\[ F_{\text{cm} \hat{z}_1} = F_{\hat{z}_2 \text{cm} \hat{z}_1} + F_{\hat{z}_3 \text{cm} \hat{z}_1} \]

\[ = \frac{1}{4\pi\varepsilon_0} \frac{\hat{z}_2 \cdot \hat{z}_1}{|\hat{z}_2|^2} \hat{V}_{\hat{z}_1} + \frac{1}{4\pi\varepsilon_0} \frac{\hat{z}_3 \cdot \hat{z}_1}{|\hat{z}_3|^2} \hat{V}_{\hat{z}_3} \]

\[ = \left( 9.0 \times 10^9 \text{ N m}^2/\text{C}^2 \right) \hat{i} \left( \frac{(4 \times 10^9 \text{ C})(5 \times 10^9 \text{ C})}{(0.5 \text{ m})^2} \right) \]

\[ + \left( \frac{(4 \times 10^9 \text{ C})(-6 \times 10^9 \text{ C})}{(0.2 \text{ m})^2} \right) \]

\[ = -4.68 \times 10^{-6} \hat{i} \text{ N} \]

\[ \vec{E}_{\text{at } \hat{z}_2} = \vec{E}_{\hat{z}_2 \text{ at } \hat{z}_2} + \vec{E}_{\hat{z}_3 \text{ at } \hat{z}_2} \]

\[ = \frac{1}{4\pi\varepsilon_0} \frac{\hat{z}_1}{|\hat{z}_2|^2} \hat{V}_{\hat{z}_2} + \frac{1}{4\pi\varepsilon_0} \frac{\hat{z}_3}{|\hat{z}_2|^2} \hat{V}_{\hat{z}_3} \]

\[ = \left( 9.0 \times 10^9 \text{ N m}^2/\text{C}^2 \right) \left( \frac{4 \times 10^9 \text{ C}}{(0.5 \text{ m})^2} \hat{i} + \frac{-6 \times 10^9 \text{ C}}{(0.3 \text{ m})^2} \hat{i} \right) \]
\[ U_{23} = \frac{1}{4\pi \varepsilon_0} \frac{z_1 \cdot z_3}{|V_{13}|} + \frac{1}{4\pi \varepsilon_0} \frac{z_2 \cdot z_3}{|V_{23}|} \]

\[ = \left( 9.0 \times 10^9 \text{Nm}^2/\text{C}^2 \right) \left( -6 \times 10^{-9} \right) \left( \frac{4 \times 10^{-9}}{0.2 \text{m}} + \frac{5 \times 10^{-9}}{0.3 \text{m}} \right) \]

\[ = -1.98 \times 10^{-6} \text{J} \]

\[ \hat{\text{i}} \]
2-(a) The charges on all three capacitors are the same. The total potential drop \( V_{ab} \) equals to the sum of the potential drops across all three capacitors. Thus

\[
V_{ab} = \frac{Q_1}{C_1} + \frac{Q_2}{C_2} + \frac{Q_3}{C_3}
\]

\[
= \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}
\]

\[
= Q \left( \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \right)
\]

\[
= \frac{Q}{2 \mu F}
\]

\[
\therefore Q_3 = Q = (2 \mu F) \cdot V_{ab} = 72 \mu C
\]

2-(b) \( U_{\text{series}} = U_1 + U_2 + U_3 = \frac{Q_1^2}{2C_1} + \frac{Q_2^2}{2C_2} + \frac{Q_3^2}{2C_3} \)

\[
= \frac{Q^2}{2} \left( \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \right)
\]

\[
= \frac{1}{4 \mu F} \left( 72 \mu C \right)^2 = 1.296 \times 10^{-3} \text{ J}
\]
2-(c) The potential difference across the three capacitors (now in parallel) is the same, i.e., \( V_{a'b'} \).

The new potential difference \( V_{a'b'} \) is related to the total charge on all the capacitors:
\[
\frac{Q'_{1} + Q'_{2} + Q'_{3}}{C_{123}} = \frac{3Q}{3C} = \frac{216 \mu C}{20 \mu F} = 10.8 \, V
\]
3-(a) In order for the entire system to have no net charge, the insulating shell must carry a total charge of $-Q$.

Since the volume of the insulating shell is simply

$$\frac{4}{3} \pi \left( (2R)^3 - (R)^3 \right) = \frac{28}{3} \pi R^3$$

the value of $\delta$ must be

$$\delta = \frac{\frac{-Q}{\frac{28}{3} \pi R^3}}{\frac{3Q}{28 \pi R^3}} = -\frac{3Q}{28 \pi R^3}$$

3-(b) Inside the solid conducting sphere,

$$\vec{E}(\mid \vec{r} \mid < R) = 0$$

Outside the insulating shell,

$$\vec{E}(\mid \vec{r} \mid > 2R) = \frac{1}{4\pi \varepsilon_0} \frac{Q + (-Q)}{r^2} \hat{r} = 0$$

Inside the insulating shell,

$$\vec{E}(R < \mid \vec{r} \mid < 2R) = \frac{1}{4\pi \varepsilon_0} \frac{\hat{r}}{r^2} \left( Q + \delta \frac{4}{3} \pi (R^3 - R^3) \right)$$
\[ \frac{Q}{4\pi\varepsilon_0} \frac{\hat{y}}{r^2} \left( 1 - \frac{y^3 - R^3}{7R^3} \right) \]

\[ = \frac{Q}{4\pi\varepsilon_0} \hat{y} \left( \frac{8}{7} \frac{1}{r^2} - \frac{y}{7R^3} \right) \]

3-(c) The potential of the conducting sphere referenced to infinity is the line integration of the electric field from the surface of the solid sphere to infinity.

\[ V_{\text{solid}} = \int_{R}^{2R} \vec{E} \cdot d\vec{l} = \int_{R}^{2R} \vec{E} \cdot d\vec{l} + \int_{2R}^{\infty} \vec{E} \cdot d\vec{l} \]

\[ = \frac{Q}{4\pi\varepsilon_0} \int_{R}^{2R} \left( \frac{8}{7} \frac{1}{r^2} - \frac{y}{7R^3} \right) dv \]

\[ = \frac{Q}{4\pi\varepsilon_0 \cdot R} \left( \frac{5}{14} \right) \]

\[ \text{**} \]
\textbf{4 - (a)}

\[ \text{\vec{F}}_{cm} \text{\vec{AB}} \]

\[ = I \text{\vec{AB}} \times \vec{B} \]

\[ = (4A)(0.2\text{m})(\hat{i} + \hat{j}) \times (0.5\text{T}\hat{k}) \]

\[ = (4A)(0.2\text{m})(0.5\text{T}) \hat{k} \]

\[ = 0.4 \text{ N} \hat{k} \]

\[ \text{\vec{F}}_{cm} \text{\vec{BC}} \]

\[ = I \text{\vec{BC}} \times \vec{B} \]

\[ = (4A)(0.2\text{m})(0.5\text{T}) \]

\[ \cdot (\hat{i} - \hat{j}) \times (\hat{j}) \]

\[ = 0.4 \text{ N} \hat{k} \]

\begin{align*}
\vec{AB} &= 0.2\text{m} \hat{i} + 0.2\text{m} \hat{j} \\
\vec{BC} &= 0.2\text{m} \hat{i} - 0.2\text{m} \hat{j} \\
\vec{CA} &= 0.4\text{m} (-\hat{i}) \\
\vec{B} &= 0.5\text{T} \hat{j}
\end{align*}

\[ \text{\vec{F}}_{cm} \text{\vec{CA}} = I \text{\vec{CA}} \times \vec{B} = (4A)(0.4\text{m})(0.5\text{T})(-\hat{i}) \times \hat{j} \]

\[ = -0.8 \text{ N} \hat{k} \]
If the wire loop is pivoted about CA (the base of the triangle), the force on CA will not exert a torque on the loop.

The torques due the forces on $\overrightarrow{AB}$ and $\overrightarrow{BC}$ can be evaluated by choosing the center of the base as the origin, and the position vectors of the center of $\overrightarrow{AB}$ and $\overrightarrow{BC}$ segments are designated as $\overrightarrow{r_{AB}}$ and $\overrightarrow{r_{BC}}$.

Then the torque on $\overrightarrow{AB}$ segment is

$$\overrightarrow{\tau_{AB}} = \overrightarrow{r_{AB}} \times \overrightarrow{F_{on \ AB}} = (0.1 \text{ m})(-\hat{i} + \hat{j}) \times (0.4 \text{ N})\hat{k}$$

$$= 0.04 \text{ N.m}(\hat{i} + \hat{j})$$

Similarly, the torque on $\overrightarrow{BC}$ segment is

$$\overrightarrow{\tau_{BC}} = \overrightarrow{r_{BC}} \times \overrightarrow{F_{on \ BC}} = (0.1 \text{ m})(0.4 \text{ N})(\hat{i} + \hat{j}) \times \hat{k}$$
\[ \vec{r}_{\beta c} = 0.04 \text{ N.m} \ (\hat{i} - \hat{j}) \]

The total torque on the loop is then

\[ \vec{T}_{\text{on loop}} = \vec{T}_{\text{A13}} + \vec{T}_{\beta c} \]

\[ = 0.08 \text{ N.m} \]

\[ \times \]

4-(c) Treating the wire loop as a magnetic dipole, the dipole moment is

\[ \vec{\mu} = I \cdot A \cdot (-\hat{k}) \]
\[ = (4A)(0.4 \text{ m})(0.2 \text{ m}) \cdot \frac{1}{2} (-\hat{k}) \]
\[ = 0.16 \text{ A.m}^2 (-\hat{k}) \]

The magnetic torque on the loop is then

\[ \vec{T} = \vec{\mu} \times \vec{B} = (0.16 \text{ A.m}^2)(0.5 \text{ T}) (-\hat{k}) \times \hat{j} \]
\[ = 0.08 \text{ N.m} \ \hat{i} \quad \text{(Same as Part (b))} \]
5-(a) \[ R_{ab} = \frac{1}{\frac{1}{10\Omega} + \frac{1}{20\Omega} + \frac{1}{60\Omega} + \frac{1}{60\Omega} + \frac{1}{30\Omega}} \]

\[ = 10\Omega \]

5-(b) Since \( V_{ab} = I \cdot R_{ab} = (2A) \cdot (10\Omega) = 20 \text{ volts} \),
the current through 10 Ω and 20 Ω resistors connected in series is

\[ I_{10\Omega} = I_{20\Omega} = \frac{V_{ab}}{10\Omega + 20\Omega} = \frac{2}{3} A \]

Thus, the power dissipated in 20 Ω is

\[ P_{20\Omega} = (I_{20\Omega})^2 \cdot (20 \Omega) = \left(\frac{2}{3} A\right)^2 \cdot (20 \Omega) \]

\[ = 8.89 \text{ watts} \]

5-(c) The equivalent circuit is now reduced to

The circuit diagram is as follows:

\[ I = 2A \]

\[ V = 5V \]

\[ I_1 = \frac{5V}{15\Omega + 5\Omega} \]

The diagram shows a network of resistors and voltages with currents flowing through them.
Loop #1: e→a→b→d→e (clockwise)

\[-5V + (2A)(10Ω) + 10V + (2A)(5Ω) - 35I_1 = 0\]

\[35I_1 = (-5V) + 20V + 10V + 10V = 35V\]

\[I_1 = 1A\]

Loop #2: e→d→f→e (clockwise)

\[(35Ω)(1A) - \varepsilon + (2A + 1A)(15Ω) = 0\]

\[\varepsilon = (35Ω)(1A) + (3A)(15Ω) = 80V\]
6-(a) The magnetic field in the region of the rectangular loop points into the paper, i.e., along \(-\hat{\mathbf{k}}\).

On the \(AB\) segment, the magnetic field is
\[
\vec{B}_{AB} = (-\hat{\mathbf{k}}) \frac{\mu_0 \mathbf{I}}{2\pi r}
\]
the motional emf is from \(A\) to \(B\),
\[
\mathcal{E}_{AB} = \int_{A}^{B} (\vec{v} \times \vec{B}) \cdot d\vec{l} = \frac{\mu_0 I}{2\pi r} \mathbf{v} \cdot \mathbf{b}
\]

On the \(CD\) segment, the magnetic field is
\[
\vec{B}_{CD} = (-\hat{\mathbf{k}}) \frac{\mu_0 \mathbf{I}}{2\pi (r+a)}
\]
the motional emf is from \(D\) to \(C\)
\[
\mathcal{E}_{DC} = \int_{D}^{C} (\vec{v} \times \vec{B}) \cdot d\vec{l} = \frac{\mu_0 I}{2\pi (r+a)} \mathbf{v} \cdot \mathbf{b}
\]
If we designate the loop direction as \( A \rightarrow B \rightarrow C \rightarrow D \) (clockwise), then the total induced motional emf is

\[
\mathcal{E} = \mathcal{E}_{AB} + \mathcal{E}_{DC} = \mathcal{E}_{AB} - \mathcal{E}_{DC}
\]

\[
= \frac{M_0 I v s}{2\pi} \left( \frac{1}{r} - \frac{1}{r+a} \right) > 0
\]

6.(6) Choose the surface normal as the unit vector into the paper \((-\hat{n})\) so that the loop direction \( A \rightarrow B \rightarrow C \rightarrow D \) (clockwise) and \((-\hat{n})\) obey the right-hand rule.

Then the magnetic flux through the flat surface that covers the rectangular loop is

\[
\Phi_B = \int \mathbf{B} \cdot d\mathbf{A} = \int_0^b \int_0^a \frac{M_0 I}{2\pi (v+a)} dv\,dx
\]

\[
= \frac{M_0 I b}{2\pi} \ln \frac{r+a}{r}
\]
As the wire loop moves to the right at a velocity \( \vec{v} = \vec{V} = (dv/dt) \hat{\mathbf{x}} \), the induced current along the loop direction (clockwise) can be calculated using Faraday's induction law:

\[
\mathbf{E}_{ABCD} = -\frac{d}{dt} \Phi_B
\]

\[
= -\frac{d}{dt} \left( \frac{M_0 I_0}{2 \pi} \frac{\ln \frac{r+a}{r}}{r} \right)
\]

\[
= \frac{M_0 I_0}{2 \pi} \left( \frac{1}{r} - \frac{1}{r+a} \right) \frac{dr}{dt}
\]

\[
= \frac{M_0 I_0 \cdot V}{2 \pi} \left( \frac{1}{r} - \frac{1}{r+a} \right)
\]

The same as found in Part (a)
6-cc) The induced emf produces a current in the wire loop ABCD along the direction of the total emf (clockwise)

\[ I_{ABCD} = \frac{E_{ABCD}}{R} = \frac{\mu_0 I \cdot b \cdot R}{2\pi r} \frac{a}{V(V+\alpha)} \]

The AB segment experiences a magnetic force

\[ \vec{F}_{\text{on AB}} = I_{ABCD} \cdot \vec{AB} \times \vec{B}_{AB} \]

\[ = I_{ABCD} \cdot b \cdot \frac{\mu_0 I}{2\pi r} \left( \hat{j} \times (-\hat{k}) \right) \]

\[ = I_{ABCD} \cdot \frac{\mu_0 I \cdot b}{2\pi r} \left( -\hat{i} \right) \]

The CD segment also experiences a magnetic force

\[ \vec{F}_{\text{on CD}} = I_{ABCD} \cdot \vec{CD} \times \vec{B}_{CD} \]

\[ = I_{ABCD} \cdot \frac{\mu_0 I}{2\pi (V+\alpha)} \left( (-\hat{i}) \times (-\hat{k}) \right) \]

\[ = I_{ABCD} \cdot \frac{\mu_0 I \cdot b}{2\pi (V+\alpha)} \cdot \hat{i} \]
So the total force on the wire loop is

\[ \vec{F}_{\text{on loop}} = \vec{F}_{\text{on AB}} + \vec{F}_{\text{on CD}} \]

(theforces on BC and DA cancel each other)

\[ = \int_{ABC0} \frac{\mu_0 I b}{2\pi} \left( \frac{1}{r} - \frac{1}{r + a} \right) \hat{r} \]  

\[ = \int_{ABC0} \frac{\mu_0 I b - a}{2\pi r (r + a)} \hat{r} \]

\[ = \frac{\mu_0 I a b}{R} \left( \frac{2a}{2\pi (r + a) r} \right)^2 \hat{r} \]

*
\[ \vec{B}_p = \vec{B}_p(8A) + \vec{B}_p(20A) + \vec{B}_p(10A) \]

\[ = \frac{\mu_0}{2\pi (0.1m)} \left( -10A - 8A + 20A \right) \hat{z} \]

\[ = \left( 2 \times 10^{-7} \text{T m/A} \right) \cdot \frac{2A}{0.1m} \cdot \hat{z} \]

\[ = 4 \times 10^{-6} \text{T} \hat{z} \]

7-(c) The current I must produce a magnetic field into the paper (i.e., along \(- \hat{k}\)), thus it needs to flow downward. In this case, \[ \vec{B}_p = \frac{\mu_0}{2\pi (0.1m)} \left( -10A + 20A - 8A - I \right) \hat{z} = 0 \]
7-(c) Only in regions (I) and (IV) that the magnetic fields from the two wire wires can cancel each to yield a zero magnetic field.

In region (I) let \( \{x, y(x)\} \) be the points that \( B(x, y) = 0 \). Since

\[
B(x, y) = \frac{M_0}{2\pi} \hat{z} \left( -\frac{10}{x} + \frac{20}{y(x)} \right) = 0
\]

Thus \( y(x) = 2x \), namely, all the points in region (I) with \( \{x, y\} = \{x, 2x\} \) have zero magnetic field. Or on a straight line in region (I) given by \( y(x) = 2x \), the magnetic field is zero.
In region (III), again let \( \{x, y(x)\} \) be the points that \( B(x, y) = 0 \). Since

\[
\overrightarrow{B}(x, y) = \frac{\mu_0}{2\pi} \hat{z} \left( \frac{10}{|x|} - \frac{20}{|y|} \right) = 0
\]

Thus \( |y(x)| = |x| \cdot 2 \). Since \( |y(x)| = -y \), \( |x| = -x \). Therefore we have

\[ y(x) = 2x \]

This means that again all the points in region III with \( \{x, y\} = \{x, 2x\} \) have zero magnetic field. These points form a straight line given by

\[ y(x) = 2x \] in region (III)