Solution to Physics 9C A MT #2

1-(a) \[ C_{23} = C_{20} + C_{30} = 20 \, \mu F \]

\[ C_{1234} = C_{10}^{-1} + C_{23}^{-1} + C_{40}^{-1} = \frac{1}{10 \mu F} + \frac{1}{20 \mu F} + \frac{1}{20 \mu F} = \frac{1}{5 \mu F} \]

\[ \therefore C_{1234} = 5 \, \mu F \]

1-(b) The charges stored on \( C_{10} \) and on \( C_{23} \) are the same as the charge stored on the network capacitor \( C_{1234} \)

\[ Q_{10} = Q_{23} = Q_{1234} \]

Since \( Q_{1234} = V_{ab} \cdot C_{1234} \)

\[ Q_{1234} = (60 \, V) \cdot (5 \, \mu F) = 300 \, \mu C \]

The potential difference across \( C_{10} \) is

\[ \Delta V_{10} = \frac{Q_{10}}{C_{10}} = \frac{300 \, \mu C}{10 \, \mu F} = 30 \, V \]

The potential difference across \( C_{20} \) is the same as that across \( C_{23} \).
Thus

\[ \Delta V_{20} = \Delta V_{23} = \frac{Q_{23}}{C_{23}} = \frac{300 \mu C}{20 \mu F} \]

\[ = 15 \text{ V} \]

1- \(c\) The energies stored on \(C_{20}\) and \(C_{30}\) are as follows

\[ U_{20} = \frac{C_{20}}{2} (\Delta U_{20})^2 = \frac{1}{2} (5 \mu F) \cdot (15 \text{ V})^2 \]

\[ = 562.5 \mu J \]

\[ U_{30} = \frac{C_{30}}{2} (\Delta U_{30})^2 = \frac{C_{30}}{2} (\Delta U_{23})^2 \]

\[ = 3 \ U_{20} = 1.69 \times 10^{-3} \text{ J} \]

1- \(d\) We need to learn the charge \(Q_{1234}\) after the dielectric is inserted in \(C_{10}\).

\[ C_1 = k \cdot C_{10} = 20 \mu F \]

\[ C_{1234}^{-1} = \frac{1}{C_1} + \frac{1}{C_{23}} + \frac{1}{C_{40}} = \frac{3}{20 \mu F} \]

\[ C_{1234} = \frac{20 \mu F}{3} \]
Thus,

\[ C_{1234} = \frac{20 \mu F}{3} = 6.67 \mu F \]

\[ Q_{1234} = V_{234} C_{1234} = (60 V) \left( \frac{20 \mu F}{3} \right) \]

\[ = 400 \mu C \]

\[ \Delta V_1 = \frac{Q_{1234}}{C_1} = \frac{400 \mu C}{20 \mu F} = 20 V \]

1-(e) In this case, the charges on all capacitors remain unchanged.

The potential drop across \( C_1 \) is simply

\[ \Delta V_1 = \frac{Q_{1234}}{C_1} = \frac{300 \mu C}{20 \mu F} = 15 V \]

The potential drop across \( C_{40} \) is

\[ \Delta V_{40} = \frac{Q_{1234}}{C_{40}} = \frac{300 \mu C}{20 \mu F} = 15 V \]

Because the charge on \( C_{40} \) is the same as the charge on the network capacitor in (4).
Assign the current counter-clockwise.
Choose a counter-clockwise loop as shown. Starting from a:

\[ \oint \mathbf{E} \cdot d\mathbf{l} = 0 \Rightarrow V_{ae} + V_{ed} + V_{dc} + V_{eb} + V_{ba} \]
aebdca

\[= -8 + 3I + 4 + 5I + 2I = 0 \]

\[\therefore 10I = 4 \]

\[I = 0.4 \text{ A} \]

2-(c) \[ P_{5R} = I^2 \cdot (52) = (0.4 \text{ A})^2 \cdot (52) = 0.8 \text{ W} \]

2-(c) \[ V_{ad} = V_{ae} + V_{ed} = -8 + I \cdot (32) = -6.8 \text{ V} \]

Equivalently:

\[ V_{ad} = V_{ae} + V_{eb} + V_{ed} = -I \cdot (22 + 52) - 4 = -6.8 \text{ V} \]
3-(a) \[ R_{13} = 4\Omega;\ R_{48} = 12\Omega \]

\[ \frac{1}{R_{1348}} = \frac{1}{R_{13}} + \frac{1}{R_{48}} = \frac{1}{4\Omega} + \frac{1}{12\Omega} = \frac{3}{3\Omega} \]

\[ \therefore R_{1348} = 3\Omega \]

\[ R_{ab} = R_5 + R_{1348} = 5\Omega + 3\Omega = 8\Omega \]

3-(b) The current through the 5\Omega resistor is the current through \( R_{ab} = R_{1345} \).

\[ I_{through} = \frac{E}{R_{ab}} = \frac{16V}{8\Omega} = 2A \]

\[ \therefore P_{5\Omega} = I_{through} \cdot (5\Omega) = 20 \text{ Watts} \]

3-(c) The potential drop between \( f \) and \( g \), namely, \( V_{fg} \) is simply the product of \( I_{through} \) times \( 5\Omega \).

\[ V_{fg} = I_{through} \cdot R_{1348} = (2A) \cdot (5\Omega) = 6V \]

The current through the 8\Omega resistor is the one that passes \( R_{48} \).
\[ I_{eq} = \frac{V_{ss}}{R_{eq}} = \frac{6V}{12\Omega} = 0.5A \]

\[ P_{8\Omega} = I_{eq}^2 (8\Omega) = 2 \text{ Watts} \]

3-(d) The power through \( R_2 \) is:

\[ P_{R2} = I_{eq}^2 (4\Omega) = 1 \text{ Watt} \]

The current through \( L_2 \) and \( R_2 \) is:

\[ I_{L2} = I_{eq} - I_{eq} = 0.5A \]

\[ P_{in} = I_{L2}^2 (L_2) = 2.25 \text{ Watts} \]

\[ P_{in} = I_{L2}^2 (R_2) = 3 \cdot P_{in} = 6.75 \text{ Watts} \]

The total power dissipated in the system is:

\[ P_{total} = P_{R2} + P_{in} + P_{in} + P_{L2} + P_{R2} = 32 \text{ Watts} \]

\[ P_{out} = I_{out}^2 \cdot R_{eq} = (2A)^2 \cdot (8\Omega) = 32 \text{ Watts} \]
Assign the currents and loops as shown.

Loop #1:
\[ \oint \vec{E} \cdot d\vec{l} = 0 ; \quad -40 + 6I_1 + 4(I_1 - I_2) = 0 \]
\[ \text{acba} \]
\[ 5I_1 - 2I_2 = 20 \quad \ldots \quad (1) \]

Loop #2:
\[ \oint \vec{E} \cdot d\vec{l} = 0 ; \quad -4(I_1 - I_2) - 20 + 2I_2 + 14 = 0 \]
\[ \text{acdea} \]
\[ -2I_1 + 3I_2 = 3 \quad \ldots \quad (2) \]

\[ 3 \times (1) + 2 \times (2) : \quad 11I_1 = 66 \quad \therefore I_1 = +6A. \]
From (1):

\[ I_2 = \frac{1}{2} (5I_1 - 20) = \frac{1}{2} (80 + 20) = 5 \text{A} \]

4-(a). \[ V_{bd} = V_{bc} + V_{cd} = 6I_1 - 20 = 16 \text{V} \]

Equivalently,

\[ V_{bd} = V_{ba} + V_{ae} + V_{ed} = 40\text{V} - 14\text{V} - 2I_2 \]

\[ = 40\text{V} - 14\text{V} - 10\text{V} = 16\text{V} \]