\vec{l} \times \vec{r}$ is directed into the paper for all infinitesimal segments that make up this semicircular piece, so $\vec{B}$ is directed into the paper and $B = \int dB$ (the vector sum of the $d\vec{B}$ is obtained by adding their magnitudes since they are in the same direction).

$\left|d\vec{l} \times \vec{r}\right| = r\,dl \sin\theta$. The angle θ between $d\vec{l}$ and $\vec{r}$ is 90° and $r = R$, the radius of the semicircle. Thus $\left|d\vec{l} \times \vec{r}\right| = R\,dl$

$$dB = \frac{\mu_0}{4\pi}\frac{I\left|d\vec{l} \times \vec{r}\right|}{r^3} = \frac{\mu_0 I_1}{4\pi}\frac{R}{R^3}dl = \left(\frac{\mu_0 I_1}{4\pi R^2}\right)dl$$

$$B = \int dB = \left(\frac{\mu_0 I_1}{4\pi R^2}\right)\int dl = \left(\frac{\mu_0 I_1}{4\pi R^2}\right)(\pi R) = \frac{\mu_0 I_1}{4R}$$

(We used that $\int dl$ is equal to $\pi R,$ the length of wire in the semicircle.) We have shown that the two straight sections make zero contribution to $\vec{B},$ so $B_1 = \mu_0 I_1 / 4R$ and is directed into the page.

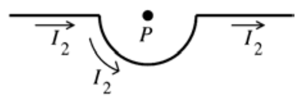

Figure 28.31e

For current in the direction shown in Figure 28.31e, a similar analysis gives $B_2 = \mu_0 I_2 / 4R,$ out of the paper

$\vec{B}_1$ and $\vec{B}_2$ are in opposite directions, so the magnitude of the net field at P is $B = |B_1 - B_2| = \dfrac{\mu_0 |I_1 - I_2|}{4R}.$

EVALUATE: When $I_1 = I_2,$ $B = 0.$

28.56. **IDENTIFY:** The net magnetic field is the vector sum of the fields due to each wire.

SET UP: $B = \dfrac{\mu_0 I}{2\pi r}.$ The direction of $\vec{B}$ is given by the right-hand rule.

EXECUTE: **(a)** The currents are the same so points where the two fields are equal in magnitude are equidistant from the two wires. The net field is zero along the dashed line shown in Figure 28.56a.
(b) For the magnitudes of the two fields to be the same at a point, the point must be 3 times closer to the wire with the smaller current. The net field is zero along the dashed line shown in Figure 28.56b.
(c) As in (a), the points are equidistant from both wires. The net field is zero along the dashed line shown in Figure 28.56c.

EVALUATE: The lines of zero net field consist of points at which the fields of the two wires have opposite directions and equal magnitudes.

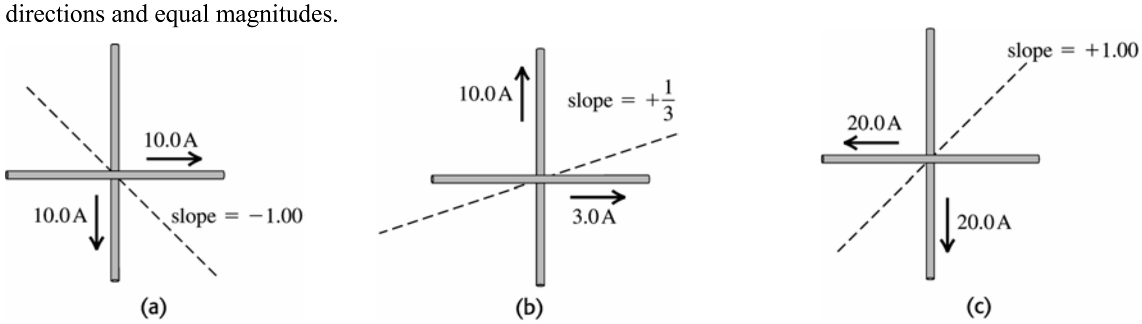

Figure 28.56

28.66. **IDENTIFY:** Apply $d\vec{B} = \dfrac{\mu_0}{4\pi}\dfrac{Id\vec{l} \times \hat{r}}{r^2}.$

SET UP: The two straight segments produce zero field at P. The field at the center of a circular loop of radius R is $B = \dfrac{\mu_0 I}{2R},$ so the field at the center of curvature of a semicircular loop is $B = \dfrac{\mu_0 I}{4R}.$

EXECUTE: The semicircular loop of radius a produces field out of the page at P and the semicircular loop of radius b produces field into the page. Therefore, $B = B_a - B_b = \dfrac{1}{2}\left(\dfrac{\mu_0 I}{2}\right)\left(\dfrac{1}{a} - \dfrac{1}{b}\right) = \dfrac{\mu_0 I}{4a}\left(1 - \dfrac{a}{b}\right),$ out of page.

EVALUATE: If $a = b,$ $B = 0.$

28.69. **IDENTIFY:** Apply $d\vec{B} = \dfrac{\mu_0}{4\pi}\dfrac{Id\vec{l} \times \hat{r}}{r^2}.$

SET UP: The contribution from the straight segments is zero since $d\vec{l} \times \vec{r} = 0.$ The magnetic field from the curved wire is just one quarter of a full loop.

EXECUTE: $B = \frac{1}{4}\left(\dfrac{\mu_0 I}{2R}\right) = \dfrac{\mu_0 I}{8R}$ and is directed out of the page.

EVALUATE: It is very simple to calculate B at point P but it would be much more difficult to calculate B at other points.

28.70. **IDENTIFY:** Apply $d\vec{B} = \dfrac{\mu_0}{4\pi}\dfrac{Id\vec{l} \times \hat{r}}{r^2}.$

SET UP: The horizontal wire yields zero magnetic field since $d\vec{l} \times \vec{r} = 0$. The vertical current provides the magnetic field of half of an infinite wire. (The contributions from all infinitesimal pieces of the wire point in the same direction, so there is no vector addition or components to worry about.)

EXECUTE: $B = \frac{1}{2}\left(\frac{\mu_0 I}{2\pi R}\right) = \frac{\mu_0 I}{4\pi R}$ and is directed out of the page.

EVALUATE: In the equation preceding Eq.(28.8) the limits on the integration are 0 to a rather than $-a$ to a and this introduces a factor of $\frac{1}{2}$ into the expression for B.

29.21. **IDENTIFY:** A conductor moving in a magnetic field may have a potential difference induced across it, depending on how it is moving.
SET UP: The induced emf is $\mathcal{E} = vBL \sin\phi$, where ϕ is the angle between the velocity and the magnetic field.
EXECUTE: **(a)** $\mathcal{E} = vBL \sin\phi = (5.00 \text{ m/s})(0.450 \text{ T})(0.300 \text{ m})(\sin 90°) = 0.675 \text{ V}$
(b) The positive charges are moved to end b, so b is at the higher potential.
(c) $E = V/L = (0.675 \text{ V})/(0.300 \text{ m}) = 2.25 \text{ V/m}$. The direction of $\vec{E}$ is from, b to a.
(d) The positive charge are pushed to b, so b has an excess of positive charge.
(e) (i) If the rod has no appreciable thickness, $L = 0$, so the emf is zero. (ii) The emf is zero because no magnetic force acts on the charges in the rod since it moves parallel to the magnetic field.
EVALUATE: The motional emf is large enough to have noticeable effects in some cases.

29.25. **IDENTIFY and SET UP:** $\mathcal{E} = vBL$. Use Lenz's law to determine the direction of the induced current. The force F_{ext} required to maintain constant speed is equal and opposite to the force F_I that the magnetic field exerts on the rod because of the current in the rod.
EXECUTE: **(a)** $\mathcal{E} = vBL = (7.50 \text{ m/s})(0.800 \text{ T})(0.500 \text{ m}) = 3.00 \text{ V}$

(b) $\vec{B}$ is into the page. The flux increases as the bar moves to the right, so the magnetic field of the induced current is out of the page inside the circuit. To produce magnetic field in this direction the induced current must be counterclockwise, so from b to a in the rod.

(c) $I = \frac{\mathcal{E}}{R} = \frac{3.00 \text{ V}}{1.50\ \Omega} = 2.00 \text{ A}$. $F_I = ILB\sin\phi = (2.00 \text{ A})(0.500 \text{ m})(0.800 \text{ T})\sin 90° = 0.800 \text{ N}$. $\vec{F}_I$ is to the left. To keep the bar moving to the right at constant speed an external force with magnitude $F_{ext} = 0.800$ N and directed to the right must be applied to the bar.
(d) The rate at which work is done by the force F_{ext} is $F_{ext}v = (0.800 \text{ N})(7.50 \text{ m/s}) = 6.00 \text{ W}$. The rate at which thermal energy is developed in the circuit is $I^2R = (2.00 \text{ A})(1.50\ \Omega) = 6.00 \text{ W}$. These two rates are equal, as is required by conservation of energy.
EVALUATE: The force on the rod due to the induced current is directed to oppose the motion of the rod. This agrees with Lenz's law.

29.26. **IDENTIFY:** Use Faraday's law to calculate the induced emf. Ohm's law applied to the loop gives I. Use Eq.(27.19) to calculate the force exerted on each side of the loop.
SET UP: The loop before it starts to enter the magnetic field region is sketched in Figure 29.26a.

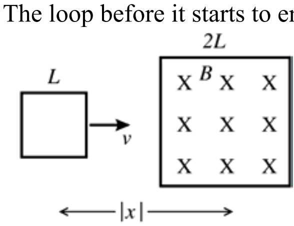

Figure 29.26a

EXECUTE: For $x < -3L/2$ or $x > 3L/2$ the loop is completely outside the field region. $\Phi_B = 0$, and $\frac{d\Phi_B}{dt} = 0$.

Thus $\mathcal{E} = 0$ and $I = 0$, so there is no force from the magnetic field and the external force F necessary to maintain constant velocity is zero.
SET UP: The loop when it is completely inside the field region is sketched in Figure 29.26b.

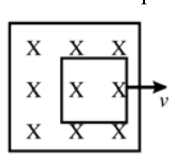

Figure 29.26b

EXECUTE: For $-L/2 < x < L/2$ the loop is completely inside the field region and $\Phi_B = BL^2$.

But $\frac{d\Phi_B}{dt} = 0$ so $\mathcal{E} = 0$ and $I = 0$. There is no force $\vec{F} = I\vec{l} \times \vec{B}$ from the magnetic field and the external force F necessary to maintain constant velocity is zero.

SET UP: The loop as it enters the magnetic field region is sketched in Figure 29.26c.

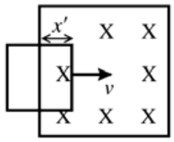

Figure 29.26c

EXECUTE: For $-3L/2 < x < -L/2$ the loop is entering the field region. Let x' be the length of the loop that is within the field.

Then $|\Phi_B| = BLx'$ and $\left|\frac{d\Phi_B}{dt}\right| = Blv.$ The magnitude of the induced emf is $|\mathcal{E}| = \left|\frac{d\Phi_B}{dt}\right| = BLv$ and the induced current is $I = \frac{|\mathcal{E}|}{R} = \frac{BLv}{R}.$ Direction of *I*: Let $\vec{A}$ be directed into the plane of the figure. Then Φ_B is positive. The flux is positive and increasing in magnitude, so $\frac{d\Phi_B}{dt}$ is positive. Then by Faraday's law $\mathcal{E}$ is negative, and with our choice for direction of $\vec{A}$ a negative $\mathcal{E}$ is counterclockwise. The current induced in the loop is counterclockwise.

SET UP: The induced current and magnetic force on the loop are shown in Figure 29.26d, for the situation where the loop is entering the field.

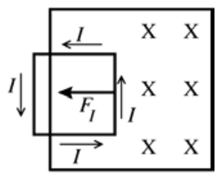

Figure 29.26d

EXECUTE: $\vec{F}_I = I\vec{l} \times \vec{B}$ gives that the force $\vec{F}_I$ exerted on the loop by the magnetic field is to the left and has magnitude $F_I = ILB = \left(\frac{BLv}{R}\right)LB = \frac{B^2L^2v}{R}.$

The external force $\vec{F}$ needed to move the loop at constant speed is equal in magnitude and opposite in direction to $\vec{F}_I$ so is to the right and has this same magnitude.

SET UP: The loop as it leaves the magnetic field region is sketched in Figure 29.26e.

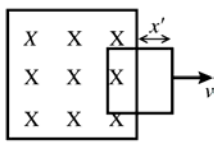

Figure 29.26e

EXECUTE: For $L/2 < x < 3L/2$ the loop is leaving the field region. Let x' be the length of the loop that is outside the field.

Then $|\Phi_B| = BL(L - x')$ and $\left|\frac{d\Phi_B}{dt}\right| = BLv.$ The magnitude of the induced emf is $|\mathcal{E}| = \left|\frac{d\Phi_B}{dt}\right| = BLv$ and the induced current is $I = \frac{|\mathcal{E}|}{R} = \frac{BLv}{R}.$ Direction of *I*: Again let $\vec{A}$ be directed into the plane of the figure. Then Φ_B is positive and decreasing in magnitude, so $\frac{d\Phi_B}{dt}$ is negative. Then by Faraday's law $\mathcal{E}$ is positive, and with our choice for direction of $\vec{A}$ a positive $\mathcal{E}$ is clockwise. The current induced in the loop is clockwise.

SET UP: The induced current and magnetic force on the loop are shown in Figure 29.26f, for the situation where the loop is leaving the field.

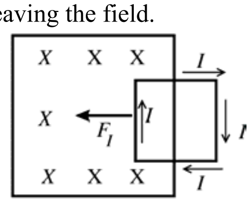

Figure 29.26f

EXECUTE: $\vec{F}_I = I\vec{l} \times \vec{B}$ gives that the force $\vec{F}_I$ exerted on the loop by the magnetic field is to the left and has magnitude $F_I = ILB = \left(\frac{BLv}{R}\right)LB = \frac{B^2L^2v}{R}.$

The external force $\vec{F}$ needed to move the loop at constant speed is equal in magnitude and opposite in direction to $\vec{F}_I$ so is to the right and has this same magnitude.

(a) The graph of *F* versus *x* is given in Figure 29.26g.

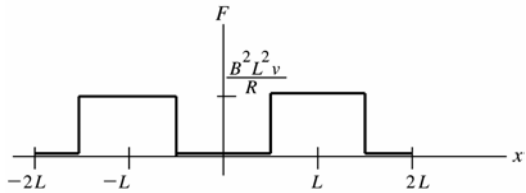

Figure 29.26g

(b) The graph of the induced current I versus x is given in Figure 29.26h.

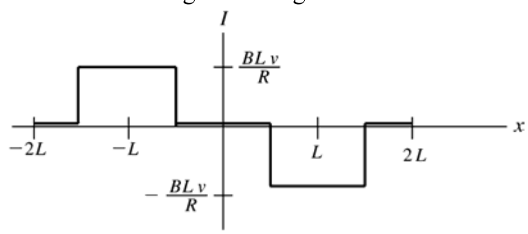

Figure 29.26h

EVALUATE: When the loop is either totally outside or totally inside the magnetic field region the flux isn't changing, there is no induced current, and no external force is needed for the loop to maintain constant speed. When the loop is entering the field the external force required is directed so as to pull the loop in and when the loop is leaving the field the external force required is directed so as to pull the loop out of the field. These directions agree with Lenz's law: the force on the induced current (opposite in direction to the required external force) is directed so as to oppose the loop entering or leaving the field.

29.56. **IDENTIFY:** Apply Newton's 2nd law to the bar. The bar will experience a magnetic force due to the induced current in the loop. Use $a = dv/dt$ to solve for v. At the terminal speed, $a = 0$.

SET UP: The induced emf in the loop has a magnitude BLv. The induced emf is counterclockwise, so it opposes the voltage of the battery, $\mathcal{E}$.

EXECUTE: **(a)** The net current in the loop is $I = \frac{\mathcal{E} - BLv}{R}$. The acceleration of the bar is $a = \frac{F}{m} = \frac{ILB\sin(90°)}{m} = \frac{(\mathcal{E} - BLv)LB}{mR}$. To find $v(t)$, set $\frac{dv}{dt} = a = \frac{(\mathcal{E} - BLv)LB}{mR}$ and solve for v using the method of separation of variables:

$$\int_0^v \frac{dv}{(\mathcal{E} - BLv)} = \int_0^t \frac{LB}{mR}dt \rightarrow v = \frac{\mathcal{E}}{BL}(1 - e^{-B^2L^2t/mR}) = (10 \text{ m/s})(1 - e^{-t/3.1\text{ s}})$$

The graph of v versus t is sketched in Figure 29.56. Note that the graph of this function is similar in appearance to that of a charging capacitor.

(b) Just after the switch is closed, $v = 0$ and $I = \mathcal{E}/R = 2.4$ A, $F = ILB = 2.88$ N and $a = F/m = 3.2 \text{ m/s}^2$.

(c) When $v = 2.0$ m/s, $a = \frac{[12 \text{ V} - (1.5 \text{ T})(0.8 \text{ m})(2.0 \text{ m/s})](0.8 \text{ m})(1.5 \text{ T})}{(0.90 \text{ kg})(5.0\ \Omega)} = 2.6 \text{ m/s}^2$.

(d) Note that as the speed increases, the acceleration decreases. The speed will asymptotically approach the terminal speed $\frac{\mathcal{E}}{BL} = \frac{12 \text{ V}}{(1.5 \text{ T})(0.8 \text{ m})} = 10$ m/s, which makes the acceleration zero.

EVALUATE: The current in the circuit is counterclockwise and the magnetic force on the bar is to the right. The energy that appears as kinetic energy of the moving bar is supplied by the battery.

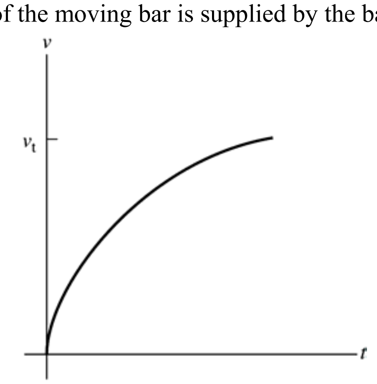

Figure 29.56

29.65. **(a) IDENTIFY:** Use Faraday's law to calculate the induced emf, Ohm's law to calculate I, and Eq.(27.19) to calculate the force on the rod due to the induced current.

SET UP: The force on the wire is shown in Figure 29.65.

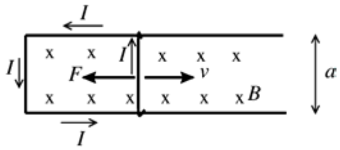

Figure 29.65

EXECUTE: When the wire has speed v the induced emf is $\mathcal{E} = Bva$ and the induced current is $I = \mathcal{E}/R = \dfrac{Bva}{R}$

The induced current flows upward in the wire as shown, so the force $\vec{F} = I\vec{l} \times \vec{B}$ exerted by the magnetic field on the induced current is to the left. $\vec{F}$ opposes the motion of the wire, as it must by Lenz's law. The magnitude of the force is $F = IaB = B^2a^2v/R$.

(b) Apply $\sum \vec{F} = m\vec{a}$ to the wire. Take $+x$ to be toward the right and let the origin be at the location of the wire at $t = 0$, so $x_0 = 0$.

$\sum F_x = ma_x$ says $-F = ma_x$

$$a_x = -\frac{F}{m} = -\frac{B^2a^2v}{mR}$$

Use this expression to solve for $v(t)$:

$$a_x = \frac{dv}{dt} = -\frac{B^2a^2v}{mR} \text{ and } \frac{dv}{v} = -\frac{B^2a^2}{mR}dt$$

$$\int_{v_0}^{v} \frac{dv'}{v'} = -\frac{B^2a^2}{mR}\int_0^t dt'$$

$$\ln(v) - \ln(v_0) = -\frac{B^2a^2t}{mR}$$

$$\ln\left(\frac{v}{v_0}\right) = -\frac{B^2a^2t}{mR} \text{ and } v = v_0 e^{-B^2a^2t/mR}$$

Note: At $t = 0$, $v = v_0$ and $v \to 0$ when $t \to \infty$

Now solve for $x(t)$:

$$v = \frac{dx}{dt} = v_0 e^{-B^2a^2t/mR} \text{ so } dx = v_0 e^{-B^2a^2t/mR}dt$$

$$\int_0^x dx' = \int_0^t v_0 e^{-B^2a^2t/mR}dt'$$

$$x = v_0\left(-\frac{mR}{B^2a^2}\right)\left[e^{-B^2a^2t'/mR}\right]_0^t = \frac{mRv_0}{B^2a^2}\left(1 - e^{-B^2a^2t/mR}\right)$$

Comes to rest implies $v = 0$. This happens when $t \to \infty$.

$t \to \infty$ gives $x = \dfrac{mRv_0}{B^2a^2}$. Thus this is the distance the wire travels before coming to rest.

EVALUATE: The motion of the slide wire causes an induced emf and current. The magnetic force on the induced current opposes the motion of the wire and eventually brings it to rest. The force and acceleration depend on v and are constant. If the acceleration were constant, not changing from its initial value of $a_x = -B^2a^2v_0/mR$, then the stopping distance would be $x = -v_0^2/2a_x = mRv_0/2B^2a^2$. The actual stopping distance is twice this.