26.43. **IDENTIFY:** The capacitors, which are in parallel, will discharge exponentially through the resistors. **SET UP:** Since $V$ is proportional to $Q$, $V$ must obey the same exponential equation as $Q$.

\[ V = V_0 e^{-t/RC} \]

The current is \( I = (V_0/R) e^{-t/RC} \).

**EXECUTE:** (a) Solve for time when the potential across each capacitor is 10.0 V:

\[ t = -RC \ln(V/V_0) = -(80.0 \, \Omega)(35.0 \, \mu F) \ln(10/45) = 4210 \, \mu s = 4.21 \, ms \]

(b) \( I = (V_0/R) e^{-t/RC} \). Using the above values, with \( V_0 = 45.0 \, V \), gives \( I = 0.125 \, A \).

**EVALUATE:** Since the current and the potential both obey the same exponential equation, they are both reduced by the same factor (0.222) in 4.21 ms.

26.74. **IDENTIFY** and **SET UP:** Just after the switch is closed, the charge on the capacitor is zero, the voltage across the capacitor is zero and the capacitor can be replaced by a wire in analyzing the circuit. After a long time the current to the capacitor is zero, so the current through \( R_3 \) is zero. After a long time the capacitor can be replaced by a break in the circuit.

**EXECUTE:** (a) Ignoring the capacitor for the moment, the equivalent resistance of the two parallel resistors is

\[ \frac{1}{R_{eq}} = \frac{1}{6.00 \, \Omega} + \frac{1}{3.00 \, \Omega} = \frac{3}{6.00 \, \Omega} \]

\( R_{eq} = 2.00 \, \Omega \). In the absence of the capacitor, the total current in the circuit (the current through the 8.00-\( \Omega \) resistor) would be

\[ i = \frac{\varepsilon}{R} = \frac{42.0 \, V}{8.00 \, \Omega + 2.00 \, \Omega} = 4.20 \, A \]

Of which 2/3, or 2.80 A, would go through the 3.00-\( \Omega \) resistor and 1/3, or 1.40 A, would go through the 6.00-\( \Omega \) resistor. Since the current through the capacitor is given by

\[ i = \frac{V}{R} e^{-t/RC} \]

at the instant \( t = 0 \) the circuit behaves as through the capacitor were not present, so the currents through the various resistors are as calculated above.

(b) Once the capacitor is fully charged, no current flows through that part of the circuit. The 8.00-\( \Omega \) and the 6.00-\( \Omega \) resistors are now in series, and the current through them is \( i = \varepsilon/R = (42.0 \, V)/(8.00 \, \Omega + 6.00 \, \Omega) = 3.00 \, A \). The voltage drop across both the 6.00-\( \Omega \) resistor and the capacitor is thus

\[ V = iR = (3.00 \, A)(6.00 \, \Omega) = 18.0 \, V \]

(There is no current through the 3.00-\( \Omega \) resistor and no voltage drop across it.)

The charge on the capacitor is

\[ Q = CV = (4.00 \times 10^{-6} \, F)(18.0 \, V) = 7.2 \times 10^{-5} \, C \]

**EVALUATE:** The equivalent resistance of \( R_2 \) and \( R_3 \) in parallel is less than \( R_1 \), so initially the current through \( R_1 \) is larger than its value after a long time has elapsed.

26.78. (a) **IDENTIFY:** With \( S \) open and after equilibrium has been reached, no current flows and the voltage across each capacitor is 18.0 V. When \( S \) is closed, current \( I \) flows through the 6.00-\( \Omega \) and 3.00-\( \Omega \) resistors.

**SET UP:** With the switch closed, \( a \) and \( b \) are at the same potential and the voltage across the 6.00-\( \Omega \) resistor equals the voltage across the 6.00-\( \mu F \) capacitor and the voltage is the same across the 3.00-\( \mu F \) capacitor and 3.00-\( \Omega \) resistor.

**EXECUTE:** (a) With an open switch: \( V_{ab} = \varepsilon = 18.0 \, V \).

(b) Point \( a \) is at a higher potential since it is directly connected to the positive terminal of the battery.

(c) When the switch is closed 18.0 V = \( I(6.00 \, \Omega + 3.00 \, \Omega) \). \( I = 2.00 \, A \) and

\[ V'_a = (2.00 \, A)(3.00 \, \Omega) = 6.00 \, V \]

(d) Initially the capacitor’s charges were \( Q_5 = CV = (3.00 \times 10^{-6} \, F)(18.0 \, V) = 5.40 \times 10^{-5} \, C \)

and \( Q_6 = CV = (6.00 \times 10^{-6} \, F)(18.0 \, V) = 1.08 \times 10^{-4} \, C \). After the switch is closed

\( Q_5 = CV = (3.00 \times 10^{-6} \, F)(18.0 \, V - 12.0 \, V) = 1.80 \times 10^{-5} \, C \)

and \( Q_6 = CV = (6.00 \times 10^{-6} \, F)(18.0 \, V - 6.0 \, V) = 7.20 \times 10^{-5} \, C \). Both capacitors lose \( 3.60 \times 10^{-5} \, C \).

**EVALUATE:** The voltage across each capacitor decreases when the switch is closed, because there is then current through each resistor and therefore a potential drop across each resistor.
27.1. **IDENTIFY and SET UP:** Apply Eq. (27.2) to calculate $\vec{F}$. Use the cross products of unit vectors from Section 1.10.

**EXECUTE:**

$\vec{v} = (+4.19 \times 10^4 \text{ m/s})\hat{i} + (-3.85 \times 10^4 \text{ m/s})\hat{j}$

(a) $\vec{B} = (1.40 \text{ T})\hat{k}$

$\vec{F} = q\vec{v} \times \vec{B} = (-1.24 \times 10^{-6} \text{ C})(1.40 \text{ T})[(+4.19 \times 10^4 \text{ m/s})\hat{i} \times \hat{i} - (-3.85 \times 10^4 \text{ m/s})\hat{j} \times \hat{i}]$

$\hat{i} \times \hat{i} = 0$, $\hat{j} \times \hat{i} = -\hat{k}$

$\vec{F} = (-1.24 \times 10^{-6} \text{ C})(1.40 \text{ T})(-3.85 \times 10^4 \text{ m/s})(-\hat{k}) = (6.68 \times 10^{-4} \text{ N})\hat{k}$

**EVALUATE:** The directions of $\vec{v}$ and $\vec{B}$ are shown in Figure 27.1a.

The right-hand rule gives that $\vec{v} \times \vec{B}$ is directed out of the paper ($+z$-direction).

The charge is negative so $\vec{F}$ is opposite to $\vec{v} \times \vec{B}$.

Figure 27.1a

$\vec{F}$ is in the $-z$-direction. This agrees with the direction calculated with unit vectors.

(b) **EXECUTE:** $\vec{B} = (1.40 \text{ T})\hat{k}$

$\vec{F} = q\vec{v} \times \vec{B} = (-1.24 \times 10^{-6} \text{ C})(1.40 \text{ T})[(+4.19 \times 10^4 \text{ m/s})\hat{i} \times \hat{k} - (-3.85 \times 10^4 \text{ m/s})\hat{j} \times \hat{k}]$

$\hat{i} \times \hat{k} = -\hat{j}$, $\hat{j} \times \hat{k} = \hat{i}$

$\vec{F} = (-7.27 \times 10^{-4} \text{ N})(-\hat{j}) + (6.68 \times 10^{-4} \text{ N})\hat{i} = [(6.68 \times 10^{-4} \text{ N})\hat{i} + (7.27 \times 10^{-4} \text{ N})\hat{j}]$

**EVALUATE:** The directions of $\vec{v}$ and $\vec{B}$ are shown in Figure 27.1b.

The direction of $\vec{F}$ is opposite to $\vec{v} \times \vec{B}$ since $q$ is negative. The direction of $\vec{F}$ computed from the right-hand rule agrees qualitatively with the direction calculated with unit vectors.

Figure 27.1b

27.4. **IDENTIFY:** Apply Newton’s second law, with the force being the magnetic force.

**SET UP:** $\vec{j} \times \vec{i} = -\vec{k}$

**EXECUTE:** $\vec{F} = m\vec{a} = q\vec{v} \times \vec{B}$ gives $\vec{a} = \frac{q\vec{v} \times \vec{B}}{m}$ and

$\vec{a} = \frac{(1.22 \times 10^{-8} \text{ C})(3.0 \times 10^4 \text{ m/s})(1.63 \text{ T})(j \times i)}{1.81 \times 10^{-3} \text{ kg}} = -(0.330 \text{ m/s}^2)\hat{k}$.

27.15. **(a) IDENTIFY:** Apply Eq. (27.2) to relate the magnetic force $\vec{F}$ to the directions of $\vec{v}$ and $\vec{B}$. The electron has negative charge so $\vec{F}$ is opposite to the direction of $\vec{v} \times \vec{B}$. For motion in an arc of a circle the acceleration is toward the center of the arc so $\vec{F}$ must be in this direction. $a = v^2/R$.

**SET UP:**

As the electron moves in the semicircle, its velocity is tangent to the circular path. The direction of $\vec{v} \times \vec{B}$ at a point along the path is shown in Figure 27.15.

Figure 27.15
EXECUTE: For circular motion the acceleration of the electron \( \ddot{r}_{\text{rad}} \) is directed in toward the center of the circle. Thus the force \( \vec{F}_B \) exerted by the magnetic field, since it is the only force on the electron, must be radially inward. Since \( q \) is negative, \( \vec{F}_B \) is opposite to the direction given by the right-hand rule for \( \vec{v}_0 \times \vec{B} \). Thus \( \vec{B} \) is directed into the page. Apply Newton’s second law to calculate the magnitude of \( \vec{B} \):
\[
\sum \vec{F} = m\vec{a} \Rightarrow \sum F_{\text{rad}} = ma \quad F_B = m(\ddot{r}/R)
\]
\[
F_B = |q|vB \sin \phi = |q|vB, \quad \text{so} \quad |q|vB = m(\ddot{r}/R)
\]
\[
B = \frac{m}{|q|} \left( \frac{(9.109 \times 10^{-31} \text{ kg})(1.41 \times 10^8 \text{ m/s})}{(1.002 \times 10^{-19} \text{ C})(0.050 \text{ m})} \right) = 1.60 \times 10^{-3} \text{ T}
\]

(b) **IDENTIFY** and **SET UP**: The speed of the electron as it moves along the path is constant. \( (\vec{F}_B \) changes the direction of \( \vec{v} \) but not its magnitude.) The time is given by the distance divided by \( v_0 \).

EXECUTE: The distance along the semicircular path is \( \pi R \), so \( t = \frac{\pi R}{v_0} = \frac{\pi(0.050 \text{ m})}{1.41 \times 10^6 \text{ m/s}} = 1.11 \times 10^{-7} \text{ s} \).

EVALUATE: The magnetic field required increases when \( v \) increases or \( R \) decreases and also depends on the mass to charge ratio of the particle.

27.22. **IDENTIFY**: For motion in an arc of a circle, \( a = \frac{v^2}{R} \) and the net force is radially inward, toward the center of the circle.

**SET UP**: The direction of the force is shown in Figure 27.22. The mass of a proton is \( 1.67 \times 10^{-27} \text{ kg} \).

**EXECUTE**: (a) \( \vec{F} \) is opposite to the right-hand rule direction, so the charge is negative. \( \vec{F} = n\vec{a} \) gives
\[
|q|vB \sin \phi = \frac{m \ddot{r}}{R} \quad \phi = 90^\circ \quad \text{and} \quad \frac{|q|vB}{m} = \frac{(1.60 \times 10^{-19} \text{ C})(0.250 \text{ T})(0.475 \text{ m})}{12(1.67 \times 10^{-27} \text{ kg})} = 2.84 \times 10^6 \text{ m/s}.
\]

(b) \( F_B = |q|vB \sin \phi = \frac{(1.60 \times 10^{-19} \text{ C})(2.84 \times 10^6 \text{ m/s})(0.250 \text{ T})}{12(1.67 \times 10^{-27} \text{ kg})} = 3.41 \times 10^{-13} \text{ N} \).

**EVALUATE**: (c) The magnetic force is always perpendicular to the path and does no work. The particles move with constant speed.

Figure 27.22

27.24. **IDENTIFY**: The magnetic force on the beam bends it through a quarter circle.

**SET UP**: The distance that particles in the beam travel is \( s = R \phi \), and the radius of the quarter circle is \( R = mv/qB \).

**EXECUTE**: Solving for \( R \) gives \( R = s/\phi = s/(\pi/2) = 1.18 \text{ cm}/(\pi/2) = 0.751 \text{ cm} \). Solving for the magnetic field: \( B = mv/qR = (1.67 \times 10^{-27} \text{ kg})(1200 \text{ m/s})/(1.60 \times 10^{-19} \text{ C})(0.00751 \text{ m}) = 1.67 \times 10^{-3} \text{ T} \).

**EVALUATE**: This field is about 10 times stronger than the Earth’s magnetic field, but much weaker than many laboratory fields.

27.66. **IDENTIFY**: Apply \( \vec{F} = q\vec{v} \times \vec{B} \).

**SET UP**: \( \vec{v} = v\hat{k} \)

**EXECUTE**: (a) \( \vec{F} = -qvB_x \hat{i} + qvB_y \hat{j} \). But \( \vec{F} = 3F_0 \hat{j} + 4F_0 \hat{k} \), so \( 3F_0 = -qvB_x \) and \( 4F_0 = qvB_y \).

Therefore, \( B_x = -\frac{3F_0}{qv} \), \( B_y = \frac{4F_0}{qv} \), and \( B_z \) is undetermined.

(b) \( B = \frac{F_0}{qv} = \sqrt{B_x^2 + B_y^2 + B_z^2} = \frac{F_0}{qv} \sqrt{9 + 16 + \left( \frac{qv}{F_0} \right)^2 B_y^2} = F_0 \sqrt{\frac{25}{F_0^2} B^2} \), so \( B_z = \pm \frac{\sqrt{14}}{v} F_0 \).

**EVALUATE**: The force doesn’t depend on \( B_z \), since \( \vec{v} \) is along the \( z \)-direction.