1-(a) The total electric force on $Q_2$ is the vector sum of the forces from the three $-Q$ charges.

\[
\vec{F}_{\text{on } Q_2} = k \frac{Q_1 Q}{r_1^2} \hat{r}_1 + k \frac{Q_2 Q}{r_2^2} \hat{r}_2 + k \frac{Q_3 Q}{r_3^2} \hat{r}_3
\]

\[
= (-k) \frac{2Q^2}{L^2} \hat{i} + (-k) \frac{2Q^2}{(L\sqrt{2})^2} \hat{i} + \hat{j} + (-k) \frac{2Q^2}{L^2} \hat{j}
\]

\[
= \frac{2kQ^2}{L^2} \left(1 + \frac{1}{2\sqrt{2}}\right) \hat{i} + \frac{2kQ^2}{L^2} \left(1 + \frac{1}{2\sqrt{2}}\right) \hat{j}
\]

\[
\text{Diagram:}
\]

\[
Q_1 = -2, \quad Q_2 = 2Q, \quad Q_3 = -2
\]

\[
\text{Coordinate axes:}
\]

\[
\text{x-axis (i')}
\]

\[
\text{y-axis (j')}
\]
1. Work done on 29 is equal to the potential energy of 29 at the "vacant" corner minus the potential energy of 29 at the center of the square.

\[ W_e = U_{29 \text{ (corner)}} - U_{29 \text{ (center)}} \]

\[ U_{29 \text{ (corner)}} = \frac{KQ\phi_1}{V_1} + \frac{KQ\phi_2}{V_2} + \frac{KQ\phi_3}{V_3} + \text{const.} \]

\[ = (-) \frac{Kz^2}{L} + (-) \frac{Kz^2}{\sqrt{2}L} + (-) \frac{Kz^2}{L} + \text{const.} \]

\[ = (-) \frac{2Kz^2}{L} \left( 2 + \frac{1}{\sqrt{2}} \right) + \text{const.} \]

\[ U_{29 \text{ (center)}} = (-) \frac{Kz^2}{(L/2)} + (-) \frac{Kz^2}{(L/2)} + (-) \frac{Kz^2}{(L/2)} \]

\[ + \text{const} \]

\[ = (-) \frac{2Kz^2}{L} \left( 3\sqrt{2} \right) + \text{const} \]

\[ W_e = (-) \frac{2Kz^2}{L} \left[ 2 + \frac{1}{\sqrt{2}} - 3\sqrt{2} \right] = 1.5\sqrt{2} \times \frac{2Kz^2}{L} > 0 \]
2. The electric force on \(-2\) is the vector sum of the electric forces produced by four straight line segments of charges, each being \(Q/4\).

The magnitude of the electric force from any one of the four segments is

\[
|F| = \frac{K \cdot (Q/4)}{R(R^2 + (L/2)^2)^{3/2}} \cdot (-2) \quad R = \sqrt{z^2 + (L/2)^2}
\]

By symmetry, only the \(z\)-component of the total electric force on \(-2\) survives the vector summation.

As a result, the net electric force on \(-2\) is along \(z\)-axis, and equals 4 times the \(z\)-component from one of the four segments

\[
F_{\text{Total on } -2} = \frac{K \cdot (Q/4)}{R(R^2 + (L/2)^2)} \cdot (-2) \cdot \cos \theta \cdot z \times 4
\]

\[
= \frac{K \cdot 9 \cdot Q \cdot z}{(z^2 + (L/2)^2) \left(2 \cdot z^2 + 2 \cdot (L/2)^2\right)^{3/2}} \cdot (-2)
\]
3-(a) By spherical symmetry and properties of electric charges on a conductor, there is a uniformly distributed charge $Q'$ on the inner surface of radius $2R$, and a uniformly distributed charge $3Q - Q'$ on the outer surface of radius $3R$.

Since the electric field inside the conducting shell is the superposition of the fields by $Q$ on the thin shell, and $Q'$ on the inner surface of the thick conducting shell, and should be zero, we have

$$Q' = -Q$$

$$3Q - Q' = 4Q.$$  

Between the thin shell and the thick conducting shell, only the charge on the thin shell contributes a non-zero electric field, and

$$\vec{E}(\vec{r}; R < |\vec{r}| < 2R) = \frac{KQ}{r^2} \hat{r}.$$  

$\Box$
The total electric field outside the conducting shell is the vector sum of the electric fields from three spherical shells at $R$, $2R$ and $3R$. The fields from the thin shell and the inner surface of the conducting shell cancel each other, so the net electric field

$$\vec{E}(\vec{r}; |\vec{r}| > 3R) = \frac{kQ}{r^2} \hat{r}$$

This means that the electric potential at $\vec{r}$ relative to infinity is

$$V(\vec{r}; |\vec{r}| > 3R) = \frac{kQ}{r}$$

Since the electric field inside the conductor is zero, the electric potential inside is a constant, and should equal

$$V(\vec{r}; |\vec{r}| = 3R)$$

So,

$$V_{\text{inside the conductor}} = V(\vec{r}; |\vec{r}| = 3R) = \frac{kQ}{3R}$$
3-(c) When the thin shell with \( \mathcal{Q} \) is removed, all the \( \mathcal{Q} \) charge on the thick conducting shell is now uniformly distributed on the outer surface of the conductor.

We have

\[ \vec{E}(\vec{r}; |\vec{r}| > 3R) = \frac{\mathcal{K}(3\mathcal{Q})}{r^2} \]

And the electric potential in this region relative to infinity

\[ V(\vec{r}; |\vec{r}| > 3R) = \frac{\mathcal{K}(3\mathcal{Q})}{r} \]

So, inside the conducting shell,

\[ V = V(\vec{r}; |\vec{r}| = 3R) = \frac{\mathcal{K}\mathcal{Q}}{R} \]

\( \times \)
4. In order to stop the "electrically charged" missile from striking the surface of the ship, the ship surface needs to be positively charged at least to $Q_{\text{min}}$ such that the initial kinetic energy of the missile completely converts into its electric potential energy at the surface of the ship. Let the potential energy at the missile's initial position be zero, we require

$$U_{\text{missile (at the ship surface)}} = K_{\text{missile (initial)}}$$

Namely

$$R \frac{Q_{\text{min}}^2}{R} = \frac{m}{\varepsilon_0} V_o^2$$

$$Q_{\text{min}} = \frac{m \cdot R \cdot V_o^2}{2 \varepsilon_0 R} = \frac{9 \times 10^4 \times (100)^2}{2 \times 9 \times 10^9 \times 0.5}$$

$$= 10 \text{ C}$$