Physics 9C-C Homework Assignment#4 (due on 2/7/06)

1. 24-18

\[ C_{eq} = \left( \frac{1}{C_1} + \frac{1}{C_2} \right)^{-1} = \left( \frac{d_1}{C_{eq,1}} + \frac{d_2}{C_{eq,2}} \right)^{-1} = \frac{C_{eq}}{d_1 + d_2}. \]

So the combined capacitance for two capacitors in series is the same as that for a capacitor of area \( A \) and separation \( (d_1 + d_2) \).

2. 24-20

a) and b) The equivalent resistance of the combination is 6.0 \( \mu F \), therefore the total charge on the network is: \( Q = C_{eq} V_{eq} (6.0 \mu F)(36 \text{ V}) = 2.16 \times 10^{-4} \text{ C}. \) This is also the charge on the 9.0 \( \mu F \) capacitor because it is connected in series with the point b. So:

\[ V_9 = \frac{Q_9}{C_9} = \frac{2.16 \times 10^{-4} \text{ C}}{9.0 \times 10^{-6} \text{ F}} = 24 \text{ V}. \]

Then \( V_3 = V_{11} = V_{12} + V_6 = V - V_9 = 36 \text{ V} - 24 \text{ V} = 12 \text{ V}. \)

\[ \Rightarrow Q_3 = C_3 V_3 = (3.0 \mu F)(12 \text{ V}) = 3.6 \times 10^{-5} \text{ C}. \]

\[ \Rightarrow Q_{11} = C_{11} V_{11} = (11 \mu F)(12 \text{ V}) = 1.32 \times 10^{-4} \text{ C}. \]

\[ \Rightarrow Q_6 = Q_{12} = Q - Q_3 - Q_{11} \\
= 2.16 \times 10^{-4} \text{ C} - 3.6 \times 10^{-5} \text{ C} - 1.32 \times 10^{-4} \text{ C}. \\
= 4.8 \times 10^{-5} \text{ C}. \]

So now the final voltages can be calculated:

\[ V_6 = \frac{Q_6}{C_6} = \frac{4.8 \times 10^{-5} \text{ C}}{6.0 \times 10^{-6} \text{ F}} = 8 \text{ V}. \]

\[ V_{12} = \frac{Q_{12}}{C_{12}} = \frac{4.8 \times 10^{-5} \text{ C}}{12 \times 10^{-6} \text{ F}} = 4 \text{ V}. \]

c) Since the 3 \( \mu F \), 11 \( \mu F \) and 6 \( \mu F \) capacitors are connected in parallel and are in series with the 9 \( \mu F \) capacitor, their charges must add up to that of the 9 \( \mu F \) capacitor. Similarly, the charge on the 3 \( \mu F \), 11 \( \mu F \) and 12 \( \mu F \) capacitors must add up to the same as that of the 9 \( \mu F \) capacitor, which is the same as the whole network. In short, charge is conserved for the whole system. It gets redistributed for capacitors in parallel and it is equal for capacitors in series.

3. 24-25

\[ E = \frac{V}{d} = \frac{(400 \text{ V})}{(0.005 \text{ m})} = 8.00 \times 10^4 \text{ V/m}. \]

And \( u = \frac{1}{2} \epsilon_0 E^2 = \frac{1}{2} \epsilon_0 (8.00 \times 10^4 \text{ V/m})^2 = 0.0283 \text{ J/m}^3. \)
4. 24-28

a) \( Q = CV_0 \).

b) They must have equal potential difference, and their combined charge must add up to the original charge. Therefore:

\[
V = \frac{Q_1}{C_1} = \frac{Q_2}{C_2}
\]

and also \( Q_1 + Q_2 = Q = CV_0 \)

\[
C_1 = C \quad \text{and} \quad C_2 = \frac{C}{2} \quad \text{so} \quad \frac{Q_1}{C} = \frac{Q_2}{(C/2)} \implies Q_2 = \frac{Q_1}{2}
\]

\[
\Rightarrow Q = \frac{3}{2} Q_1 \implies Q_1 = \frac{2}{3} Q \quad \text{so} \quad V = \frac{Q_1}{C} = \frac{2}{3} \frac{Q}{C} = \frac{2}{3} V_0
\]

c) \( U = \frac{1}{2} \left( \frac{Q_1^2}{C_1} + \frac{Q_2^2}{C_2} \right) = \frac{1}{2} \left[ \frac{\left( \frac{3}{2} Q \right)^2}{C} + \frac{2 \left( \frac{1}{2} Q \right)^2}{C} \right] = \frac{1}{3} \frac{Q^2}{C} = \frac{1}{3} CV_0^2
\]

d) The original \( U \) was \( \frac{1}{2} CV_0^2 \) \( \implies \Delta U = \frac{1}{6} CV_0^2 \).

e) Thermal energy of capacitor, wires, etc., and electromagnetic radiation.

5. 24-36

a) \( u = \frac{1}{2} \varepsilon_0 E^2 = \frac{1}{2} \varepsilon_0 \left( \frac{1}{4\pi \varepsilon_0} \frac{q}{r^2} \right)^2 = \frac{1}{32\pi^2 \varepsilon_0} \frac{(8.00 \times 10^{-9} \text{ C})^2}{(0.120 \text{ m})^4} = 1.11 \times 10^{-4} \text{ J/m}^3 \).

b) If the charge was –8.00 nC, the electric field energy would remain the same since \( U \) only depends on the square of \( E \)

6. 24-53

a) The power output is 600 W, and 95\% of the original energy is converted.

\[
\Rightarrow E = Pt = (2.70 \times 10^5 \text{ W}) (1.48 \times 10^{-3} \text{ s}) = 400 \text{ J} \quad \therefore E_0 = \frac{400 \text{ J}}{0.95} = 421 \text{ J}.
\]

b) \( U = \frac{1}{2} CV^2 \) \( \implies C = \frac{2U}{V^2} = \frac{2(421 \text{ J})}{(125 \text{ V})^2} = 0.054 \text{ F} \)
7. 24-56

Originally: \( Q_1 = C_1 V_1 = (9.0 \, \mu F) (28 \, V) = 2.52 \times 10^{-4} \, C; \, Q_2 = C_2 V_2 = (4.0 \, \mu F) \times (28 \, V) = 1.12 \times 10^{-4} \, C \), and \( C_{eq} = C_1 + C_2 = 13.0 \, \mu F \). So the original energy stored is

\[ U = \frac{1}{2} C_{eq} V^2 = \frac{1}{2} (13.0 \times 10^{-6} \, F) (28 \, V)^2 = 5.10 \times 10^{-3} \, J. \]

Disconnect and flip the capacitors, so now the total charge is \( Q = Q_2 - Q_1 = 1.4 \times 10^{-4} \, C \), and the equivalent capacitance is still the same, \( C_{eq} = 13.0 \, \mu F \). So the new energy stored is:

\[ U = \frac{Q^2}{2 C_{eq}} = \frac{(1.4 \times 10^{-4} \, C)^2}{2(13.0 \times 10^{-6} \, F)} = 7.54 \times 10^{-4} \, J \]

\[ \Rightarrow \Delta U = 7.45 \times 10^{-4} \, J - 5.10 \times 10^{-3} \, J = -4.35 \times 10^{-3} \, J. \]

8. 24-59

a) \( \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2 + \left( \frac{1}{C_3} + \frac{1}{C_4} \right)} + \frac{1}{C_5} \Rightarrow C_1 = C_5 = 2C_2 \) and

\[ C_2 = C_3 = C_4 \text{ so } \frac{1}{C_{eq}} = \frac{2}{C_1} + \frac{2}{3C_2} = \frac{5}{3} C_2 \Rightarrow C_{eq} = \frac{3}{5} C_2 = 2.52 \, \mu F. \]

b) \( Q = CV = (2.52 \, \mu F)(220 \, V) = 5.54 \times 10^{-4} \, C = Q_1 = Q_5 \)

\[ \Rightarrow V_1 = V_5 = (5.54 \times 10^{-4} \, C) / (8.4 \times 10^{-6} \, F) = 66 \, V. \]

So \( V_2 = 220 - 2(66) = 88 \, V \Rightarrow Q_2 = (88 \, V)(4.2 \, \mu F) = 3.70 \times 10^{-4} \, C. \) Also \( V_3 = V_4 = \frac{1}{2} (88 \, V) = 44 \, V \Rightarrow Q_3 = Q_4 = (44 \, V)(4.2 \, \mu F) = 1.85 \times 10^{-4} \, C. \)

9. 24-60

a) With the switch open: \( C_{eq} = \left( \frac{1}{3 \, \mu F} + \frac{1}{6 \, \mu F} \right)^{-1} + \left( \frac{1}{3 \, \mu F} + \frac{1}{6 \, \mu F} \right)^{-1} = 4.00 \, \mu F \)

\[ \Rightarrow Q_{total} = C_{eq} V = (4.00 \, \mu F) (210 \, V) = 8.4 \times 10^{-4} \, C. \] By symmetry, each capacitor carries \( 4.20 \times 10^{-4} \, C \). The voltages are then just calculated via \( V = Q/C. \)

So: \( V_{ad} = Q/C_3 = 140 \, V, \) and \( V_{ac} = Q/C_6 = 70 \, V \Rightarrow V_{ed} = V_{ad} - V_{ac} = 70 \, V. \)

b) When the switch is closed, the points \( c \) and \( d \) must be at the same potential, so the equivalent capacitance is:

\[ C_{eq} = \left( \frac{1}{(3 + 6) \, \mu F} + \frac{1}{(3 + 6) \, \mu F} \right)^{-1} = 4.5 \, \mu F. \]
\[ Q_{\text{total}} = C_{\text{eq}} V = (4.50 \ \mu\text{F})(210 \ \text{V}) = 9.5 \times 10^{-4} \ \text{C}, \]
and each capacitor has the same potential difference of 105 V (again, by symmetry).

c) The only way for the sum of the positive charge on one plate of \( C_2 \) and the negative charge on one plate of \( C_1 \) to change is for charge to flow through the switch. That is, the quantity of charge that flows through the switch is equal to the charge in \( Q_2 - Q_1 = 0 \).

With the switch open, \( Q_1 = Q_2 \) and \( Q_2 - Q_1 = 0 \). After the switch is closed, \( Q_2 - Q_1 = 315 \ \mu\text{C}; 315 \ \mu\text{C} \) of charge flowed through the switch.

10. **24-61**

a) \[ C_{\text{eq}} = \left( \frac{1}{8.4 \ \mu\text{F}} + \frac{1}{8.4 \ \mu\text{F}} + \frac{1}{4.2 \ \mu\text{F}} \right)^{-1} = 2.1 \ \mu\text{F} \]

\[ \Rightarrow Q = C_{\text{eq}} V = (2.1 \ \mu\text{F})(36 \ \text{V}) = 7.50 \times 10^{-5} \ \text{C}. \]

b) \[ U = \frac{1}{2} CV^2 = \frac{1}{2} (2.1 \ \mu\text{F})(36 \ \text{V})^2 = 1.36 \times 10^{-3} \ \text{J}. \]

c) If the capacitors are all in parallel, then:

\[ C_{\text{eq}} = (8.4 \ \mu\text{F} + 8.4 \ \mu\text{F} + 4.2 \ \mu\text{F}) = 21 \ \mu\text{F} \]

and \( Q = 3(7.56 \times 10^{-5} \ \text{C}) = 2.27 \times 10^{-4} \ \text{C} \),

and \( V = \frac{Q}{C} = (2.27 \times 10^{-4} \ \text{C})/(21 \ \mu\text{F}) = 10.8 \ \text{V} \).

d) \[ U = \frac{1}{2} CV^2 = \frac{1}{2} (21 \ \mu\text{F})(10.8 \ \text{V})^2 = 1.22 \times 10^{-3} \ \text{J}. \]

11. **24-62**

a) \[ C_{\text{eq}} = \left( \frac{1}{4.0 \ \mu\text{F}} + \frac{1}{6.0 \ \mu\text{F}} \right)^{-1} = 2.4 \times 10^{-6} \ \text{F} \]

\[ \Rightarrow Q = C_{\text{eq}} V = (2.4 \times 10^{-6} \ \text{F})(600 \ \text{V}) = 1.58 \times 10^{-3} \ \text{C} \]

and \( V_2 = Q/C_2 = (1.58 \times 10^{-3} \ \text{C})/(4.0 \ \mu\text{F}) = 395 \ \text{V} \)

\[ \Rightarrow V_3 = 660 \ \text{V} - 395 \ \text{V} = 265 \ \text{V}. \]

b) Disconnecting them from the voltage source and reconnecting them to themselves we must have equal potential difference, and the sum of their charges must be the sum of the original charges:
\[ Q_1 = C_1 V \text{ and } Q_2 = C_2 V \Rightarrow 2Q = Q_1 + Q_2 = (C_1 + C_2) V \]

\[ \Rightarrow V = \frac{2Q}{C_1 + C_2} = \frac{2(1.58 \times 10^{-3} \text{ C})}{10.0 \times 10^{-6} \text{ F}} = 316 \text{ V}. \]

\[ \Rightarrow Q_1 = (4.00 \times 10^{-6} \text{ F})(316 \text{ V}) = 1.26 \times 10^{-3} \text{ C}. \]

\[ \Rightarrow Q_2 = (6.00 \times 10^{-6} \text{ F})(316 \text{ V}) = 1.90 \times 10^{-3} \text{ C}. \]

12. 24-70

a) \[ u = \frac{1}{2} \varepsilon_0 E^2 = \frac{1}{2} \varepsilon_0 \left( \frac{\lambda}{2\pi \varepsilon_0 r} \right)^2 = \frac{\lambda^2}{8\pi^2 \varepsilon_0 r^2}. \]

b) \[ U = \int udV = 2\pi L \int urdr = \frac{L\lambda^2}{4\pi \varepsilon_0} \int_{r_a}^{r_b} \frac{dr}{r} \Rightarrow \frac{U}{L} = \frac{\lambda^2}{4\pi \varepsilon_0} \ln(r_b / r_a). \]

c) Using Equation (24.9):

\[ U = \frac{Q^2}{2C} = \frac{Q^2}{4\pi \varepsilon_0 L} \ln(r_b / r_a) = \frac{\lambda^2 L}{4\pi \varepsilon_0} \ln(r_b / r_a) = U \text{ of part (b)}. \]