Physics 9C-C Homework Assignment#9 (due on 3/14/06)

1. 28-24

There is no magnetic field at the center of the loop from the straight sections. The magnetic field from the semicircle is just half that of a complete loop:

\[ B = \frac{1}{2} B_{\text{loop}} = \frac{1}{2} \left( \frac{\mu_0 I}{2R} \right) = \frac{\mu_0 I}{4R}, \]

into the page.

2. 28-25

As in Exercise 28.24, there is no contribution from the straight wires, and now we have two oppositely oriented contributions from the two semicircles:

\[ B = (B_1 - B_2) = \frac{1}{2} \left( \frac{\mu_0 I}{2R} \right) |I_1 - I_2|, \]

into the page. Note that if the two currents are equal, the magnetic field goes to zero at the center of the loop.

3. 28-64

\[ B = B_a - B_b = \frac{1}{2} \left( \frac{\mu_0 I}{2} \right) \left( \frac{1}{a} - \frac{1}{b} \right) = \frac{\mu_0 I}{4a} \left( 1 - \frac{a}{b} \right), \text{ out of the page.} \]

4. 28-67

The contributions from the straight segments is zero since \( d\vec{l} \times \vec{r} = 0 \). The magnetic field from the curved wire is just one quarter of a full loop:

\[ \Rightarrow B = \frac{1}{4} \left( \frac{\mu_0 I}{2R} \right), \]

and is out of the page.
5. **28-76**

a) \( I_0 = \int \int \int \left[ \frac{b}{r} e^{(r-a)/\delta} \right] r dr d\theta = 2\pi b \left[ e^{(r-a)/\delta} \right]_0^\infty = 2\pi b \delta \left( 1 - e^{-a/\delta} \right) \Rightarrow I_0 = 2\pi (600 \text{ A/m}) (0.025 \text{ m}) (1 - e^{(0.050/0.025)}) = 81.5 \text{ A.} \)

b) For \( r \geq a \Rightarrow \int \int \int \left[ \mathbf{B} \cdot d\mathbf{A} \right] = B2\pi r = \mu_0 I_{\text{encl}} = \mu_0 I_0 \Rightarrow B = \frac{\mu_0 I_0}{2\pi r}.

c) \( r \leq a \Rightarrow I(r) = \int \int \int \left[ \mathbf{J} \cdot d\mathbf{A} = \int \int \left[ \frac{b}{r^2} e^{(r-a)/\delta} \right] r' dr' d\theta = 2\pi b \int_0^r e^{(r-a)/\delta} dr = 2\pi b \delta e^{(r-a)/\delta} \right]_0^r \Rightarrow I(r) = I_0 \left( \frac{e^{r/\delta} - 1}{e^{a/\delta} - 1} \right).

d) For \( r \leq a \Rightarrow \int \int \int \left[ \mathbf{B} \cdot d\mathbf{A} \right] = B(r)2\pi r = \mu_0 I_{\text{encl}} = \mu_0 I_0 \Rightarrow B = \frac{\mu_0 I_0 (e^{r/\delta} - 1)}{2\pi (e^{a/\delta} - 1)}.

e) At
\[
\begin{align*}
r &= \delta = 0.025 \text{ m} \Rightarrow B &= \frac{\mu_0 I_0 (e - 1)}{2\pi \delta (e^{a/\delta} - 1)} = \frac{\mu_0 (81.5 \text{ A}) (e - 1)}{2\pi (0.025 \text{ m}) (e^{0.050/0.025} - 1)} = 1.75 \times 10^{-4} \text{ T.}
\end{align*}
\]
At \( r = a = 0.050 \text{ m} \Rightarrow B = \frac{\mu_0 I_0 (e^{a/\delta} - 1)}{2\pi a (e^{a/\delta} - 1)} = \frac{\mu_0 (81.5 \text{ A})}{2\pi (0.050 \text{ m})} = 3.26 \times 10^{-4} \text{ T.}
\]
At \( r = 2a = 0.100 \text{ m} \Rightarrow B = \frac{\mu_0 I_0}{2\pi r} = \frac{\mu_0 (81.5 \text{ A})}{2\pi (0.100 \text{ m})} = 1.63 \times 10^{-4} \text{ T.}

6. **29-2**

a) Before: \( \Phi_B = NBA = (200)(6.0 \times 10^{-5} \text{ T})(12 \times 10^{-4} \text{ m}^2) = 1.44 \times 10^{-5} \text{ T} \cdot \text{m}^2; \) after: 0

b) \( |\varepsilon| = \frac{\Delta \Phi_B}{\Delta t} = \frac{\Delta NBA}{\Delta t} = \frac{(200)(6.0 \times 10^{-5} \text{ T})(1.2 \times 10^{-3} \text{ m}^2)}{0.040 \text{ s}} = 3.6 \times 10^{-4} \text{ V.} \)

7. **29-5**

From Exercise (29.3),
\[
Q = \frac{NBA}{R} \Rightarrow B = \frac{QR}{NA} = \frac{(3.56 \times 10^{-5} \text{ C})(60.0 \Omega + 45.0 \Omega)}{(120)(3.20 \times 10^{-3} \text{ m}^2)} = 0.0973 \text{ T.}
\]
8. 29-16

a) If the magnetic field is increasing into the page, the induced magnetic field must oppose that change and point opposite the external field’s direction, thus requiring a counterclockwise current in the loop.

b) If the magnetic field is decreasing into the page, the induced magnetic field must oppose that change and point in the external field’s direction, thus requiring a clockwise current in the loop.

c) If the magnetic field is constant, there is no changing flux, and therefore no induced current in the loop.

9. 29-17

a) When the switch is opened, the magnetic field to the right decreases. Therefore the second coil’s induced current produces its own field to the right. That means that the current must pass through the resistor from point $a$ to point $b$.

b) If coil $B$ is moved closer to coil $A$, more flux passes through it toward the right. Therefore the induced current must produce its own magnetic field to the left to oppose the increased flux. That means that the current must pass through the resistor from point $b$ to point $a$.

c) If the variable resistor $R$ is decreased, then more current flows through coil $A$, and so a stronger magnetic field is produced, leading to more flux to the right through coil $B$. Therefore the induced current must produce its own magnetic field to the left to oppose the increased flux. That means that the current must pass through the resistor from point $b$ to point $a$.

10. 29-22

a) $\varepsilon = vBL = (5.00 \text{ m/s})(0.450 \text{ T})(0.300 \text{ m}) = 0.675 \text{ V}$.

b) The potential difference between the ends of the rod is just the motional emf $V = 0.675 \text{ V}$.

c) The positive charges are moved to end $b$, so $b$ is at the higher potential.

d) $E = \frac{V}{L} = \frac{0.675 \text{ V}}{0.300 \text{ m}} = 2.25 \text{ V/m}$.

e) b
11. 29-25
For the loop pulled through the region of magnetic field,

a)

\[ F = F_0 \]

b)

\[ \Phi = \varepsilon \mu \]

Where \( \varepsilon = vBL = IR \Rightarrow I_0 = \frac{vBL}{R} \) and \( F_0 = ILB = \frac{vB^2L^2}{R} \).

12. 29-27

\[ \varepsilon = \frac{d\Phi}{dt} = \frac{d}{dt}(BA) = \frac{d}{dt}(\mu_0 nLA) = \mu_0 nA \frac{dl}{dt} \quad \text{and} \quad \oint \mathbf{E} \cdot d\mathbf{l} = \varepsilon \Rightarrow \]

\[ E = \frac{\varepsilon}{2\pi r} = \frac{\mu_0 nA}{2\pi} \frac{dl}{dt} = \frac{\mu_0 n r}{2} \frac{dl}{dt}. \]

a) \( r = 0.50 \ \text{cm} \Rightarrow E = \frac{\mu_0 (900 \ \text{m}^{-1})(0.0050 \ \text{m})}{2} (60 \ \text{A} / \text{s}) = 1.70 \times 10^{-4} \ \text{V} / \text{m}. \)

b) \( r = 1.00 \ \text{cm} \Rightarrow E = 3.39 \times 10^{-4} \ \text{V} / \text{m}. \)
13. 29-30

\[ \varepsilon = \frac{d\Phi_B}{dt} = \frac{d}{dt}(BA) = \frac{d}{dt}(\mu_0 nIA) = \mu_0 nA \frac{dl}{dt} \Rightarrow \frac{dl}{dt} = \frac{E \cdot 2\pi r}{\mu_0 nA} \]

\[ \Rightarrow \frac{dl}{dt} = (8.00 \times 10^{-6} \text{ V/m})2\pi(0.0350)\mu(0.0110 \text{ m})^2 = 9.21 \text{ A/s}. \]

14. 29-46

a) \[ I = \frac{\varepsilon}{R} = \frac{1}{R} \frac{d\Phi_B}{dt} = \frac{1}{R} \frac{d}{dt}(BA \cos \omega t) = \frac{BA \omega \sin \omega t}{R}. \]

b) \[ P = I^2 R = \frac{B^2 A^2 \omega^2 \sin^2 \omega t}{R}. \]

c) \[ \mu = IA = \frac{BA^2 \omega \sin \omega t}{R}. \]

d) \[ \tau = \mu B \sin \phi = \mu B \sin \omega t = \frac{B^2 A^2 \omega \sin^2 \omega t}{R}. \]

e) \[ P = \tau \omega = \frac{B^2 A^2 \omega^2 \sin^2 \omega t}{R}, \text{ which is the same as part (b)}. \]

15. 29-56

The bar will experience a magnetic force due to the induced current in the loop. According to Example 29.6, the induced voltage in the loop has a magnitude \( BLv \), which opposes the voltage of the battery, \( \varepsilon \). Thus, the net current in the loop is \( I = \frac{\varepsilon - BLv}{R} \). The acceleration of the bar is \( a = \frac{F}{m} = \frac{BLv(90^\circ)}{m} = \frac{(\varepsilon - BLv)LB}{mR} \).

a) To find \( v(t) \), set \( \frac{dv}{dt} = a = \frac{(\varepsilon - BLv)LB}{mR} \) and solve for \( v \) using the method of separation of variables:

\[ \int_0^v \frac{dv}{(\varepsilon - BLv)} = \int_0^v \frac{LB}{mR} dt \rightarrow v = \frac{\varepsilon}{BL} \left( 1 - e^{-\frac{BLv}{mR}} \right) = (10 \text{ m/s}) \left( 1 - e^{-\frac{1}{11}} \right). \]

Note that the graph of this function is similar in appearance to that of a charging capacitor.
b) $I = \frac{\epsilon}{R} = 2.4 \, \text{A}; \; F = ILB = 2.88 \, \text{N}; \; a = F/m = 3.2 \, \text{m/s}^2$

c) When $v = 2.0 \, \text{m/s}$, $a = \frac{[12 \, \text{V} - (1.5 \, \text{T})(0.8 \, \text{m})(2.0 \, \text{m/s})] (0.8 \, \text{m})(1.5 \, \text{T})}{(0.90 \, \text{kg})(5.0\, \Omega)} = 2.6 \, \text{m/s}^2$

d) Note that as the velocity increases, the acceleration decreases. The velocity will asymptotically approach the terminal velocity $\frac{\epsilon}{BL} = \frac{12 \, \text{V}}{(1.5 \, \text{T})(0.8 \, \text{m})} = 10 \, \text{m/s}$, which makes the acceleration zero.

16. 29-65

a) $I = \frac{\epsilon}{R} = \frac{vB\alpha}{R} \Rightarrow F = IaB = \frac{vB^2a^2}{R}$.

develops a later

b) $F = ma = m \frac{dv}{dt} = \frac{vB^2a^2}{R} \Rightarrow \int v \, dv' = \frac{B^2a^2}{mR} \int_0^t dt' \Rightarrow v = v_0 e^{-\left(\frac{B^2a^2}{mR}\right) t} = \frac{dx}{dt} = \int_0^x dx'$

$\int_0^x dx' = v_0 \int_0^e e^{-\left(\frac{B^2a^2}{mR}\right) t} dt' \Rightarrow x = \left[-\frac{mRv_0}{B^2a^2} e^{-\left(\frac{B^2a^2}{mR}\right) t}\right]_0^x = \frac{mRv_0}{B^2a^2}$.

17. 29-67

At point $a$, $\frac{\epsilon}{dt} = A \frac{dB}{dt} = \pi r^2 \frac{dB}{dt}$ and $F = qE = qE = \frac{q}{2\pi r} = \frac{QR dB}{2 \, dt} \rightarrow \text{left.}$

At point $b$, the field is the same magnitude as at $a$ since they are the same distance from the center. So $F = \frac{q}{2 \, dt} dB$, but upward.

At point $c$, there is no force by symmetry arguments: one cannot have one direction picked out over any other, so the force must be zero.