1-(a) The electric force on the small sphere centers from the positively charged sheet $\Sigma$. The sheet produces a uniform electric field where the sphere resides.

$$\vec{E}_\Sigma = \frac{\sigma}{2\varepsilon_0} \hat{x}$$

Thus,

$$\vec{F}_{\text{on sphere}} = \vec{F}_{\text{on sphere}} = q \vec{E}_\Sigma = \frac{q\sigma}{2\varepsilon_0} \hat{x}$$

1-(b) The sphere also experiences a gravitational force due to its mass

$$\vec{F}_{\text{on sphere}} = mg \; (\uparrow \hat{y})$$

and the tension along the fiber $T$. In equilibrium,

$$\vec{T} + \vec{F}_{\text{on sphere}} + \vec{F}_{\text{on sphere}} = 0$$

$$\therefore \quad T \cos \theta = |\vec{F}_{\text{on sphere}}| = mg$$

$$T \sin \theta = |\vec{F}_{\text{on sphere}}| = \frac{q\sigma}{2\varepsilon_0} \quad \Rightarrow \quad tan \theta = \frac{75}{2mg \varepsilon_0}$$
2- (a) The charges on all four surfaces should uniformly distributed over the respective surfaces. Let the charges be \( Q_a, Q_b, Q_c, Q_d \). From charge conservation
\[
Q_a + Q_b = +2Q
\]
\[
Q_c + Q_d = +4Q
\]

Now, since the electric field in the region (inside the small conducting shell) \( a < r < b \) is effectively only contributed by \( Q_a \)
\[
\vec{E}(\vec{r}, a < r < b) = \frac{1}{4\pi\varepsilon_0} \frac{Q_a}{r^2} \hat{r}
\]
and must be zero, we arrive at
\[
Q_a = 0
\]

Thus \( Q_b = +2Q \)

Now since the electric field in the region \( c < r < d \) (inside the large conducting shell) is effectively only contributed by \( Q_b = +2Q \) and \( Q_c \), and must be zero,
\[
\vec{E}(\vec{r}, c < r < d) = \frac{1}{4\pi\varepsilon_0} \frac{Q_b + Q_c}{r^2} = 0
\]
\[
\Rightarrow Q_c = -Q_b = -2Q; \quad Q_d = 4Q - Q_c = +6Q
\]
2-(b) For \( r < a \), \( \vec{E}(\vec{r}) = 0 \).

For \( b < r < c \), the electric field is effectively contributed by \( \Phi_b = +2\Phi \) only,

\[
\vec{E}(\vec{r}, b < r < c) = \frac{1}{4\pi\varepsilon_0} \frac{2\Phi}{|\vec{r}|^2} \hat{r}
\]

2-(c) The electric potential at \( r = d \) relative to the infinity equals to the line integral of the electric field from \( r = d \) to \( r = +\infty \). Since the electric field in the region of \( d < r < +\infty \) is effectively contributed only by \( \Phi_d = +6\Phi \), and

\[
\vec{E}(\vec{r}, d < r < +\infty) = \frac{1}{4\pi\varepsilon_0} \frac{r}{|\vec{r}|^3} (6\Phi)
\]

\[
V(r=d) - V(r=+\infty) = \left( \int \vec{E} \cdot d\vec{l} \right)_{d} = \frac{1}{4\pi\varepsilon_0} \frac{6\Phi}{d}
\]

\*
3-(a) The electric potential at point a relative to infinity equals to the line integral of the total electric field produced by the two charged spheres from a to infinity. We can choose the path of the line integral always outside the two charged spheres. In this case the electric fields produced by these two charged spheres are the same as by two point charges $q_1 = +7.5\ \mu C$ placed at the center of the positively charged sphere, and $q_2 = -7.5\ \mu C$ placed at the center of the negatively charged sphere.

As a result

$$V_a = V_a^{(+) + V_a^{(-)}} = \frac{1}{4\pi \varepsilon_0} \frac{q_1}{R} + \frac{1}{4\pi \varepsilon_0} \frac{q_2}{L-R}$$

$$= \left(9 \times 10^9 \text{ N m}^2/\text{C}^2\right) \left(75 \times 10^{-6} \text{ C}\right) \left(\frac{1}{0.25 \text{ m}} - \frac{1}{0.75 \text{ m}}\right)$$

$$= 1.8 \times 10^5 \text{ V}.$$
3. (c) By symmetry or similar calculation,

\[ V_y = V_{5\text{, relative to infinity}} \]

\[ V_5 = V_{(+1)\text{, relative to infinity}} - V_{(-1)\text{, relative to infinity}} \]

As a result,

\[ V_5 = -1.8 \times 10^{-5} V \]

\[ V_c - V_y = 3.6 \times 10^{-5} V \]
4-(a) The 3 outer $C_1$ capacitors are in series, and thus their equivalence capacitance is

$$C_{eq} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_1} + \frac{1}{C_1}} = \frac{C_1}{3} = 2.3 \mu F.$$ 

The net capacitance between $e$ and $d$ is that of $C_2$ and $C_{eq}$ in parallel

$$C_{ed} = C_2 + C_{eq} = 4.6 \mu F + 2.3 \mu F = 6.9 \mu F.$$ 

The equivalent capacitance along the segment of $ecd$ is given by

$$C_{ef} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_{ed}} + \frac{1}{C_1}} = \frac{C_1}{3} = 2.3 \mu F.$$ 

The net capacitance between $e$ and $f$ is that of $C_2$ (between $e$ and $f$) and $C_{ef}$ in parallel

$$C_{ef} = C_2 + C_{ef} = 6.9 \mu F.$$ 

Finally, the network capacitance between $a$ and $b$ is

$$C_{ab} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_{ef}} + \frac{1}{C_1}} = \frac{C_1}{3} = 2.3 \mu F.$$
The charges on \( C_1 \), between \( a \) and \( f \) and \( C_2 \), are the same as flat on the wire.

\[ Q_{ab} = (420 \text{ V}) \cdot 6.9 \mu \text{F} = 966 \mu \text{C} \]

Thus,

\[ Q_{ab} \text{ on } C_1 \text{ (flat)} = 966 \mu \text{C} \]

And,

\[ Q_{ab} \text{ on } C_2 \text{ (flat)} = 966 \mu \text{C} \]

This means flat the potential difference between \( e \) and \( f \) is

\[ V_{ef} = \frac{Q_{ab}}{C_2} = \frac{966 \mu \text{C}}{6.9 \mu \text{F}} = 140 \text{ V} \]

Thus the charge on \( C_2 \) between \( e \) and \( f \) is

\[ Q_{ef} = V_{ef} \cdot C_2 = (140 \text{ V}) (4.6 \mu \text{F}) = 644 \mu \text{C} \]
4 - (c) Since $V_{ef} = 140V$, and $C_1 = C_{ad} = 6.9\mu F$.

$V_{ef}$ is equally distributed over $C_1$ and $C_2$.

Thus, $V_{ad} = V_{en} + V_{cf} = \frac{1}{3} V_{ef} = 46.7V$.
5-(a) Immediately after the switch is closed, the bar is a still resistor with \[ R = 6 \Omega. \] This makes the equivalent resistance between \( a \) and \( b \)

\[
R_{eq} = \frac{1}{\frac{1}{R} + \frac{1}{R_1}} = 3\Omega
\]

Thus the total current through \( R_{ab} \) is the same as that through the battery

\[
I_{eq} = \frac{36V}{R + R_{eq}} = \frac{36V}{9\Omega} = 4A,
\]

and goes from \( a \) to \( b \) (top to bottom).

The current through the bar is one half of \( I_{ab} \), and also flows from top to bottom.

\[
I_{bar} = \frac{I_{eq}}{2} = 2A.
\]

5-(b) Since \( I_{bar} \) flows from the top to the bottom along the bar of \( L \),

\[
F_{\text{bar}} = I_{bar}(L \cdot 9) \times B
\]

\[
= (2A)(9.5m)(2T)(9 \times 10^{-2})
\]
\( F_{\text{on bar}} = 6.0 \text{ N } \hat{x} \) (to the right)

5. (c) Under \( F_{\text{on bar}} \), the bar is accelerated towards the right (\( \hat{x} \)), and gains a velocity \( \vec{v} = v \hat{x} \).

As a result of the bar being moving with a velocity \( \vec{v} \) in the same magnetic field \( \vec{B} = B \hat{z} \), an emf is produced inside the bar that points upward (from bottom to top), with a magnitude \( E_{\text{induce}} = BLv \). This emf reduces the current that flows through the bar.

A terminal velocity is reached, \( v_t = v_t \hat{x} \), when the net emf \( BLv_t \) equals the potential drop between a and b such that no net current flows through the bar (and the acceleration stops as well). In this case, the current produced by \( E = 36V \), only flows through \( R_1 \) and \( R_2 \),

\[
I = \frac{E}{R_1 + R_2} = \frac{36V}{12\Omega} = 3A
\]

Thus the potential difference between a and b
\[ V_{as} = I \cdot R_1 = 18 \text{ V} \]

and it equals to the "terminal" metic \( BLV_t \)

\[ V_t = \frac{V_{as}}{\beta L} = \frac{18 \text{ V}}{(2T)(1.5\text{ m})} = 6 \text{ m/sec} \]

**t = 0:**
\[ \Sigma = 36 \text{ V} \]

**t = \infty:**
\[ \Sigma = 36 \text{ V} \]

\[ E_{\text{induced}} = BLV_t = V_a - V_b = I_{as} \cdot R_1 \]
6-(a). Assign the currents as shown in the figure.
Along loop #1 (C1).
\[ 0 \quad 10I - 24 + 12I_1 = 0 \]
\[ I_1 = 12 \]
Along loop #2 (C2).
\[ 0 \quad -12I_1 + (I - I_1)10 + 12 = 0 \]
\[ I_1 = 6 \]
\[ I_1 - I_1 = 17I_1 = 18 \quad I = \frac{18}{17} \text{ A (as assigned)} \]
\[ 11 \times 0 + 6 \times 0 \quad 85I = 96 \quad I = \frac{96}{85} \text{ A (as assigned)} \]
\[ I - I_1 = \frac{6}{85} \text{ A (as assigned)} \]
The current through 12V battery is \( \frac{6}{85} \) A in the direction as assigned to \( I - I_1 \).
6 (c) The power dissipation is

\[ P(120) = I_1^2 (120) = 13.45 \text{ Watt} \]
The magnetic field produced by the section L is given by

\[ B_x = 0, \]

\[ B_y = \frac{\mu_0 I dl \times x}{\pi}, \]

\[ B_z = \frac{\mu_0 I dl}{\pi} \times \frac{(-a)}{y}. \]

Thus, \( B \times dS = \mu_0 I dl \times \frac{(-a)}{y} \),

\[ \int B \cdot dS = \mu_0 I dl \times \int \frac{(-a)}{y} \, dS. \]

The magnetic field is given by the section L is shown in the diagram.
\[ R_2 = \int \frac{\mu_0 I}{4\pi} \frac{\mathbf{l} d\mathbf{l}_2 \times \hat{\mathbf{v}}_2}{|\mathbf{v}_2|^2} \]  

(4.2)  

\[ = \frac{\mu_0 I}{4\pi R^2} \int d\mathbf{l}_2 \times \hat{\mathbf{v}}_2 \]  

(4.2)  

\[ = \frac{\mu_0 I}{4\pi R^2} \int d\mathbf{l}_2 (-\hat{\mathbf{r}}) \]  

(4.2)  

\[ = \frac{\mu_0 I}{4\pi R^2} (-\hat{\mathbf{r}}) \frac{2\pi R}{4} \]  

\[ = -\frac{\mu_0 I}{8R} \hat{\mathbf{r}} \]
8 - (a) Left $B = -BR$

Orientations:
1. $w_1 = NIA A$, $z_1 = m x B = 0$
2. $w_2 = NIA A$, $z_2 = m x B = -NIA A x$
3. $w_3 = NIA A$, $z_3 = m x B = NIA B x$

8 - (b) With $B = -BR$, $U = -w x B$

Orientations:
1. $w_4 = NIA A$, $U_{(w)} = -w x B = NIA A x$
2. $w_5 = NIA A$, $U_{(w)} = -w x B = NIA A x$
3. $w_6 = NIA A$, $U_{(w)} = -w x B = NIA B x$